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Quasi B-Open Sets in Bitopological Spaces

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Abstract

Since the notion of generalized closed set in topological spaces have appeared, many topologists have started looking for more general ones and the goal was to find new decompositions of continuity. In this paper, the relatively new notion of quasi-b-open sets is introduced and investigated. Hence, the notion of quasi-b-continuity between bitopological spaces is defined and a decomposition is provided. Moreover, we investigate a group of quasi-b-homeomorphisms and define several new bitopological spaces.

Keywords: B-open set, quasi-b-open set, bitopological space, quasi-b-continuity, group.

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1 Introduction

One of the most significant concepts in topology is the notion of b-closed sets that was studied by several authors such as [1, 2, 3]. A subset A of a topological space (X,τ) is called *b-open* if $A \subseteq \text{int}(cl(A)) \cup cl(\text{int}(A))$, where *int()* and *cl()* denote the interior and the closure operations, respectively. The complement of a b-open set is called *b-closed*. The collection of all b-open (resp., b-closed) sets of (X,τ) will be denoted by $BO(X)$ (resp., $BC(X)$). The notions of semi-open sets and semi-continuity between topological spaces were studied in [4, 5] and in [6], quasi-semi-open sets were explored. In recent years a number of other generalizations of open sets have been studied, for example [7, 8, 9]

The idea of bitopological spaces was first appeared in [10]. A *bitopological space* (X,τ_1,τ_2) (simply, a *space*) is a non-empty set X with two topologies τ_1 and τ_2 on *X*. The topological notions of preopen [11] semi-open [4 and α-open [9 were generalized to bitopological spaces in [8]. In [7] several other notions of generalized open sets were generalized to bitopological spaces. Analogous to [7,8] and based on the notion of b-open sets in topological spaces, the notion of quasi-b-open set in bitopological spaces is introduced and explored. It is used to generate two new

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topological spaces starting from a given bitopological space and to define and study the notion of quasi-b-continuity in bitopological spaces. Several characterizations and a decomposition of this type of maps are also provided. Finally as an application, the group of quasi b-homeomorphism is formed. For the undefined topological concepts in this paper, we refer the reader to [12, 13].

2 Quasi-b-open sets

In this section, the relatively new notion of quasi-b-open set is introduced and investigated.

Definition 2.1 *A subset* $A \subseteq X$ *is* quasi-b-open *in* (X, τ_1, τ_2) *if* $A = U \cup V$ *for some* $U \in BO(X, \tau_1)$ *and* $V \in BO(X, \tau_2)$. *The complement of a quasi-b-open set is quasi-b-closed.*

By $QBO(X, \tau_1, \tau_2)$ $(resp., QBC(X, \tau_1, \tau_2))$ we denote the class of all quasi-b-open (resp., quasi-b-closed) subsets of (X, τ_1, τ_2). We next recall the following lemma from [2].

Lemma 2.2 *In a topological space:*

(i) Arbitrary union of b-open sets is b-open .

(ii) The intersection of an open set and a b-open set is a b-open set.

We remark that the intersection of two b-open sets need not be b-open, see [2]. The proof of the following result follows immediately from Lemma 2.2.

Theorem 2.3 *For a space* (X, τ_1, τ_2) :

(i) $BO(X, \tau_i) \subseteq QBO(X, \tau_1, \tau_2)$ *holds for all* $i \in \{1, 2\}$.

(ii) Arbitrary union of quasi-b-open sets is quasi-b-open.

(iii) If a subset A *is open in* (X, τ_1, τ_2) *(i.e.,* $A \in \tau_1 \cap \tau_2$*) and a subset* $B \in QBO(X, \tau_1, \tau_2)$, *then* $A \cap B \in QBO(X, \tau_1, \tau_2)$.

Definition 2.4 The *quasi b-closure* of a subset A of (X, τ_1, τ_2) is defined to be $qbCl(A) = \cap\{F : F \in QBC(X, \tau_1, \tau_2), A \subseteq F\}$. A subset A of (X, τ_1, τ_2) is *quasi-b-generalized closed* (simply, *quasi-bg-closed*) if $qbCl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in QBO(X, \tau_1, \tau_2)$. The complement of a quasi-b-generalized closed set is called *quasi-b-generalized open* (simply, *quasi-bg-open*).

Clearly $x \in qbCl(A)$ if each $V \in QBO(X, \tau_1, \tau_2)$ containing x meets A and A is quasi-b-closed if and only if $A = qbCl(A)$.

Theorem 2.5 *If A is quasi-bg-closed, then $qbCl(A) - A$ does not contain any nonempty quasi-b-closed subsets.*

Proof Let F be a quasi-b- closed subset of (X, τ_1, τ_2) such that $F \subseteq qbCl(A) - A$. Then $A \subseteq X - F$. Since A is a quasi-bg-closed set and $X - F$ is quasi-b-open, then $qbCl(A) \subseteq X - F$ and so $F \subseteq X - qbCl(A)$. Therefore $F \subseteq (X - qbCl(A)) \cap qbCl(A) = \phi$ and thus $F = \phi$.■

Next, two new operations are given and several of their properties are also provided. It turns out that each operation generates a new set and each collection of these sets forms a topology on the original set.

Definition 2.6 *The* quasi-Λ_b *and* quasi-$\vee_b$ *for a subset A of (X, τ_1, τ_2) are defined to be*

$$\Lambda_{qb}(A) = \cap\{G : A \subseteq G, G \in QBO(X, \tau_1, \tau_2)\} \text{ and}$$

$$\vee_{qb}(A) = \cup\{F : F \subseteq A, F \in QBC(X, \tau_1, \tau_2)\}.$$

A is called Λ_{qb} (resp. $\vee_{qb}$)-set if $\Lambda_{qb}(A) = A$ (resp. $\vee_{qb}(A) = A$).

Lemma 2.7 *For subsets A, B and A_i of (X, τ_1, τ_2) where $i \in I$, the following properties hold:*

(1) $A \subseteq \Lambda_{qb}(A)$.

(2) If $A \subseteq B$, *then* $\Lambda_{qb}(A) \subseteq \Lambda_{qb}(B)$.

(3) If $A \in QBO(X, \tau_1, \tau_2)$, *then* $A = \Lambda_{qb}(A)$.

(4) $\Lambda_{qb}(\cap\{A_i : i \in I\}) \subseteq \cap\{\Lambda_{qb}(A_i) : i \in I\}$

(5) $\Lambda_{qb}(\mathrm{X} \setminus A)) = \mathrm{X} \setminus \vee_{qb}(A)$.

(6) $\Lambda_{qb}(\Lambda_{qb}(A)) = \Lambda_{qb}(A)$.

(7) $\Lambda_{qb}(\cup\{A_i : i \in I\}) = \cup\{\Lambda_{qb}(A_i) : i \in I\}$.

Proof: (1), (2), (3), (4) and (5) are immediate consequences of Definition 2.6. We only prove (6) and (7).

(6) By Definition 2.6, $\Lambda_{qb}(A) \subseteq \Lambda_{qb}(\Lambda_{qb}(A))$. For the converse, let $x \notin \Lambda_{qb}(A)$. Then there exists $G \in QBO(X,\tau_1,\tau_2)$ such that $A \subseteq G$ and $x \notin G$. Since

$$\Lambda_{qb}(\Lambda_{qb}(A)) = \cap\{G : \Lambda_{qb}(A) \subseteq G, G \in QBO(X,\tau_1,\tau_2)\},$$

we have $x \notin \Lambda_{qb}(\Lambda_{qb}(A))$. Thus by contrapositivity, $\Lambda_{qb}(\Lambda_{qb}(A)) \subseteq \Lambda_{qb}(A)$. Therefore (6) holds.

(7) Let $A = \cup\{A_i : i \in I\}$. By (2) and since $A_i \subseteq A$, $\Lambda_{qb}(A_i) \subset \Lambda_{qb}(A)$ for all $i \in I$ and so $\cup\{\Lambda_{qb}(A_i) : i \in I\} \subseteq \Lambda_{qb}(A)$. To prove the converse inclusion, let $x \notin \cup\{\Lambda_{qb}(A_i) : i \in I\}$. Then for each $i \in I$ there exists $G_i \in QBO(X,\tau_1,\tau_2)$ such that $A_i \subseteq G_i$ and $x \notin G_i$. If $G = \cup\{G_i : i \in I\}$, then $G \in QBO(X,\tau_1,\tau_2)$ with $A \subseteq G$ and $x \notin G$. Hence $x \notin \Lambda_{qb}(A)$. Thus by contrapositivity, $\Lambda_{qb}(A) \subseteq \cup\{\Lambda_{qb}(A_i) : i \in I\}$. Therefore (7) holds. ■

The proof of the following result follows by a similar manner to that of Lemma 2.7 and thus omitted.

Lemma 2.8 *For subsets* A, B *and* A_i *of* (X,τ_1,τ_2) *where* $i \in I$, *the following properties hold:*

(1) $\vee_{qb}(A) \subseteq A$.

(2) If $A \subseteq B$, *then* $\vee_{qb}(A) \subseteq \vee_{qb}(B)$.

(3) If $A \in QBC(X,\tau_1,\tau_2)$, *then* $A = \vee_{qb}(A)$.

(4) $\vee_{qb}(\cap\{A_i : i \in I\}) = \cap\{\vee_{qb}(A_i) : i \in I\}$.

(5) $\vee_{qb}(\vee_{qb}(A)) = \vee_{qb}(A)$.

(6) $\cup\{\vee_{qb}(A_i) : i \in I\} \subseteq \vee_{qb}(\cup\{A_i : i \in I\})$.

Theorem 2.9 *In a apace* (X, τ_1, τ_2), *the class of all* Λ_{qb} *(resp.,* $\vee_{\text{qb}}$*)-sets is a topological space.*

Proof We only prove the Λ_{qb} case. It is obvious from Definition 2.6 that X and ϕ are Λ_{qb}-sets . Let A and B be quasi - Λ_{b}-sets, then $\Lambda_{\text{qb}}(A) = A$ and $\Lambda_{\text{qb}}(B) = B$. Thus by Lemma 2.2, $\Lambda_{\text{qb}}(A \cap B) \subseteq \Lambda_{\text{qb}}(A) \cap \Lambda_{\text{qb}}(B) = A \cap B \subseteq \Lambda_{\text{qb}}(A \cap B)$. Hence $A \cap B$ is a Λ_{qb}-set. Finally, let $\{A_i : i \in i\}$ be a family of Λ_{qb}-set in (X, τ_1, τ_2) and $A = \cup\{A_i : i \in i\}$. Then by Lemma 2.7, we have $\Lambda_{\text{qb}}(A) = \cup\{\Lambda_{\text{qb}}(A_i) : i \in I\} = \cup\{A_i : i \in I\} = A.$. ■

The following example is given in details as it will be needed throughout the rest of this paper.

Example 2.10 Let $X = \{a,b,c,d\}$, $\tau_1 = \{X, \phi, \{a,b\}, \{b\}, \{b,c,d\}\}$ and $\tau_2 = \{X, \phi, \{b\}\}$. Then

(1) $QBO(X, \tau_1, \tau_2) = \{X, \phi, \{b\}, \{a,b\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, \{a,c,d\}\}$.

(2) The set of all Λ_{qb}-sets is

$$\{X, \phi, \{b\}, \{a,b\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, \{a,c,d\}\} .$$

(3) The set of all $\vee_{\text{qb}}$-sets is

$$\{X, \phi, \{a\}, \{c\}, \{d\}, \{a,c\}, \{a,d\}, \{c,d\}, \{a,c,d\}\} \cup \{A \subseteq X : b \in A\} .$$

Definition 2.11 *A subset* A *of* (X, τ_1, τ_2) *is called* λ_{qb}-closed *if* $A = L \cap F$, *where* L *is a* Λ_{qb}*-set and* F *is quasi-b-closed. The complement of a* λ_{qb}*-closed is* λ_{qb}-open.

In Example 2.10, every subset of X is a λ_{qb}-closed set.

Theorem 2.12 *For a subset* A *of* (X,τ_1,τ_2)*, the following are equivalent:*

(i) A *is* λ_{qb} *-closed*.

(ii) $A = L \cap qbCl(A)$ *for some* Λ_{qb} *-set* L *in* (X,τ_1,τ_2) .

(iii) $A = \Lambda_{\text{qb}}(A) \cap qbCl(A)$.

Proof $(1)\Rightarrow(2)$: Let $A = L \cap F$ where L is a Λ_{qb}-set and F is quasi-b-closed. Since $F = qbCl(F)$, $A \subseteq L \cap qbCl(A) \subseteq L \cap qbCl(F) = A$ and thus the result is obtained.

$(2)\Rightarrow(3)$: Since $A \subseteq \Lambda_{\text{qb}}(A) \subseteq L$ and $A \subseteq qbCl(A)$, we have

$$A \subseteq \Lambda_{\text{qb}}(A) \cap qbCl(A) \subseteq L \cap qbCl(A) = A .$$

Hence $A = \Lambda_{\text{qb}}(A) \cap qbCl(A)$.

$(3)\Rightarrow(1)$: Since $\Lambda_{\text{qb}}(A)$ is a quasi- Λ_{qb} -set and $qbCl(A)$ is quasi-b-closed , A is a λ_{qb} -closed set. ■

The following is a relatively new notion of generalized closed sets that will be used to find a characterization for quasi-bg- closed sets in Theorem 2.14.

The proof of the following result follows from definitions.

Lemma 2.13 *Every quasi-b-closed is both quasi-bg- closed and* λ_{qb} *-closed. Moreover, a subset* A *of* (X,τ_1,τ_2) *is quasi-bg-closed if and only if* $qbCl(A) \subseteq \Lambda_{\text{qb}}(A)$.

In Example 2.10, the class of all quasi-b-closed sets is quasi-bg-closed

Theorem 2.14 *A subset* A *of* (X,τ_1,τ_2) *is quasi-b-closed if and only if* A *is quasi-bg-closed and* λ_{qb} *-closed.*

Proof Since every quasi-b-closed set is quasi-bg-closed and X is a Λ_{qb}-set, then $A = X \cap A$ is λ_{qb} -closed.

Conversely by Lemma 2.13, $qbCl(A) \subseteq \Lambda_{qb}(A)$ and A is λ_{qb} -closed. Then by Theorem 2.12, $A = \Lambda_{qb}(A) \cap qbCl(A) = qbCl(A)$ and hence A is quasi-b-closed. ■

3 Quasi b-spaces.

In this section, we introduce several new classes of bitopological spaces; namely quasi-b-T_0, $T_{\frac{1}{2}}$, T_1 and T_2 spaces. We show that the class of quasi-b-T_2 spaces is stronger than that of quasi-b-T_1 spaces which is stronger than the class of quasi-b-$T_{\frac{1}{2}}$, while the class of quasi-b-T_0 is the weakest one. Moreover, several characterizations of these spaces are provided.

Definition 3.1 *A space* (X,τ_1,τ_2) *is* quasi-b-T_0 *if for every two distinct points* x, y *of* X, *there exists* $A \in QBO(X,\tau_1,\tau_2)$ *such that (*$x \in A$ *and* $y \notin A$*) or (*$x \notin A$ *and* $y \in A$*).*

Definition 3.2 *A space* (X,τ_1,τ_2) *is* quasi-b-$T_{\frac{1}{2}}$ *if every quasi-bg-closed set is quasi-b-closed.*

Definition 3.3 *A space* (X,τ_1,τ_2) *is* quasi-b-T_1 *if for every two distinct points* x, y *of* X, *there exist* $A, B \in QBO(X,\tau_1,\tau_2)$ *such that each set contains one and only one element of* x *and* y.

Definition 3.4 *A space* (X,τ_1,τ_2) *is* quasi-b-T_2 *if for every two distinct points* x, y *of* X, *there exist disjoint sets* $A, B \in QBO(X,\tau_1,\tau_2)$ *such that* $x \in A$ *and* $y \in B$.

The proofs of the following two results, in which *quasi-b-*T_0 and *quasi-b-*T_1 spaces are characterized, are straightforward and thus omitted.

Theorem 3.5 *For a space* (X,τ_1,τ_2)*, the following are equivalent:*

(i) X *is quasi-b-*T_0.

(ii) For every two distinct points x, y *of X, there exists*

$A \in QBO(X,\tau_1,\tau_2) \cup QBC(X,\tau_1,\tau_2)$ such that $x \in A$ and $y \notin A$.

Separation axioms stand among the most common and to a certain extent the most important and interesting concepts in topology. One of the most well-known low separation axiom is the one which requires that singletons are closed. Analogously for the case of bitopological spaces, we have the following result:

Theorem 3.6 A space (X,τ_1,τ_2) *is*

(i) quasi-b-T_0 *if and only if for every two distinct points* x and y *of* X $qbCl\{x\} \neq qbCl\{y\}$.

(ii) quasi b-T_1 *if and only if singletons are quasi-b-closed.*

Next, we characterize quasi-b-T_0 via λ_{qb}-closed notion.

Theorem 3.7 *For a space* (X,τ_1,τ_2)*, the following are equivalent:*

(i) X *is quasi-b-*T_0.

(ii) Every singleton is λ_{qb}*-closed*.

Proof (i)$\Rightarrow$(ii): Let $x \in X$. By Theorem 3.5 for each $x \neq y$, there exists $A_y \in QBO(X,\tau_1,\tau_2) \cup QBC(X,\tau_1,\tau_2)$ such that $x \in A_y$ and $y \notin A_y$. Set

$L = \cap\{A_y \in QBO(X,\tau_1,\tau_2) : y \neq x\}$

and $A = \cap\{A_y \in QBC(X,\tau_1,\tau_2) : y \neq x\}$. Then L is a Λ_{qb}-set, A is quasi-b-closed and $\{x\} = L \cap A$ or $\{x\} = L$ or $\{x\} = A$. In all cases, $\{x\}$ is λ_{qb}-closed.

(ii) $\Rightarrow$(i): Let x, y be two distinct points of X. By (ii), $\{x\} = L \cap A$ where L is a Λ_{qb}-set and A is a quasi-b-closed set. If $y \notin A$, then $X - A$ is a quasi-b-open set contains y but not x. If $y \notin L$, then $y \notin A_y$ for some quasi-b-open set A_y containing x. Thus X is quasi-b-T_0.■

In Example 2.10, every singleton is λ_{qb} -closed and thus (X,τ_1,τ_2) is quasi-b-T_0.

Theorem 3.8 *For a space* (X,τ_1,τ_2)*, the following are equivalent:*

(1) (X,τ_1,τ_2) *is quasi-b-*$T_{\frac{1}{2}}$.

(2) Every singleton $\{x\}$ *is quasi-b-open or quasi-b-closed.*

(3) Every subset is λ_{qb} *-closed.*

Proof (1) $\Rightarrow$(2) Let $x \in X$. If $\{x\}$ is not quasi-b-closed, then $X-\{x\}$ is not quasi-b-open. Since X is the only quasi-b-open set containing $X-\{x\}$, $\{x\}$ is quasi-bg-closed. As (X,τ_1,τ_2) is quasi-b-$T_{\frac{1}{2}}$, $X-\{x\}$ is quasi-b-closed and thus $\{x\}$ is quasi-b-open.

(2)$\Rightarrow$(3) Let A be a subset of *X*. By assumption, every singleton $\{x\}$ is quasi-b-open or quasi-b-closed. Let $(X \setminus A)_{QBO} = \{x \in X \setminus A : \{x\} \in QBO(X,\tau_1,\tau_2)\}$,

$B = X \setminus \left(A \cup (X \setminus A)_{QBO}\right)$ and $L = \cap\{X \setminus \{x\} : x \in B\}$. Then L is a Λ_{qb} -set,

$F = \cap\{X \setminus \{x\} : x \in (X \setminus A)_{QBO}\}$ is a quasi-b-closed set and $A = L \cap F$. Therefore A is λ_{qb} -closed.

(3)$\Rightarrow$(1) Let A be a quasi-bg-closed set. By assumption and Theorem 2.14, A is quasi-b-closed. Therefore (X,τ_1,τ_2) is quasi-b-$T_{\frac{1}{2}}$. ■

In Example 2.10, every subset is λ_{qb}-closed and thus (X,τ_1,τ_2) is quasi-b-$T_{\frac{1}{2}}$.

Corollary 3.9 *For a space* (X,τ_1,τ_2)*, the following are equivalent:*

(i) X *is quasi-b-*$T_{\frac{1}{2}}$.

(ii) Every generalized Λ_{qb}*-set is* Λ_{qb}*-set.*

Theorem 3.10 (i) *Every quasi-b-*T_2 *space is quasi-b-*T_1

(ii) *Every quasi-b-*T_i *space is quasi-b-*$T_{i-\frac{1}{2}}$ *for* $i=\frac{1}{2},1$.

Proof (i) follows from Definition 3.3 and Definition 3.4, while quasi-b-T_1 space is quasi-b-$T_{\frac{1}{2}}$ follows by combining Theorem 3.6 and Theorem 3.8 together. Let (X,τ_1,τ_2) be a quasi-b-$T_{\frac{1}{2}}$ space. We shall prove that for any two distinct points $x,y\in X$, $qbCl\{x\}\neq qbCl\{y\}$. By Theorem 3.8, we have the following three cases:

Case 1. $\{x\}$ and $\{y\}$ are quasi-b-closed sets. Then $\{x\}=qbCl\{x\}\neq qbCl\{y\}=\{y\}$.

Case 2. $\{x\}$ is τ_i-b-open and $\{y\}$ is quasi-b-closed, where $i\in\{1\,,2\}$. Since a τ_i-b-open set is quasi-b-open , $x\notin qbCl\{y\}=\{y\}$ and hence $qbCl\{x\}\neq qbCl\{y\}$.

Case 3. $\{x\}$ is τ_i-b-open and $\{y\}$ is τ_j-b-open where $i,j\in\{1\,,2\}$. Since $\{x\},\{y\}\in QBO(X,\tau_1,\tau_2)$, we have $x\notin qbCl\{y\}$ and hence $qbCl\{x\}\neq qbCl\{y\}$.

In all cases we have $qbCl\{x\}\neq qbCl\{y\}$. Therefore (X,τ_1,τ_2) is a *quasi-b-*T_0 *space.*■

Conjecture 3.11 *A quasi-b-*$T_{\frac{1}{2}}$ *space need not be a quasi-b-*T_1 *space as shown next, while we leave it as an open question to find an example of a quasi-b-*T_0 *space that is not a quasi-b-*$T_{\frac{1}{2}}$ *space and another one of a quasi-b-*T_1 *space that is not a quasi-b-*T_2 *space.*

If our conjecture is true, the converses of all parts of Theorem 3.3 will not be true.

Example 3.12 Let $X=\{a,b,c\},\tau_1=\{X,\phi,\{a\}\}$ and $\tau_2=\{X,\phi,\{a\},\{a,b\},\{a,c\}\}$. Clearly, $BO(X,\tau_1)=BO(X,\tau_2)=QBO(X,\tau_1,\tau_2)=\tau_2$. Thus (X,τ_1,τ_2) is a quasi-b-$T_{\frac{1}{2}}$ space that is not quasi-b-T_1.

4 Quasi-b-homeomorphisms group

In this section, we investigate a group of quasi-b-homeomorphisms on a bitopological space. We begin by defining the notion of quasi-b-homeomorphisms.

Definition 4.1 *A map* $f:(X,\tau_1,\tau_2)\to(Y,J_1,J_2)$ *is* quasi-b- *(resp.,* λ_{qb}*-,* quasi-bg-*) continuous if the inverse image of every* $V\in J_1\cup J_2$ *is quasi-b-open (resp.,* λ_{qb}*-open, quasi-bg-open) set.*

Clearly, a map $f:(X,\tau_1,\tau_2)\to(Y,J_1,J_2)$ is quasi-b- (resp., λ_{qb}-, quasi-bg-) continuous if the inverse image of every closed subset with respect to J_1 or J_2 is quasi-b-closed (resp., λ_{qb}-closed, quasi-bg-closed) set. Note that if $\tau_1=\tau_2$ and $J_1=J_2$, then quasi-b-continuity implies quasi-continuity in topological spaces. Theorem 2.14 provides the following immediate decomposition of quasi-b-continuity in bitopological spaces:

Theorem 4.2 *A map* $f:(X,\tau_1,\tau_2)\to(Y,J_1,J_2)$ *is quasi-b-continuous if and only if* f *is* λ_{qb}*-continuous and quasi-bg-continuous.*

Proof Let $f:(X,\tau_1,\tau_2)\to(Y,J_1,J_2)$ be quasi-b-continuous and *V* be a closed subset with respect to J_1 or J_2. Then *V* is quasi-b-closed. Thus by Theorem 2.14, *V* is quasi-bg-closed and λ_{qb}-closed. Hence *f* is both quasi-bg-continuous and λ_{qb}-continuous.

Conversely, let *f* be quasi-bg-continuous and λ_{qb}-continuous and *V* be a closed subset with respect to J_1 or J_2. Then $f^{-1}(V)$ is quasi-bg-closed and λ_{qb}-closed. Hence by Theorem 2.14, $f^{-1}(V)$ is quasi-b-closed. Therefore, *f* is quasi-b-continuous. ■

Definition 4.3 *A map* $f:(X,\tau_1,\tau_2)\to(Y,J_1,J_2)$ *is called* quasi-b-irresolute *if the inverse image of every quasi-b-open set is a quasi-b-open set.*

Definition 4.4 *A bijection* $f:(X,\tau_1,\tau_2)\to(Y,J_1,J_2)$ *is called* quasi-b-homeomorphism *(simply,* quasi-bh*) if both* f *and* f^{-1} *are quasi-b-irresolute maps.*

Definition 4.5 *For a space* (X,τ_1,τ_2) *and a subset* H *of* X *,* $quasi-bh(X,H,\tau_1,\tau_2)$ *is the set of all maps* $f:(X,\tau_1,\tau_2)\rightarrow(\mathrm{X},\tau_1,\tau_2)$ *such that* f *is a quasi-b-homeomorphism and* $f(H)=H$ *. and* $quasi-bh(X,\tau_1,\tau_2)$ *is defined to be* $quasi-bh(X,\phi,\tau_1,\tau_2)$.

Clearly, the binary operation

$$\beta: quasi-bh(X,\tau_1,\tau_2)\times quasi-bh(X,\tau_1,\tau_2)\rightarrow quasi-bh(X,\tau_1,\tau_2)$$

is well defined by $\beta(f,g)=g\circ f$ for every f and $g\in quasi-bh(X,\tau_1,\tau_2)$.

Theorem 4.6 $quasi-bh(X,\tau_1,\tau_2)$ *is a group and* $quasi-bh(X,H,\tau_1,\tau_2)$ *is a subgroup of* $quasi-bh(X,\tau_1,\tau_2)$ *for every* $H\subseteq X$.

Proof. Clearly the this operation is associative. Let f and g be elements in $quasi-bh(X,\tau_1,\tau_2)$. Then *f*, f^{-1}, g and g^{-1} are quasi-b-irresolute. Hence $g\circ f$ and $(g\circ f)^{-1}$ are quasi-b-irresolute maps. Thus this operation is closed. Since the identity map is quasi-b-irresolute, it belongs to $quasi-bh(X,\tau_1,\tau_2)$ and it is the identity element in this set. Moreover for any *f* in $quasi-bh(X,\tau_1,\tau_2)$, f^{-1} is quasi-b-irresolute and hence f^{-1} belongs to $quasi-bh(X,\tau_1,\tau_2)$. Therefore, $quasi-bh(X,\tau_1,\tau_2)$ is a group with this binary operation.

Since $quasi-bh(X,H,\tau_1,\tau_2)\subseteq quasi-bh(X,\tau_1,\tau_2)$, it remains to show that for any $f,g\in quasi-bh(X,H,\tau_1,\tau_2)$, $g\circ f^{-1}\in quasi-bh(X,H,\tau_1,\tau_2)$. But if

$f,g\in quasi-bh(X,H,\tau_1,\tau_2)$, then *f*, f^{-1}, g and g^{-1} are quasi-b-irresolute maps that map *H* into *H*. Hence $g\circ f^{-1}$ and $f\circ g^{-1}$ are quasi-b-irresolute maps. Moreover, $g\circ f^{-1}(H)=g(H)=H$. Therefore $quasi-bh(X,H,\tau_1,\tau_2)$ is a subgroup of $quasi-bh(X,\tau_1,\tau_2)$ for every $H\subseteq X$. ■

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شبه المجموعات المفتوحة من النمط ب في الفضاءات التوبولوجية المزدوجة

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ملخص

منذ ظهور فكرة المجموعات المغلقة العامة (Generalized closed sets) في الفضاءات التوبولوجيه، بدأ االعديد من المهتمين البحث عن مجموعات أعم بهدف ايجاد تراكيب جديده للاقترانات المتصله (Decompositions of continuity). إن الهدف من هذا البحث أن نقدم وندرس اصنافا جديده نسبيا من المجموعات المسماه بشبه المجموعات المفتوحة من النمط ب (Quasi b-open sets). لذلك فاننا نعرف ما يسمى بالاقترانات شبه المتصلة من النمط ب (Quasi b-continuity) في الفضاءات التوبولوجية المزدوجة (Bitopological spaces) ونعطي احدى التراكيب. بالاضافة الى ذلك فاننا نجد زمرة جديده من هذه الاقترانات والعديد من الفضاءات التوبولوجية المزدوجة.

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