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SOFT VECTOR SPACES AND SOFT TOPOLOGICAL VECTOR SPACES

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Abstract. Molodtsov introduced the concept of soft set theory, which can be used as a generic mathematical tool for dealing with uncertainty. In fact, it is free from the difficulties that have troubled the usual theoretical approaches. Roy and Samanta [13, 14, 15] defined several basic notions on soft set and studied many properties. In continuation of [13, 14, 15], here we have introduced soft topological vector space and studied few basic properties related to soft topological vector spaces. For introduction of soft topological vector space, there is need to define soft vector space and soft product topological space, which has been encountered in this paper.

1. Introduction

In 1999, Molodstov [9] initiated a novel concept known as soft set as a new mathematical tool for dealing with uncertainties. He pointed out that the important existing theories viz. probability theory, fuzzy set theory, intuitionistic fuzzy set theory, rough set theory etc. which can be considered as mathematical tools for dealing with uncertainties, have their own difficulties. These theories cannot be successfully used to solve complicated problems in the fields of engineering, social science, economics, medical science etc. He further pointed out that the reason for these difficulties is,

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possibly, the inadequacy of the parameterization tool of the theory. The soft set theory introduced by Molodstov is free of the difficulties present in these theories. The absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable.

Subsequently, many works on fuzzy soft rings and fuzzy soft ideals [5], Soft group [6], soft topological spaces [8, 10], fuzzy soft topological spaces [11], fuzzy soft open and closed sets [12], vector sum and scalar multiplication of soft sets [13], balanced and absorbing soft sets [14] have been done in this field. In 2007, H. Aktas et al. [2] worked on some mathematical aspects of soft sets and soft groups. In 2010, U. Acar et al. [1] introduced the concept of soft rings. In 2010, N. Cagman et al. [3] applied this concepts to solve a few decision making problems. In 2015, M. Chiney et al. [4] introduced the notion of vector soft topology. They assumed the universal set as a usual vector space. But in this paper, only the parameter set is assumed to be a usual vector space. The concept of fuzzy vector space was given for the first time in [7] by A. K. Katsaras et.al. in 1977 and also the notion of fuzzy topological vector space was given in that paper. In the present time, researchers are trying to explore these concepts on soft sets. In this paper, we have introduced the notion of soft topological vector space, which is a continuation of the works of [13, 14, 15]. That's why, for organizing this paper, in Section 2, some definitions and results from the papers [3, 10, 13, 14, 15] are highlighted. In Section 3, few properties of vector sum and scalar multiplications on soft sets are being developed for the introduction of soft vector space which is described in the Section 4. In Section 5, there is a notion of soft product topological spaces and in the last section, that is, Section 6, soft topological vector space is introduced taking help of soft vector space and soft product topological space.

2. Preliminaries

In this paper, U refers to an initial universe, E is the set of parameters, $P(U)$ is the power set of U and $A \subseteq E$.

Definition 2.1. [3] A soft set F_A on the universe U is defined to be a set of ordered pairs $F_A = \{(e, F_A(e)) : e \in E, F_A(e) \in P(U)\}$ where $F_A : E \to P(U)$ such that $F_A(e) = \emptyset$ if e is not an element of A. The set of all soft sets over (U, E) is denoted by $S(U)$.

Definition 2.2. [3] Let $F_A \in S(U)$. If $F_A(e) = \emptyset$, for all $e \in E$ then F_A is called a empty soft set, denoted by Φ. $F_A(e) = \emptyset$ means that there is no element in U related to the parameter $e \in E$.

Definition 2.3. [3] Let $F_A, G_B \in S(U)$. We say that F_A is a soft subsets of G_B and we write $F_A \sqsubseteq G_B$ if and only if $F_A(e) \subseteq G_B(e)$ for all $e \in E$.

Definition 2.4. [3] Let $F_A, G_B \in S(U)$. Then F_A and G_B are said to be soft equal, denoted by $F_A = G_B$ if $F_A(e) = G_B(e)$ for all $e \in E$.

Definition 2.5. [3] Let $F_A, G_B \in S(U)$. Then the soft union of F_A and G_B is also a soft set $F_A \sqcup G_B = H_{A\cup B} \in S(U)$, defined by $H_{A\cup B}(e) = (F_A \sqcup G_B)(e) = F_A(e) \cup G_B(e)$ for all $e \in E$.

Let $F_{A_i} \in S(U)$, $i \in I$, an indexed set. The arbitrary soft union of the soft sets $\{F_{A_i} : i \in I\}$ is a soft set $\sqcup_{i\in I} F_{A_i} \in S(U)$, defined by $\sqcup_{i\in I} F_{A_i}(e) = \cup_{i\in I} F_{A_i}(e)$ for all $e \in E$.

Definition 2.6. [3] Let $F_A, G_B \in S(U)$. Then the soft intersection of F_A and G_B is also a soft set $F_A \sqcap G_B = H_{A\cap B} \in S(U)$, defined by $H_{A\cap B}(e) = (F_A \sqcap G_B)(e) = F_A(e) \cap G_B(e)$ for all $e \in E$.

Definition 2.7. [14] Let U be an initial universe and $f : X \to Y$ be a mapping, where X and Y are sets of parameters. If F_A is a soft set over (U, X) then $f(F_A)$, a soft set over (U, Y), is defined by

$$f(F_A)(y) = \begin{cases} \cup_{x \in f^{-1}(y)} F_A(x) & if \quad f^{-1}(y) \neq \emptyset \\ \emptyset & otherwise. \end{cases}$$

Definition 2.8. [14] Let U be an initial universe and $f : X \to Y$ be a mapping, where X and Y are set of parameters. If G_B is a soft set over (U, Y) then $f^{-1}(G_B)$, a soft set over (U, X), is defined by $f^{-1}(G_B)(x) = G_B(f(x))$.

Definition 2.9. [10] A soft topology τ on soft set F_A is a family of soft subsets of F_A satisfying the following properties

(i) $\Phi, F_A \in \tau$

(ii) If $G_B, H_C \in \tau$ then $G_B \sqcap H_C \in \tau$

(iii) If $F_{A_\alpha} \in \tau$ for all $\alpha \in \Lambda$, an index set then $\sqcup_{\alpha \in \Lambda} F_{A_\alpha} \in \tau$.

If τ is a soft topology on a soft set F_A, the pair (F_A, τ) is called the soft topological space and the member of τ is called soft open set in (F_A, τ).

Definition 2.10. Let (F_A, τ) be a soft topological space. A soft subset G_B of F_A is said to be a soft closed set in (F_A, τ) if $F_A - G_B \in \tau$, where $(F_A - G_B)(e) = F_A(e) \setminus G_B(e)$ for all $e \in E$.

Definition 2.11. [10] A collection Ω of some members of a soft topology τ is said to be soft subbase for τ if and only if the collection of all finite intersections of members of Ω is a soft base for τ.

Theorem 2.1. [10] *A collection Ω of some soft subsets of F_A is a soft subbase for a suitable soft topology τ on F_A if and only if*

(i) $\Phi \in \Omega$ or Φ is the intersection of a finite number of members of Ω.

(ii) $F_A = \sqcup \Omega$.

Definition 2.12. [13] Let U be a universal set and E be a usual vector space over $\mathbb{R}$ or $\mathbb{C}$ and $F_{A_1}, F_{A_2}, \cdots, F_{A_n}$ be soft sets over (U, E) and $f : E^n \to E$ be a function defined by $f(e_1, e_2, \cdots, e_n) = e_1 + e_2 + \cdots + e_n$. Then the vector sum $F_{A_1} + F_{A_2} + \cdots + F_{A_n}$ is defined by

$(F_{A_1} + F_{A_2} + \cdots + F_{A_n})(e) = \cup_{(e_1,e_2,\cdots,e_n)\in f^{-1}(e)}\{F_{A_1}(e_1) \cap F_{A_2}(e_2) \cap \cdots \cap F_{A_n}(e_n)\}$.

Definition 2.13. [13] If U is a universal set, E is a usual vector space over $\mathbb{R}$ or $\mathbb{C}$, t is a scalar and $g : E \to E$ is a mapping defined by $g(e) = te$ then the scalar multiplication tF_A of a soft set F_A is defined by $tF_A = g(F_A)$. That is, for $e \in E$, $tF_A(e) = g(F_A)(e) = \cup_{e' \in g^{-1}(e)} F_A(e')$.

Proposition 2.1. [13] *Let U be a universal set, E be a usual vector space over $\mathbb{R}$ or $\mathbb{C}$, t be a scalar and F_A be a soft set over (U, E). Then*

$$tF_A(e) = \begin{cases} F_A(t^{-1}e) & if \quad t \neq 0, \\ \emptyset & if \quad t = 0 \quad and \quad e \neq 0, \\ \cup_{x\in E} F_A(x) & if \quad t = 0 \quad and \quad e = 0. \end{cases}$$

Proposition 2.2. [13] *Suppose E_1 and E_2 are linear spaces over the same field $\mathbb{R}$ or $\mathbb{C}$ and $f : E_1 \to E_2$ is linear mapping. Then for any soft sets F_A and G_B over (U, E_1) and for the scalar t,*
(i) $f(F_A + G_B) = f(F_A) + f(G_B)$.
(ii) $f(tF_A) = tf(F_A)$.

Proposition 2.3. [13] *If S is an ordinary subset of a linear space E over $\mathbb{R}$ or $\mathbb{C}$, F_A is a soft set over (U, E) and $x \in E$ then*
(i) $(x + F_A)(e) = F_A(e - x)$ for all $e \in E$ where $x + F_A$ means $1_x + F_A$ and

$$1_x(e) = \begin{cases} U & if \quad e = x \\ \emptyset & if \quad e \neq x. \end{cases}$$

(ii) $x + F_A = T_x(F_A)$ for the translation mapping $T_x : E \to E$ defined by $T_x(e) = x + e$

for all $e \in E$.

(iii) $S + F_A = \sqcup_{a \in S}(a + F_A)$.

Proposition 2.4. [13] *If* $F_A, F_{A_1}, F_{A_2}, \cdots, F_{A_n}$ *are soft sets over* (U, E)*, where* E *is a linear space. Then for the scalars* $t_1, t_2, \cdots, t_n$*, the following are equivalent:*

(i) $t_1 F_{A_1} + t_2 F_{A_2} + \cdots + t_n F_{A_n} \sqsubseteq F_A$

(ii) for all $e_1, e_2, \cdots, e_n \in E$*,* $F_A(t_1 e_1 + t_2 e_2 + \cdots + t_n e_n) \supseteq \cap_{i=1}^{n} F_{A_i}(e_i)$

Proposition 2.5. [15] *If* F_A *is a soft set over* (U, E) *and* $\alpha \in K$ *then* $\alpha F_A + 0 F_A = \alpha F_A$*, where* E *is a vector space over the field* $K(= \mathbb{R}$ *or* $\mathbb{C})$.

The Definitions 2.12, 2.13 and the Propositions 2.1, 2.2, 2.3, 2.4, 2.5 could also be true if we consider E as a vector space over any field K instead of considering K as the field of real numbers $\mathbb{R}$ or the field of complex numbers $\mathbb{C}$.

3. A few properties of vector sum and scalar multiplication of soft sets

In this section, we assume that the parameter set E is a usual vector space over the field K and U, universal set.

Proposition 3.1. *If* F_A *and* F_B *are two soft sets over* (U, E) *then* $a(F_A + F_B) = aF_A + aF_B$ *for any scalar* $a \in K$.

Proof. Case1 : *If* $a \neq 0$ *then for* $e \in E$*,*

$a(F_A + F_B)(e)$

$= (F_A + F_B)(a^{-1}e)$

$= \cup_{a^{-1}e = e_1 + e_2}\{F_A(e_1) \cap F_B(e_2)\}$

$= \cup_{e = ae_1 + ae_2}\{F_A(e_1) \cap F_B(e_2)\}$

$= \cup_{e = x + y}\{F_A(a^{-1}x) \cap F_B(a^{-1}y)\}$ *where* $ae_1 = x$ *and* $ae_2 = y$

$= \cup_{e = x + y}\{aF_A(x) \cap aF_B(y)\}$

$= (aF_A + aF_B)(e)$.

Case2 : *If* $a = 0$ *then we show that* $0(F_A + F_B)(e) = (0F_A + 0F_B)(e)$ *for any* $e \in E$. *If* $e \neq 0$ *then it is easy to see that* $0(F_A + F_B)(e) = (0F_A + 0F_B)(e) = \emptyset$. *So we assume that* $e = 0$. *Then*

$0(F_A + F_B)(0)$

$= \cup_{e \in E}(F_A + F_B)(e)$

$= \cup_{e \in E}[\cup_{e=x+y}\{F_A(x) \cap F_B(y)\}]$

$= \cup_{x,\, y \in E}\{F_A(x) \cap F_B(y)\}$

$= \{\cup_{x \in E} F_A(x)\} \cap \{\cup_{y \in E} F_B(y)\}$

$= 0F_A(0) \cap 0F_B(0)$

$= (0F_A + 0F_B)(0)$.

Proposition 3.2. *If* F_A *and* F_B *are two soft sets over* (U, E) *then* $F_A + F_B = F_B + F_A$.

Proposition 3.3. *If* F_A, F_B *and* F_C *are soft sets over* (U, E) *then* $(F_A + F_B) + F_C = F_A + (F_B + F_C)$.

Proof. *Let* $e \in E$. *Then*

$\{(F_A + F_B) + F_C\}(e)$

$= \cup_{e=e_1+e_2}\{(F_A + F_B)(e_1) \cap F_C(e_2)\}$

$= \cup_{e=e_1+e_2}[\{\cup_{e_1=x+y}(F_A(x) \cap F_B(y))\} \cap F_C(e_2)]$

$= \cup_{e=e_1+e_2}[\cup_{e_1=x+y}\{F_A(x) \cap F_B(y) \cap F_C(e_2)\}]$

$= \cup_{e=x+y+e_2}\{F_A(x) \cap F_B(y) \cap F_C(e_2)\}$

$= \cup_{e=x+z}[\cup_{z=y+e_2}\{F_A(x) \cap F_B(y) \cap F_C(e_2)\}]$

$= \cup_{e=x+z}[F_A(x) \cap \{\cup_{z=y+e_2}(F_B(y) \cap F_C(e_2))\}]$

$= \cup_{e=x+z}[F_A(x) \cap (F_B + F_C)(z)\}]$

$= \{F_A + (F_B + F_C)\}(e)$

Hence $(F_A + F_B) + F_C = F_A + (F_B + F_C)$.

Proposition 3.4. *If* F_A *is a soft set over* (U, E) *then* $a(bF_A) = b(aF_A)$ *for any scalars* a, $b \in K$.

Proof. Case 1 : *If* $a = b = 0$ *then it is trivial.*

Case 2 : *If one of* a *and* b *is zero, say* $a = 0$ *then we show that* $0(bF_A)(e) = b(0F_A)(e)$ *for all* $e \in E$.

Subcase 1 : *If* $e \neq 0$ *then*

$0(bF_A)(e) = \emptyset$ *and* $b(0F_A)(e) = 0F_A(b^{-1}e) = \emptyset$. *Thus* $0(bF_A) = b(0F_A)$.

Subcase 2 : *If* $e = 0$ *then*

$0(bF_A)(0)$

$= \cup_{e\in E} bF_A(e)$

$= \cup_{e\in E} F_A(b^{-1}e)$

$= \cup_{x\in E} F_A(x)$ *as* $b^{-1}E = E$

$= 0F_A(0)$

$= b(0F_A)(0)$.

Case 3 : *If* a, $b \neq 0$ *then we show that* $a(bF_A)(e) = b(aF_A)(e)$ *for all* $e \in E$.

Subcase 1 : *If* $e = 0$ *then*

$a(bF_A)(0) = F_A(0) = b(aF_A)(0)$.

Subcase 2 : *If* $e \neq 0$ *then*

$a(bF_A)(e) = F_A(b^{-1}a^{-1}e) = F_A(a^{-1}b^{-1}e) = b(aF_A)(e)$. *This completes the proof.*

4. Soft Vector Spaces

In this section, we assume that the parameter set E is a usual vector space over the field K and U, a universal set.

Definition 4.1. A soft set F_A over (U, E) is called a soft vector space or soft linear space if the following conditions are hold:

i) $F_A + F_A \sqsubseteq F_A$

ii) $tF_A \sqsubseteq F_A$ for every scalar $t \in K$.

Example 4.1. *Let* E *be a vector space over the field* K *and* U *be a universal set and* A *be a finite dimensional sub-vector space of* E. *Also let* $\{\alpha_1, \alpha_2, \cdots, \alpha_n\}$ *be a basis* *of* A *and* $\{U_1, U_2, \cdots, U_n\}$ *be the collection of* n *subsets of* U. *Let us define a soft set* F_A *over* (U, E) *as follows:*

$$F_A(t\alpha_i) = \begin{cases} U & if\ t = 0, \\ U_i & if\ t(\neq 0) \in K \end{cases} \qquad for\ i = 1, 2, \cdots, n.$$

and for any $e \in A$ *with* $e = t_1\alpha_{k_1} + t_2\alpha_{k_2} + \cdots + t_m\alpha_{k_m}$ *for some non-zero scalars* $t_1, t_2, \cdots, t_m \in K$ *and* $\{k_1, k_2, \cdots, k_m\} \subseteq \{1, 2, \cdots, n\}$,

$F_A(e) = U_{k_1} \cap U_{k_2} \cap \cdots \cap U_{k_m}$ *and for* $e \in E - A$, $F_A(e) = \emptyset$.

We now show that F_A *is a soft vector space.*

Let $e \in E$ *and* $e = e_1 + e_2$. *Then two cases arise:*

(i) *one or both of* e_1, e_2 *belongs in* $E - A$, (ii) $e_1, e_2 \in A$.

(i) *If* e_1 *or* $e_2 \in E - A$ *then* $F_A(e_1) \cap F_A(e_2) = \emptyset \subseteq F_A(e)$.

(ii) *If* $e_1, e_2 \in A$ *then let* $e = t_1\alpha_{k_1} + t_2\alpha_{k_2} + \cdots + t_m\alpha_{k_m}$, $e_1 = c_1\alpha_{p_1} + c_2\alpha_{p_2} + \cdots + c_r\alpha_{p_r}$ *and* $e_2 = d_1\alpha_{q_1} + d_2\alpha_{q_2} + \cdots + d_s\alpha_{q_s}$ *for some non-zero scalars* $t_1, t_2, \cdots, t_m, c_1, c_2, \cdots, c_r$ *and* $d_1, d_2, \cdots, d_s \in K$. *Here it is easy to see that*

$\{\alpha_{k_1}, \alpha_{k_2}, \cdots, \alpha_{k_m}\} \subseteq \{\alpha_{p_1}, \alpha_{p_2}, \cdots, \alpha_{p_r}, \alpha_{q_1}, \alpha_{q_2}, \cdots, \alpha_{q_s}\}$. *So,* $F_A(e_1) \cap F_A(e_2) \subseteq F_A(e)$.

Therefore, if $e \in E - A$ *then by* (i) *we get* $(F_A + F_A)(e) = \emptyset \subseteq F_A(e)$ *and if* $e \in A$ *then by* (i) *and* (ii), *we get* $(F_A + F_A)(e) = \cup_{e=e_1+e_2}\{F_A(e_1) \cap F_A(e_2)\} \subseteq \cup_{e=e_1+e_2}\{F_A(e)\} = F_A(e)$. *So,* $F_A + F_A \sqsubseteq F_A$. *Thus the first assumption of soft vector space holds.*

For the second assumption, let $e \in E$ *and* $t \in K$.

Case 1 : *If* $e \in E - A$ *then* $e \neq 0$ *and* $t^{-1}e \in E - A$ *for all* $t(\neq 0) \in K$. *So,* $tF_A(e) = \emptyset \subseteq F_A(e)$ *for all* $t \in K$.

Case 2 : *If* $e \in A$ *then either* $e = 0$ *or* $e \neq 0$.

If $e = 0$ *then* $tF_A(0) \subseteq U = F_A(0)$.

If $e \neq 0$ *then* $tF_A(e) = \begin{cases} \emptyset & if\ t = 0, \\ F_A(e) & if\ t(\neq 0). \end{cases}$

So, $tF_A(e) \subseteq F_A(e)$ *for all* $e \in A$.

Thus, $tF_A \sqsubseteq F_A$ *for all* $t \in K$. *Hence* F_A *is a soft vector space over* (U, E).

Proposition 4.1. *If* F_A *is a soft set over* (U, E) *then the followings are equivalent*

i) F_A *is a soft vector space.*

ii) $aF_A + bF_A \sqsubseteq F_A$ *for all scalars* a, $b \in K$.

iii) $F_A(ax + by) \supseteq F_A(x) \cap F_A(y)$ *for all* a, $b \in K$ *and for all* x, $y \in E$.

Proof. $(i) \Rightarrow (ii)$

Suppose F_A *is a soft vector space. Then for any* $e \in E$,

$(aF_A + bF_A)(e)$

$= \cup_{e=x+y}\{aF_A(x) \cap bF_A(y)\}$

$\subseteq \cup_{e=x+y}\{F_A(x) \cap F_A(y)\}$

$= (F_A + F_A)(e)$

$\subseteq F_A(e)$.

This proves (ii).

$(ii) \Rightarrow (i)$

Let us assume that $aF_A + bF_A \sqsubseteq F_A$ *for all scalars* a, $b \in K$.

Taking $a = b = 1$ *then* $F_A + F_A \sqsubseteq F_A$.

Taking $b = 0$ *then by Proposition 2.5, we have* $aF_A = aF_A + 0F_A \sqsubseteq F_A$.

Again by Proposition 2.4, (ii) *and* (iii) *are equivalent. This completes the proof of the proposition.*

Proposition 4.2. *If* F_A *is a soft vector space over* (U, E) *then* $F_A(x) \subseteq F_A(0)$ *for all* $x \in E$.

Proof. *By the Definition 4.1, we have* $tF_A \sqsubseteq F_A$ *for every* $t \in K$. *So,* $0F_A \sqsubseteq F_A$, *that is,* $0F_A(0) \subseteq F_A(0)$ *which implies that* $\cup_{x\in E} F_A(x) \subseteq F_A(0)$. *Hence* $F_A(x) \subseteq F_A(0)$ *for all* $x \in E$.

Theorem 4.1. F_A *is a soft vector space over* (U, E) *if and only if* (i) $F_A(x + y) \supseteq F_A(x) \cap F_A(y)$ *and* (ii) $F_A(ax) \supseteq F_A(x)$ *for all* $x, y \in E$ *and* $a \in K$.

Proof. *Let* F_A *be a soft vector space over* (U, E). *Then from Proposition 4.1, we have* $F_A(ax + by) \supseteq F_A(x) \cap F_A(y)$ *for all* a, $b \in K$ *and for all* x, $y \in E$.
Taking $a = b = 1$ *then* $F_A(x + y) \supseteq F_A(x) \cap F_A(y)$.
Taking $y = 0$ *then* $F_A(ax) \supseteq F_A(x) \cap F_A(0) = F_A(x)$ *by Proposition 4.2.*
Conversely, we assume that the conditions (i) *and* (ii) *are hold. Then* $F_A(x)\cap F_A(y) \subseteq F_A(ax) \cap F_A(by) \subseteq F_A(ax + by)$ *for all* $a, b \in K$. *Therefore by Proposition 4.1,* F_A *is a soft vector space over* (U, E).

Theorem 4.2. *If* F_A *and* F_B *are soft vector spaces over* (U, E) *then* $F_A + F_B$ *and* tF_A *are soft vector spaces over* (U, E) *for any scalar* $t \in K$.

Proof. *Let* a, $b \in K$. *Then*
$a(F_A + F_B) + b(F_A + F_B)$
$= (aF_A + aF_B) + (bF_A + bF_B)$ *by Proposition 3.1*
$= (aF_A + bF_A) + (aF_B + bF_B)$ *by Propositions 3.2, 3.3*
$\sqsubseteq F_A + F_B$.
Thus by the Proposition 4.1, $F_A + F_B$ *is a soft vector space over* (U, E).
Again $a(tF_A) + b(tF_A)$
$= t(aF_A) + t(bF_A)$ *by Proposition 3.4*
$= t(aF_A + bF_A)$ *by Proposition 3.1*
$\sqsubseteq tF_A$, *as* F_A *is a soft vector space.*

Theorem 4.3. *If X and Y are two linear spaces over the same field K and $f : X \to Y$ is a linear mapping then F_A is a soft vector space over (U, X) implies that $f(F_A)$ is a soft vector space over (U, Y).*

Proof. By Proposition 2.2, we have $f(F_A) + f(F_A) = f(F_A + F_A) \sqsubseteq f(F_A)$, as $F_A + F_A \sqsubseteq F_A$.
Again, by Proposition 2.2, we have $tf(F_A) = f(tF_A) \sqsubseteq f(F_A)$ for all $t \in K$. Hence $f(F_A)$ is also a soft vector space over (U, Y).

Theorem 4.4. *Let X and Y be two linear spaces over the same field K and $f : X \to Y$ be a linear mapping. If G_B is a soft vector space over (U, Y) then $f^{-1}(G_B)$ is a soft vector space over (U, X).*

Proof. Let G_B be a soft vector space over (U, Y) and $x \in X$. Then
$(f^{-1}(G_B) + f^{-1}(G_B))(x)$
$= \cup_{x=x_1+x_2}\{f^{-1}(G_B)(x_1) \cap f^{-1}(G_B)(x_2)\}$
$= \cup_{x=x_1+x_2}\{G_B(f(x_1)) \cap G_B(f(x_2))\}$
$= \cup_{f(x)=f(x_1)+f(x_2)}\{G_B(f(x_1)) \cap G_B(f(x_2))\}$
$= (G_B + G_B)(f(x))$
$\subseteq G_B(f(x))$
$= f^{-1}(G_B)(x)$.
Thus, $f^{-1}(G_B) + f^{-1}(G_B) \sqsubseteq f^{-1}(G_B)$.
Next let $t \in K$ and $x \in X$. We now show that $tf^{-1}(G_B)(x) \subseteq f^{-1}(G_B)(x)$.
Case 1 : If $t \neq 0$ then
$tf^{-1}(G_B)(x) = f^{-1}(G_B)(t^{-1}x) = G_B(f(t^{-1}x)) = G_B(t^{-1}f(x)) = tG_B(f(x)) \subseteq G_B(f(x)) = f^{-1}(G_B)(x)$.
Case 2 : If $t = 0$ and $x \neq 0$ then it is trivial.
Case 3 : If $t = 0$ and $x = 0$ then
$0f^{-1}(G_B)(0) = \cup_{x\in X} f^{-1}(G_B)(x) = \cup_{x\in X} G_B(f(x)) \subseteq \cup_{y\in Y} G_B(y) = 0G_B(0) \subseteq$

$G_B(0) = G_B(f(0)) = f^{-1}(G_B)(0)$.

Hence $f^{-1}(G_B)$ *is a soft vector space over* (U, X).

5. Soft Product Topology

Definition 5.1. Let $\{F_{A_i} : i \in I\}$ be an indexed family of soft sets over (U, E). Then the soft product of these family of soft sets is denoted by $\prod_{i \in I} F_{A_i}$ and is defined by

$$\prod_{i\in I} F_{A_i}(x) = \begin{cases} \cap_{i\in I} F_{A_i}(x_i) & if\ x \in \prod_{i\in I} A_i, \\ \emptyset & otherwise, \end{cases}$$

where x_i is the i-th component of x.

Definition 5.2. Let $\{F_{A_i} : i \in I\}$ be an indexed family of soft sets and $F_A = \prod_{i\in I} F_{A_i}$. A box in F_A is a subset G_B of F_A of the form $\prod_{i\in I} G_{B_i}$ where $G_{B_i} \sqsubseteq F_{A_i}$, for all $i \in I$. For $j \in I$, G_{B_j} is called the $j-$th side of the box G_B.

A box G_B is said to be large box if all except finitely many of its sides are equal to the respective sets F_{A_i}'s.

Definition 5.3. Let $f : E_1 \to E_2$ be a mapping and F_A be a soft sets over (U, E_1). Then by the Definition 2.7, the mapping $f : E_1 \to E_2$ can be extended to the soft mapping $f : F_A \to F_B$ where F_B is a soft set over (U, E_2) containing $f(F_A)$. The extended soft mapping $f : F_A \to F_B$ is said to be a soft mapping induced by the mapping $f : E_1 \to E_2$.

Definition 5.4. Let $\{F_{A_i} : i \in I\}$ be an indexed family of soft sets. A wall $G_B \sqsubseteq \prod_{i\in I} F_{A_i}$ corresponding to the soft set $G_{B_j} \sqsubseteq F_{A_j}$ for some $j \in I$ is a set of the form $\prod_{i\in I} F_{B_i}$ where $F_{B_i} = F_{A_i}$ for $i \neq j$ and $F_{B_j} = G_{B_j}$.

The wall corresponding to the soft set G_{B_j} is denoted by $wall(G_{B_j})$.

Definition 5.5. Let $\{F_{A_i} : i \in I\}$ and $\{F_{B_i} : i \in I\}$ be two indexed family of soft sets. Then the soft union of $\prod_{i\in I} F_{A_i}$ and $\prod_{i\in I} F_{B_i}$ is defined by the soft set $\prod_{i\in I} F_{C_i}$, where $F_{C_i} = F_{A_i} \sqcup F_{B_i}$ for all $i \in I$.

The soft intersection of $\prod_{i \in I} F_{A_i}$ and $\prod_{i \in I} F_{B_i}$ is defined by the soft set $\prod_{i \in I} F_{C_i}$, where $F_{C_i} = F_{A_i} \sqcap F_{B_i}$ for all $i \in I$.

Theorem 5.1. *Let $\{(F_{A_i}, \tau_i) : i \in I\}$ be an indexed collection of soft topological spaces. Then the family of all wall corresponding to the soft sets $G_{B_i} \in \tau_i, i \in I$ forms a soft subbase for some topology τ on $\prod_{i \in I} F_{A_i}$.*

Proof. *Let $S = \{wall(G_{B_i}) : G_{B_i} \in \tau_i, i \in I\}$. So, it is now enough to show that S forms a soft subbase for some topology τ. Since the empty soft set $\Phi_i \in \tau_i$ for each $i \in I$, $wall(\Phi_i) \in S$ and obviously, for each $i \in I$, $wall(\Phi_i)$ forms a empty soft subset of $\prod_{i \in I} F_{A_i}$. So, S contains the empty soft subset of $\prod_{i \in I} F_{A_i}$. Again $wall(G_{B_j}) \sqcup wall(G_{B_k}) = \prod_{i \in I} F_{A_i}$ for $j \neq k$. So, $\sqcup_{i \in I} wall(G_{B_i}) = \prod_{i \in I} F_{A_i}$. Hence S forms a soft subbase for some topology τ.*

Definition 5.6. Let $\{(F_{A_i}, \tau_i) : i \in I\}$ be an indexed collection of soft topological spaces. The soft topology generated by $S = \{wall(G_{B_i}) : G_{B_i} \in \tau_i, i \in I\}$ as a subbase is called the soft product topology on $\prod_{i \in I} F_{A_i}$.

Theorem 5.2. *Let $\{(F_{A_i}, \tau_i) : i \in I\}$ be an indexed collection of soft topological spaces and τ be the soft product topology on the soft set $\prod_{i \in I} F_{A_i}$. Then the family of all large boxes with all its sides are soft open in their respective spaces is a soft base for the soft product topology τ on $\prod_{i \in I} F_{A_i}$.*

Proof. *Since the set $S = \{wall(G_{B_i}) : G_{B_i} \in \tau_i, i \in I\}$ forms a soft subbase for soft product topology τ on $\prod_{i \in I} F_{A_i}$, the collection of all finite intersection of members of S forms a soft base. Now $wall(G_{B_i}) = \prod_{j \in I} F_{B_j}$, where $F_{B_j} = F_{A_j}$ for $j \neq i$ and $F_{B_i} = G_{B_i}$. So, $wall(G_{B_i})$ forms a large box whose all sides are soft open in their respective spaces and also all sides are equal to the respective spaces except $i-th$ side. That is, any finite intersection of members of S forms a large box whose all sides are soft open.*

We now show that any large box whose all sides are soft open in their respective spaces *is a finite intersection of members of S.*

Let $G_B = \prod_{i \in I} G_{B_i}$ be a large box with $G_{B_i} \in \tau_i$ for all $i \in I$ and $G_{B_i} = F_{A_i}$ except for $i = i_1, i_2, \cdots, i_n$. Then $G_B = \sqcap_{k=1}^{n} wall(G_{B_{i_k}})$. This completes the proof of the theorem.

Definition 5.7. Let F_A be a soft set over (U, E), where E is a usual vector space over the field K and $P \subseteq K$. Then we define the soft set $P \times F_A$ over $(U,\, K \times E)$ as follows:

$P \times F_A = 1_P \times F_A$ where $1_P(e) = \begin{cases} U & if\ e \in P \\ \emptyset & if\ e \notin P. \end{cases}$

So, $(P \times F_A)(t, e) = (1_P \times F_A)(t, e) = \begin{cases} F_A(e) & if\ t \in P, \\ \emptyset & if\ t \notin P, \end{cases}$

where $(t,\, e) \in K \times E$.

Example 5.1. *Let the universal set $U = \mathbb{Z}$ be the set of all integers , the parameter set $E = \mathbb{R}$ be a euclidean vector space and $A = \mathbb{Z}$. Also let F_A be a soft set over (U, E) defined by*

$F_A(n) = \begin{cases} [n] & if\ n \in \mathbb{Z}, \\ \emptyset & if\ n \notin \mathbb{Z}, \end{cases}$

where $[n]$ denotes the set of all integers which are congruent with n modulo 5.

Let P be the set of all natural numbers. Then the soft set $P \times F_A$ over $(U, \mathbb{R} \times E)$ is

$(P \times F_A)(t, n) = \begin{cases} [n] & if\ (t,\, n) \in P \times \mathbb{Z}, \\ \emptyset & otherwise. \end{cases}$

Theorem 5.3. *If $(F_A,\, \tau)$ is soft topological space over (U, E), where E is a usual vector space over the field K and σ is a topology on K then the collection $\{P \times G_B : P \in \sigma,\, G_B \in \tau\}$ forms a soft product topology on $K \times F_A$, where 1_P belongs to the soft topology on 1_K iff $P \in \sigma$.*

6. Soft Topological Vector Spaces

In this section, we assume that the parameter sets E, E_1 and E_2 are usual vector spaces over the field K and U, a universal set.

Definition 6.1. If F_A is a soft set over (U, E) then a point $(x, u) \in E \times U$ is said to be a soft point of F_A if $u \in F_A(x)$. If (x, u) is a soft point of F_A then we write $(x, u) \in F_A$. Here E is not necessarily a vector space.

Definition 6.2. If F_A is a soft set over (U, E), where E is a vector space over the field K and $t \in K$, $(x, u) \in F_A$ then $(t, (x, u))$ is said to be a soft point of $K \times F_A$ and we write $(t, (x, u)) \in K \times F_A$.

Proposition 6.1. *If $f : F_A \to F_B$ is a soft mapping induced by the mapping $f : E_1 \to E_2$ then the image of a soft point (x, u) of F_A is a soft point $(f(x), u)$ of F_B where F_A, F_B are two soft sets over (U, E_1) and (U, E_2) respectively.*

Proof. *Let (x, u) be a soft point of F_A. This soft point is equivalent to a soft set P_x^u where* $P_x^u(e) = \begin{cases} \{u\} & if\ e = x \\ \emptyset & if\ e \neq x. \end{cases}$

Therefore $f(x, u)(e) = f(P_x^u)(e) = \cup_{e_1 \in f^{-1}(e)} P_x^u(e_1) = \begin{cases} \{u\} & if\ e = f(x) \\ \emptyset & if\ e \neq f(x). \end{cases}$

So, $f(P_x^u) = (f(x), u)$. Hence the image of a soft point (x, u) of F_A is a soft point $(f(x), u)$ of F_B.

Proposition 6.2. *If $f : K \times F_A \to F_B$ is a soft mapping induced by the mapping $f : K \times E \to E$, defined by $f(t, x) = tx$ then the image of a soft point $(t, (x, u))$ of $K \times F_A$ is a soft point $(f(t, x), u)$ of F_B where F_A, F_B are two soft sets over (U, E).*

Proof. *Let $(t, (x, u))$ be a soft point of $K \times F_A$. The soft point (x, u) is equivalent to a soft set P_x^u where* $P_x^u(e) = \begin{cases} \{u\} & if\ e = x \\ \emptyset & if\ e \neq x. \end{cases}$

If $t \neq 0$ *then* $f(t,(x,u))(e) = f(t, P^u_x)(e)$

$= \cup_{(t,e_1)\in f^{-1}(e)} P^u_x(e_1) = \begin{cases} \{u\} & if\ e = f(t,x) \\ \emptyset & if\ e \neq f(t,x). \end{cases}$

So, $f(t,(x,u)) = (f(t,x),u)$.

Again if $t = 0$ *then* $f(0,(x,u))(e) = f(0, P^u_x)(e) = 0P^u_x(e) = \begin{cases} \{u\} & if\ e = 0 \\ \emptyset & if\ e \neq 0. \end{cases}$

So, $f(0,(x,u)) = (0,u) = (f(0,x),u)$. *Hence the image of a soft point* $(t,(x,u))$ *of* F_A *is a soft point* $(f(t,x),u)$ *of* F_B.

Definition 6.3. If (F_A, τ) is a soft topological space and $H_C \sqsubseteq F_A$ then H_C is said to be a soft neighbourhood of a soft point (x,u) of F_A if there exists $G_B \in \tau$ such that $(x,u) \in G_B \sqsubseteq H_C$.

Definition 6.4. Let $f : F_A \to F_B$ be a soft mapping induced by the mapping $f : E_1 \to E_2$ where (F_A, τ_1), (F_B, τ_2) be two soft topological spaces over (U, E_1) and (U, E_2) respectively. Then the soft mapping f is said to be soft continuous on F_A if for any $G_{B_1} \in \tau_2$, $f^{-1}(G_{B_1}) \in \tau_1$.

Definition 6.5. Let $f : F_A \to F_B$ be a soft mapping induced by the mapping $f : E_1 \to E_2$ where (F_A, τ_1), (F_B, τ_2) be two soft topological spaces over (U, E_1) and (U, E_2) respectively. Then the soft mapping f is said to be soft continuous at a soft point (x,u) of F_A if for any $G_D \in \tau_2$ with $(f(x),u) \in G_D$, there exists $G_C \in \tau_1$ with $(x,u) \in G_C$ such that $f(G_C) \sqsubseteq G_D$

Theorem 6.1. *If* $f : F_A \to F_B$ *is a soft mapping induced by the mapping* $f : E_1 \to E_2$ *where* (F_A, τ_1), (F_B, τ_2) *are two soft topological spaces over* (U, E_1) *and* (U, E_2) *respectively then* f *is soft continuous at every soft point of* F_A *if and only if* f *is soft continuous on* F_A.

Proof. *At first we assume that* f *is soft continuous at every soft point of* F_A *and* $G_{B_1} \in \tau_2$. *Let the soft point* $(x,u) \in f^{-1}(G_{B_1})$. *Then* $f(x,u) = (f(x),u) \in$

G_{B_1}. *Since* f *is soft continuous, there exists* $G_{A_x^u} \in \tau_1$ *with* $(x,u) \in G_{A_x^u}$ *such that* $f(G_{A_x^u}) \sqsubseteq G_{B_1}$, *that is* $G_{A_x^u} \sqsubseteq f^{-1}(G_{B_1})$. *Therefore* $\bigsqcup_{(x,u)\in f^{-1}(G_{B_1})} G_{A_x^u} = f^{-1}(G_{B_1})$. *since each* $G_{A_x^u} \in \tau_1$, $\bigsqcup_{(x,u)\in f^{-1}(G_{B_1})} G_{A_x^u} \in \tau_1$. *Hence* $f^{-1}(G_{B_1}) \in \tau_1$.

Conversely, let f *be soft continuous on* F_A *and* $(x,u) \in F_A$. *We now show that* f *is soft continuous at the soft point* (x,u). *Let* $(f(x),u) \in G_C \in \tau_2$. *Since* f *is soft continuous,* $f^{-1}(G_C) \in \tau_1$. *Now* $(f(x),u) \in G_C$ *implies that* $(x,u) \in f^{-1}(G_C)$. *So* $(f(x),u) \in G_C$ *implies that there exists* $f^{-1}(G_C) \in \tau_1$ *with* $(x,u) \in f^{-1}(G_C)$ *such that* $f(f^{-1}(G_C)) \sqsubseteq G_C$. *Thus* f *is soft continuous at* (x,u). *Since* (x,u) *is an arbitrary soft point of* F_A, f *is soft continuous at every soft point of* F_A.

Definition 6.6. Let (F_A, τ_1) be a soft topological space over (U,E) and $f : E \times E \to E$ be a mapping defined by $f(x,y) = x+y$. Then the soft mapping $f : F_A \times F_A \to F_B$, induced by the mapping f where (F_B, τ_2) is a soft topological space over (U,E), is said to be soft continuous with respect to the first variable separately if for any soft point $(x,u) \in F_A$ and for any $G_D \in \tau_2$ with $(f(x,y),u) \in G_D$, there exists $G_C \in \tau_1$ with $(x,u) \in G_C$ such that $f(G_C, y) = y + G_C \sqsubseteq G_D$, where y is an arbitrary but a fixed element of E. Similarly, we can defined the soft continuous with respect to the second variable separately.

Definition 6.7. Let (F_A, τ_1) and (F_B, τ_2) be two soft topological spaces over (U, E_1) and (U, E_2) respectively. Also let (K, σ) be a topological space and $f : K \times E_1 \to E_2$ be a mapping defined by $f(t,x) = tx$. Then the soft mapping $f : K \times F_A \to F_B$, induced by the mapping f is said to be soft continuous at a soft point $(t,(x,u)) \in K \times F_A$ if for any $G_D \in \tau_2$ with $(f(t,x),u) \in G_D$, there exist $P \in \sigma$ containing t and $G_C \in \tau_1$ with $(x,u) \in G_C$ such that $f(P \times G_C) \sqsubseteq G_D$.
Thus following the Theorem 6.1, we have : If f is soft continuous at every point of $K \times F_A$ then f is said to be a soft continuous on $K \times F_A$.

Definition 6.8. Let (F_A, τ_1) and (F_B, τ_2) be two soft topological spaces over (U, E_1) and (U, E_2) respectively. Then the soft mapping $f : F_A \to F_B$, induced by the mapping $f : E_1 \to E_2$ is said to be soft open or open if for any $G_C \in \tau_1$, $f(G_C) \in \tau_2$.

Suppose F_A is a soft vector space over (U, E) and
$g : E \times E \to E$ defined by $g(x, y) = x + y$ and
$h : K \times E \to E$ defined by $h(t, x) = tx$. Then the image $g(F_A \times F_A)$ of the product soft set $F_A \times F_A$ is a soft set where
$g(F_A \times F_A)(x)$
$= \cup_{(y,z)\in g^{-1}(x)}\{F_A(y) \cap F_A(z)\}$
$\subseteq \cup_{(y,z)\in g^{-1}(x)} F_A(y + z)$, [by the Theorem 4.1]
$= F_A(x)$ for all $x \in E$.
Hence $g(F_A \times F_A) \sqsubseteq F_A$.
Again to find the image $h(K \times F_A)$ of the product soft set $K \times F_A$, we shall prove the following result:

Result 6.1. *Let F_A be a soft set over (U, E), where E is a vector space over the field K and $h : K \times E \to E$ defined by $h(t, x) = tx$. Then $h(P \times F_A)(x) = \cup_{t\in P} tF_A(x)$ for all $x \in E$ and $P \subseteq K$.*

Proof. *We shall prove the result by the following cases:*
Case 1: If $0 \notin P$, then
$h(P \times F_A)(x) = \cup_{(t,\, y)\in h^{-1}(x),\, t\in P}(P \times F_A)(t,\, y) = \cup_{(t,\, y)\in h^{-1}(x),\, t\in P} F_A(y)$
$= \cup_{(t,\, y)\in h^{-1}(x),\, t\in P} tF_A(ty)$
$= \cup_{t\in P} tF_A(x)$.
Case 2: If $0 \in P$,
Subcase 1: $x \neq 0$, then
$h(P \times F_A)(x) = \cup_{(t,\, y)\in h^{-1}(x),\, t(\neq 0)\in P}(P \times F_A)(t,\, y) = \cup_{(t,\, y)\in h^{-1}(x),\, t(\neq 0)\in P} F_A(y)$
$= \cup_{(t,\, y)\in h^{-1}(x),\, t(\neq 0)\in P} tF_A(ty)$

$= \cup_{t(\neq 0)\in P} tF_A(x)$

$= \cup_{t\in P} tF_A(x)$ *as* $0F_A(x) = \emptyset$ *for* $x \neq 0$.

Subcase 2: $x = 0$, *then*

$h(P \times F_A)(0) = \cup_{(t,\, y)\in h^{-1}(0),\, t\in P}(P \times F_A)(t,\, y) = \cup_{(t,\, y)\in h^{-1}(0),\, t\in P} F_A(y)$

$= \cup_{y\in E} F_A(y)$

$= 0F_A(0) = \cup_{t\in P} tF_A(0)$

Hence $h(P \times F_A)(x) = \cup_{t\in P} tF_A(x)$ *for all* $x \in E$.

So, from the result 6.1, we have $h(K\times F_A)(x) = \cup_{t\in K} tF_A(x) \subseteq \cup_{t\in K} F_A(x) = F_A(x)$ for all $x \in E$. Hence $h(K \times F_A) \sqsubseteq F_A$.

Therefore the induced mappings $g : F_A \times F_A \to F_A$ and $h : K \times F_A \to F_A$ are well defined.

Definition 6.9. A soft vector space F_A over (U, E) with a soft topology τ is said to be a soft topological vector space if

$i)$ the soft mapping $g : F_A \times F_A \to F_A$, induced by the mapping $g : E \times E \to E$ where $g(x, y) = x + y$ is soft continuous in each variable separately and

$ii)$ the soft mapping $h : K \times F_A \to F_A$, induced by the mapping $h : K \times E \to E$ where $h(t,\, x) = tx$ is soft continuous when K has a topology and $K \times F_A$ has given the soft product topology.

Example 6.1. *Let E be a vector space over the field K, A be a sub-vector space of E and (A, τ_A) be a topological vector space. Let $V \subseteq U$ and F_A be a soft set over (U, E) defined by* $F_A(e) = \begin{cases} V & if\ e \in A, \\ \emptyset & otherwise. \end{cases}$

Then it is easy to see that F_A is a soft vector space. For each $B \in \tau_A$, let us define a soft set G_B as $G_B(e) = \begin{cases} F_A(e) & if\ e \in B, \\ \emptyset & otherwise. \end{cases}$

Let $\upsilon = \{G_B : B \in \tau_A\}$. Then υ is a soft topology on F_A. So, it is enough to show that the mappings (i) g, defined in the Definition 6.9 is soft continuous from

$F_A \times F_A$ to F_A in each variable separately, (ii) h, defined in the Definition 6.9 is soft continuous from $K \times F_A$ to F_A where K has a topology σ.

Let $a \in E$, $(x,u) \in F_A$ and $(a+x,u) \in G_B \in \upsilon$. So, $u \in G_B(a+x)$. Obviously, $a+x \in B$ and $B \in \tau_A$. Since (A,τ_A) is topological sub-vector space, there exist $C, D \in \tau_A$ such that $a \in C$, $x \in D$ and $C+D \subseteq B$, that is, $a+D \subseteq B$. Since $x \in D$, $(x,u) \in G_D$. It is easy to see that $a+G_D \sqsubseteq G_B$. Thus, the mapping g is soft continuous from $F_A \times F_A$ to F_A w.r.t the second variable separately. Similarly it can be prove that the mapping g is soft continuous from $F_A \times F_A$ to F_A w.r.t the first variable separately.

Let $t \in K$, $(x,u) \in F_A$ and $(tx,u) \in G_B \in \upsilon$ then $u \in G_B(tx)$. Obviously, $tx \in B$ and $B \in \tau_A$. Since (A,τ_A) is topological sub-vector space, there exist $P \in \sigma$ containing t and $C \in \tau_A$ containing x such that $PC \subseteq B$. Since $C \in \tau_A$, $G_C \in \upsilon$ and $(x,u) \in G_C$. Now it is easy to see that $PG_C \sqsubseteq G_B$. Thus, the mapping h is soft continuous. Hence (F_A,υ) is a soft topological vector space.

Definition 6.10. Let $f : F_A \to F_B$ be a soft mapping induced by the mapping $f : E_1 \to E_2$ where (F_A,τ_1) and (F_B,τ_2) be two soft topological spaces over (U,E_1) and (U,E_2) respectively. Then the soft mapping f is said to be a soft homeomorphism if

i. $f : E_1 \to E_2$ is bijective.

ii. The soft mapping $f : F_A \to F_B$ is both soft open and soft continuous.

Theorem 6.2. *If (F_A, τ) is a soft topological vector space over (U,E) and $a \in E$ then the soft mapping $T_a : F_A \to F_A$, induced by the translation $T_a : E \to E$ where $T_a(x) = x + a$ is a soft homeomorphism.*

Proof. *It is clear that T_a is bijective.*

Since (F_A, τ) is a soft topological vector space over (U,E), T_a is soft continuous. Again, the inverse of T_a being T_{-a} defined by $T_{-a}(x) = x - a$. By similar argument

T_{-a} is also soft continuous on F_A. Let $G_B \in \tau$. Since the mapping T_{-a} is soft continuous, $T_{-a}^{-1}(G_B) \in \tau$.
Now $T_{-a}^{-1}(G_B)(x) = G_B(T_{-a}(x)) = G_B(x-a) = (a+G_B)(x)$ for all $x \in E$ (using Proposition 2.3). So, $a + G_B \in \tau$. Therefore the mapping T_a is soft open. Hence T_a is a soft homeomorphism on F_A.

Theorem 6.3. *If (F_A, τ) is a soft topological vector space over (U, E) and $t(\neq 0) \in K$ then the soft mapping $M_t : F_A \to F_A$, induced by the mapping $M_t : E \to E$ where $M_t(x) = tx$ is a soft homeomorphism.*

Proof. *It is clear that M_t is bijective.*
We now show that M_t is soft continuous on F_A. Let $(x, u) \in F_A$ and $(M_t(x), u) = (tx, u) \in G_D \in \tau$. Since the soft mapping h on F_A given in the Definition 6.9 is soft continuous, there exists $P \in \sigma$ containing t and $G_C \in \tau$ containing (x, u) such that $h(P \times G_C) \sqsubseteq G_D$, that is,
$h(P \times G_C)(e) \subseteq G_D(e)$ for all $e \in E$,,
or, $\cup_{p \in P} pG_C(e) \subseteq G_D(e)$ for all $e \in E$, by Result 6.1
or, $tG_C(e) \subseteq G_D(e)$ for all $e \in E$, as $t \in P$. Therefore, $tG_C \sqsubseteq G_D$, that is, $M_t(G_C) \sqsubseteq G_D$. Thus the soft mapping M_t is a soft continuous on F_A.
Again, the inverse of M_t being $M_{t^{-1}}$ defined by $M_{t^{-1}}(x) = t^{-1}x$. By similar argument $M_{t^{-1}}$ is also soft continuous on F_A. Let $G_B \in \tau$. Since the mapping $M_{t^{-1}}$ is soft continuous, $M_{t^{-1}}^{-1}(G_B) \in \tau$ by Definition 6.4.
Now $M_{t^{-1}}^{-1}(G_B)(e) = G_B(M_{t^{-1}}(e)) = G_B(t^{-1}e) = \cup_{y \in M_t^{-1}(e)} G_B(y) = M_t(G_B)(e)$ for all $e \in E$. So, $M_t(G_B) \in \tau$. Therefore the mapping M_t is open. Hence M_t is a soft homeomorphism on F_A.

Corollary 6.1. *In a soft topological vector space, every translation of a soft open set is a soft open set and any multiplication of a soft open set by a non-zero scalar is also a soft open set.*

Proposition 6.3. *If (F_A, τ) is a soft topological vector spaces over (U, E) then $F_A(x) = F_A(y)$ for all $x, y \in E$.*

Proof. From the Theorem 6.2, it is clear that for any $x \in E$ and $G_B \in \tau$, $x + G_B \in \tau$. Now since $F_A \in \tau$, $x + F_A \in \tau$ for all $x \in E$. Also it is obvious that $x + F_A \sqsubseteq F_A$ for all $x \in E$. Let $x, y \in E$. Then $(y - x) + F_A \sqsubseteq F_A$. So, $((y - x) + F_A)(y) \subseteq F_A(y)$, that is, $F_A(x) \subseteq F_A(y)$. Similarly, it can be shown that $F_A(y) \subseteq F_A(x)$. Thus, if (F_A, τ) is a soft topological vector spaces over (U, E) then $F_A(x) = F_A(y)$ for all $x, y \in E$.

Corollary 6.2. *If (F_A, τ) is a soft topological vector space over (U, E) and $a \in E$ then for any soft closed set G_B, $a + G_B$ is also soft closed sets.*

Proof. At first, let us consider the soft homeomorphism $T_a : F_A \to F_A$, induced by $T_a : E \to E$ where $T_a(x) = x + a$. Since G_B is closed, $F_A - G_B$ is open. So, $T_a(F_A - G_B)$ is open. Now for $x \in E$,
$T_a(F_A - G_B)(x) = \cup_{y \in T_a^{-1}(x)}(F_A - G_B)(y) = \cup_{y \in T_a^{-1}(x)}\{F_A(y) - G_B(y)\} = F_A(x-a) - G_B(x - a) = F_A(x) - (a + G_B)(x) = (F_A - (a + G_B))(x)$.
So, $F_A - (a + G_B)$ is open. That is, $a + G_B$ is soft closed set.

Corollary 6.3. *If (F_A, τ) is a soft topological vector space over (U, E) and $t(\neq 0) \in K$ then for any soft closed set G_B, tG_B is also soft closed sets.*

Proof. At first, let us consider the soft homeomorphism $M_t : F_A \to F_A$ induced by $M_t : E \to E$ where $M_t(x) = tx$. Since G_B is soft closed set, $F_A - G_B$ is soft open set. So, $M_t(F_A - G_B)$ is soft open. Now for $x \in E$,
$M_t(F_A - G_B)(x) = \cup_{y \in M_t^{-1}(x)}(F_A - G_B)(y) = \cup_{y \in M_t^{-1}(x)}\{F_A(y) - G_B(y)\} = F_A(t^{-1}x) - G_B(t^{-1}x) = F_A(x) - tG_B(x) = (F_A - tG_B)(x)$.
So, $F_A - tG_B$ is soft open. That is, tG_B is soft closed set.

Conclusion: During last few year, several researchers are trying to incorporate soft set theory in crisp mathematical analysis and as a result different notions are being grown up which are sometimes very much useful for making decision in different expert systems. In this paper, it is seen that soft open and closed sets remain invariant through translations and scalar multiplications which are also soft homomorphisms. In future it would help to establish several results of crisp topological vector space in soft topological vector space.

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