Jordan Journal of Mathematics and Statistics (JJMS) 11(2), 2018, pp 145 - 167

3-DIFFERENCE CORDIALITY OF SOME SPECIAL GRAPHS

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Abstract. Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \ldots, k\}$ be a map where k is an integer $2 \leq k \leq p$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph. In this paper we investigate 3-difference cordial labeling behavior of ladder, book, dumbbell graph, and umbrella graph.

1. Introduction

All graphs in this paper are finite, simple and undirected. Let G be a (p, q) graph where p refers the number of vertices of G and q refers the number of edge of G. The number of vertices of a graph G is called order of G, and the number of edges is called size of G. Let G_1 and G_2 be two graphs with vertex sets V_1 and V_2 and edge sets E_1 and E_2, respectively. Then their join $G_1 + G_2$ is the graph whose vertex set is $V_1 \cup V_2$ and edge set is $E_1 \cup E_2 \cup \{uv : u \in V_1 \text{ and } v \in V_2\}$. The cartesian product of two graphs G_1 and G_2 is the graph $G_1 \times G_2$ with the vertex set $V_1 \times V_2$ and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent whenever $[u_1 = v_1$ and u_2 adj $v_2]$ or $[u_2 = v_2$ and u_1 adj $v_1]$. The graph $K_{1,n}$ is called the star.The notion of difference

2000 *Mathematics Subject Classification.* 05C78.

Key words and phrases. Path, ladder, book, dumbbell graph, umbrella graph.

Received: Sept. 8, 2017 Accepted: Jan. 8, 2018 .

cordial labeling was introduced by R. Ponraj, S. Sathish Narayanan and R. Kala in [3]. Seoud and Salman [12], studied the difference cordial labeling behavior of some families of graphs and they are ladder, triangular ladder, grid, step ladder and two sided step ladder graphs etc. Recently Ponraj et al. [4], introduced the concept of k-difference cordial labeling of graphs and studied the 3-difference cordial labeling behavior of of star, m copies of star etc. In [5, 6, 7, 8, 9, 10, 11] they discussed the 3-difference cordial labeling behavior of path, cycle, complete graph, complete bipartite graph, star, bistar, comb, double comb, quadrilateral snake, wheel, helms, flower graph, sunflower graph, lotus inside a circle, closed helm, double wheel, union of graphs with the star, union of graphs with splitting graph of star, union of graphs with subdivided star, union of graphs with bistar, $P_n \cup P_n$, $(C_n \odot K_1) \cup (C_n \odot K_1)$, $F_n \cup F_n$, $K_{1,n} \odot K_2$, $P_n \odot 3K_1$, mC_4, $spl(K_{1,n})$, $DS(B_{n,n})$, $C_n \odot K_2$, $C_4^{(t)}$, $S(K_{1,n})$, $S(B_{n,n})$, $DA(T_n) \odot K_1$, $DA(T_n) \odot 2K_1$, $DA(T_n) \odot K_2$, $DA(Q_n) \odot K_1$, $DA(Q_n) \odot 2K_1$, triangular snake, alternate triangular snake, alternate quadrilateral snake, irregular triangular snake, irregular quadrilateral snake, double triangular snake, double quadrilaterla snake, double alternate triangular snake, and double alternate quadrilateral snake, $T_n \odot K_1$, $T_n \odot 2K_1$, $T_n \odot K_2$, $A(T_n) \odot K_1$, $A(T_n) \odot 2K_1$, $A(T_n) \odot K_2$, slanting ladder, book with triangular pages, middle graph of a path, shadow graph of a path, triangular ladder, the armed crown, and some more graphs. In this paper we investigate 3-difference cordial labeling behavior of ladder, book, dumbbell graph, and umbrella graph. Terms not defined here follow from Harary [2] and Gallian [1].

2. k-Difference cordial labeling

Definition 2.1. Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \ldots, k\}$ be a map. For each edge uv, assign the label $|f(u) - f(v)|$. f is called a k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph.

The graph $L_n = P_n \times P_2$ is called a ladder.

Theorem 2.1. *The ladder L_n is 3-difference cordial for all n.*

Proof. Let $V(L_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(L_n) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i : 1 \leq i \leq n\}$. Clearly, the order and size of L_n is 2n and 3n-2.

Case 1. $n \equiv 0 \pmod 4$.

Assign the label 1 to the vertices u_{12i+1}, u_{12i+3} and u_{12i+5} for all the values of i=0,1,2,...., $\lceil \frac{n}{12} \rceil - 1$. For all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$ assign the label 3 to the vertices u_{12i+2}, u_{12i+4}, u_{12i+6}, u_{12i+9}, u_{12i+10} and u_{12i+11}. Then assign the label 1 to the vertices u_{12i} for i=1,2,.....,$\lceil \frac{n}{12} \rceil - 1$. Now we assign the label 2 to the vertices u_{12i+7} and u_{12i+8} for all the values of i=0,1,2,..... $\lceil \frac{n}{12} \rceil - 1$.Now we move to the vertices v_i. Assign the label 2 to the vertices $v_{12i+1}, v_{12i+3}, v_{12i+5}, v_{12i+9}, v_{12i+10}$ and v_{12i+11} for all the values of i=0, 1, 2,....,x and assign the label 3 to the vertices v_{12i+4} and v_{12i+6} for i=0,1,2,3,...,$\lceil \frac{n}{12} \rceil - 1$. For all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$, assign the label 1 to the vertices v_{12i+2}, v_{12i+7} and v_{12i+8}. Then assign the label 1 to the vertices v_{12i} for i=1,2,3,....,$\lceil \frac{n}{12} \rceil - 1$. Clearly the vertex condition is given in table 1 and the edge condition is $e_f(0) = e_f(1) = \frac{3n-2}{2}$.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod{12}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 4 \pmod{12}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$
$n \equiv 8 \pmod{12}$	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$

TABLE 1

Case 2. $n \equiv 1 \pmod 4$.

Assign the label 2 to the vertex u_1. Then assign the label 3 to the vertices u_{12i+2}, u_{12i+3}, u_{12i+4}, u_{12i+7}, u_{12i+9} and u_{12i+11} for all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 2 to the vertices u_{12i} and u_{12i+1} for i=1,2,....,$\lceil \frac{n}{12} \rceil - 1$. For all the values of i=0,1,2,...,$\lceil \frac{n}{12} \rceil - 1$, assign the label 1 to the vertices $u_{12i+5}, u_{12i+6}, u_{12i+8}$ and u_{12i+10}. Next we move to the vertices v_i. Assign the label 1 to the vertex v_1. Then assign the label 2 to the vertices $v_{12i+2}, v_{12i+3}, v_{12i+4}, v_{12i+6}, v_{12i+8}$ and v_{12i+10} for all the values of i=0,1,2,....,x and we assign the label 3 to the vertices v_{12i+9} and v_{12i+11} for i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$. Now we assign the label 1 to the vertices v_{12i+5} and v_{12i+7} for i=0,1,2,.....,$\lceil \frac{n}{12} \rceil - 1$.For all the values of i=1,2,....,$\lceil \frac{n}{12} \rceil - 1$, assign the label 1 to the vertices v_{12i} and v_{12i+1}. Note that in this case $e_f(0) = \frac{3n-3}{2}$ and $e_f(1) = \frac{3n-1}{2}$ and the vertex condition is given in table 2.

Values of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 9 \pmod{12}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 1 \pmod{12}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$
$n \equiv 5 \pmod{12}$	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$

TABLE 2

Case 3. $n \equiv 2 \pmod 4$.

First we consider the vertices u_i. Fix the labels 1,3 to the vertices u_1, u_2 respectively. Then we assign the label 1 to the vertices u_{12i+3}, u_{12i+10} and u_{12i+11} for all the values of i=0,1,2,....,x and assign the label 1 to the vertices u_{12i+1} for i=1,2,....,$\lceil \frac{n}{12} \rceil - 1$. For all the values of i=1,2,....,$\lceil \frac{n}{12} \rceil - 1$, assign the label 2 to the vertices u_{12i+5} and u_{12i+6}. Now we assign the label 3 to the vertices $u_{12i+4}, u_{12i+7}, u_{12i+8}$ and u_{12i+9} for all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 1 to the vertices u_{12i} and u_{12i+2} for i=1,2,...,$\lceil \frac{n}{12} \rceil - 1$. Next we move to the vertices v_i. Assign the labels 2,3 to the vertices v_1, v_2 respectively. Then we assign the label 2 to the vertices $v_{12i+3}, v_{12i+7}, v_{12i+8}, v_{12i+9}$ and v_{12i+11} for i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label

2 to the vertices v_{12i+1} for i=1,2,....,$\lceil \frac{n}{12} \rceil - 1$. For all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$, assign the label 3 to the vertices v_{12i+4}. Now we assign the label 3 to the vertices v_{12i+2} for i=1,2,....,$\lceil \frac{n}{12} \rceil - 1$. Then we assign the label 1 to the vertices v_{12i+5}, v_{12i+6} and v_{12i+10} for all the values of i=0,1,2,.....,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 1 to the vertices v_{12i} for i=1,2,.....,$\lceil \frac{n}{12} \rceil - 1$. Clearly $e_f(0) = e_f(1) = \frac{3n-2}{2}$ and the vertex condition is given in table 3.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 6 \pmod{12}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 2 \pmod{12}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$
$n \equiv 10 \pmod{12}$	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$

TABLE 3

Case 4. $n \equiv 3 \pmod 4$.

Consider the vertices u_i. Assign the label 2 to the vertices u_1 and u_3 and we assign the label 3 to the vertex u_2. Then we assign the label 3 to the vertices $u_{12i+4}, u_{12i+5}, u_{12i+6}, u_{12i+9}$ and u_{12i+11} for all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 3 to the vertices u_{12i+1} for all the values of i=1,2,....,$\lceil \frac{n}{12} \rceil - 1$ .For all the values of i=1,2,....,$\lceil \frac{n}{12} \rceil - 1$, assign the label 2 to the vertices u_{12i+2} and u_{12i+3}. Now we assign the label 1 to the vertices u_{12i+7}, u_{12i+8} and u_{12i+10} for all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 1 to the vertices u_{12i} for i=1,2,.....,$\lceil \frac{n}{12} \rceil - 1$. Next we move to the vertices v_i. Assign the label 1 to the vertices v_1 and v_3 and we assign the label 3 to the vertex v_2. Then we assign the label 2 to the vertices $v_{12i+4}, v_{12i+5}, v_{12i+6}, v_{12i+8}$ and v_{12i+10} for all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 2 to the vertices v_{12i} for i=1,2,3,.....,$\lceil \frac{n}{12} \rceil - 1$. Now we assign the label 3 to the vertices v_{12i+11} for i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 3 to the vertices v_{12i+1} for all the values of i=1,2,.....,$\lceil \frac{n}{12} \rceil - 1$. For all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$,assign the label 1 to the vertices v_{12i+7} and v_{12i+9}. Finally assign the label 1 to the vertices v_{12i+2} and v_{12i+3} for all the values of i=1,2,....,$\lceil \frac{n}{12} \rceil - 1$. Clearly $e_f(0) = \frac{3n-3}{2}$ and $e_f(1) = \frac{3n-1}{2}$ and the vertex condition is given in table 4.

Values of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 3 \pmod{12}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 7 \pmod{12}$	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$
$n \equiv 11 \pmod{12}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$

TABLE 4

□

Example 2.1. A 3-difference cordial labeling of L_7 is given in figure 1.

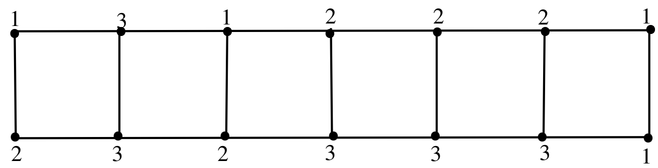

FIGURE 1

The book B_m is the graph $S_m \times P_2$ where S_m is the star with $m+1$ vertices.

Theorem 2.2. *If* $n \equiv 0, 1, 2 \pmod 4$*, the book graph is 3-difference cordial.*

Proof. Let G be a book with n pages. Let $V(G) = \{u, v\} \cup \{u_i, v_i : 1 \leq i \leq n\}$ and $E(G) = \{uv\} \cup \{uu_i, vu_i, u_iv_i : 1 \leq i \leq n\}$. Clearly, the order and size of the book B_n is 2n+2 and 3n+1. Assign the labels 1,2 to the vertices u, v respectively.

Case 1. $n \equiv 0 \pmod 4$.

First we consider the vertices u_i. Assign the label 1 to the vertices u_1, u_5 and u_7 and we assign the label 3 to the vertices u_2, u_3 and u_6. Then assign the label 2 to the vertices u_4 and u_8. For all the values of i=1,2,.....,$\lceil \frac{n}{12} \rceil - 1$, assign the label 1 to the vertices $u_{12i+1}, u_{12i+3}, u_{12i+5}$ and u_{12i+6}. Assign the label 1 to the vertices u_{12i+9} for all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$. Then assign the label 3 to the vertices u_{12i+10} for all the value of i=0,1,2,3,...,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 3 to the vertices u_{12i+2} for all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$. For all the values of i=1,2,.....,$\lceil \frac{n}{12} \rceil - 1$, assign the label 2 to the vertices $u_{12i}, u_{12i+4}, u_{12i+7}$ and u_{12i+8}. Now we assign the label 2 to the vertices u_{12i+11} for i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$ .Next we move to the vertices v_i. Assign the label 3 to the vertex v_1, v_2, v_5 and v_6 and we assign the label 2 to the vertices v_3 and v_7. Then assign the label 1 to the vertices v_4 and v_8. For all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$, assign the label 3 to the vertices $v_{12i+1}, v_{12i+2}, v_{12i+5}$ and v_{12i+8}. Assign the label 3 to the vertices v_{12i+9} and v_{12i+10} for i=0,1,2,.....,$\lceil \frac{n}{12} \rceil - 1$.Now we assign the label 2 to the vertices v_{12i+11} for all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 2 to the vertices v_{12i+3} and v_{12i+7} for i=1,2,.....,$\lceil \frac{n}{12} \rceil - 1$. For all the values of i=1,2,.... ,$\lceil \frac{n}{12} \rceil - 1$ assign the label 1 to the vertices v_{12i}, v_{12i+4} and v_{12i+6}.

Case 2. $n \equiv 1 \pmod 4$.

Assign the label 1 to the vertex u_2 and we assign the label 3 to the vertices u_3 and u_1. Then we assign the label 2 to the vertices u_4 and u_5. Now we assign the label 1 to the vertices u_{12i+6} and u_{12i+10} for all the values i=0,1,2,3,....,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 1 to the vertices u_{12i}, u_{12i+2} and u_{12i+3} for i=1,2,....,$\lceil \frac{n}{12} \rceil - 1$. For all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$ assign the label 3 to the vertices u_{12i+7} and u_{12i+11}. Then we assign the label 2 to the vertices u_{12i+8} and u_{12i+9} for i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 2 to the vertices u_{12i+1}, u_{12i+4} and u_{12i+5} for all the values i=1,2,3,....,$\lceil \frac{n}{12} \rceil - 1$. Next we move to the vertices v_i. Assign the label 3 to the vertex v_1, v_2 and v_3. Then we assign the label 2 to the vertex v_4 and assign the label 1 to the vertex v_5. Assign the label 3 to the vertices $v_{12i+6}, v_{12i+7}, v_{12i+10}$ and v_{12i+11} for all the values of i=0,1,2,.....,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 3 to the vertices v_{12i+2} and v_{12i+5} for i=1,2,.....,$\lceil \frac{n}{12} \rceil - 1$ .For all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$, assign the label 2 to the vertices v_{12i+8}. Then we assign the label 2 to the vertices v_{12i} and

v_{12i+4} for i=1,2,.....,x. Now we assign the label 1 to the vertices v_{12i+9} for all the values of i=0,1,2,.....,$\lceil \frac{n}{12} \rceil - 1$ and assign the label 1 to the vertices v_{12i+1} and v_{12i+3} for i=1,2,.....,$\lceil \frac{n}{12} \rceil - 1$.

Case 3. $n \equiv 2 \pmod 4$.

Assign the label 1 to the vertex u_1 and assign the label 3 to the vertex u_2. For all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$, assign the label 1 to the $u_{12i+3}, u_{12i+7}, u_{12i+9}$ and u_{12i+11}. Then we assign the label 1 to the vertices u_{12i} for i=1,2,....,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 3 to the vertices u_{12i+4} and u_{12i+8} for i=0,1,2,...,$\lceil \frac{n}{12} \rceil - 1$. Now we assign the label 2 to the vertices u_{12i+5}, u_{12i+6} and u_{12i+10} for i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 2 to the vertices u_{12i+1} and u_{12i+2} for all the values i=1,2,3,....,$\lceil \frac{n}{12} \rceil - 1$. Next we move to the vertices v_i. Assign the labels 3,2 to the vertex v_1, v_2 respectively. Then we assign the label 3 to the vertex $v_{12i+3}, v_{12i+4}, v_{12i+7}, v_{12i+8}$ and v_{12i+11} for all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$ and assign the label 3 to the vertex v_{12i+2} for all the values i=1,2,3,.....,$\lceil \frac{n}{12} \rceil - 1$. Assign the label 3 to the vertices $v_{12i+6}, v_{12i+7}, v_{12i+10}$ and v_{12i+11} for all the values of i=0,1,2,.....,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 3 to the vertices v_{12i+2} for i=1,2,.....,x. For all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$, assign the label 2 to the vertices v_{12i+8}. Then we assign the label 2 to the vertices v_{12i} and v_{12i+4} for i=1,2,.....,$\lceil \frac{n}{12} \rceil - 1$, assign the label 2 to the vertices v_{12i+5} and v_{12i+9}. Assign the label 2 to the vertices v_{12i+1} for i=1,2,.....,$\lceil \frac{n}{12} \rceil - 1$. Now we assign the label 1 to the vertices v_{12i+6} and v_{12i+10} for all the values of i=0,1,2,.....,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 1 to the vertices v_{12i} for all the values of i=1,2,.....,$\lceil \frac{n}{12} \rceil - 1$.

The vertex and edge conditions of all the above three cases are given in table 5 and table 6, respectively.

□

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0,6,9 \pmod{12}$	$\frac{2n}{3}$	$\frac{2n+3}{3}$	$\frac{2n+3}{3}$
$n \equiv 1,4,10 \pmod{12}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n+4}{3}$
$n \equiv 2,5,8 \pmod{12}$	$\frac{2n+2}{3}$	$\frac{2n+2}{3}$	$\frac{2n+2}{3}$

TABLE 5

Values of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 4$	$\frac{3n}{2}$	$\frac{3n+2}{2}$
$n \equiv 1 \pmod 4$	$\frac{3n+1}{2}$	$\frac{3n+1}{2}$
$n \equiv 2 \pmod 4$	$\frac{3n+2}{2}$	$\frac{3n}{2}$

TABLE 6

Example 2.2. A 3-difference cordial labeling of the book B_4 is given in figure 2.

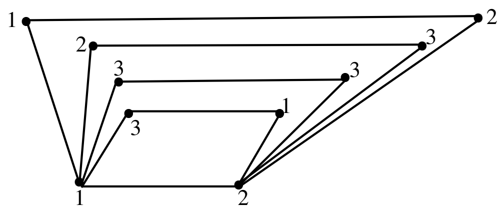

FIGURE 2

The graph obtained by joining two disjoint cycles $u_1u_2 \dots u_nu_1$ and $v_1v_2 \dots v_nv_1$ of same length with an edge u_1v_1 is called dumbbell graph Db_n.

Theorem 2.3. *The dumbbell graph Db_n is 3-difference cordial.*

Proof. It is clear that $|V(Db_n)| = 2n$ and $|E(Db_n)| = 2n + 1$.

Case 1. $n \equiv 0 \pmod 3$.

Assign the label 1 to the vertices u_{6i+1} for i=0,1,2,....,$\lceil \frac{n}{6} \rceil - 1$ and we assign the label 1 to the vertices u_{6i} for all the values of i=1,2,....,$\lceil \frac{n}{6} \rceil - 1$ .For all the values of i=0,1,2,....,$\lceil \frac{n}{6} \rceil - 1$, assign the label 3 to the vertices u_{6i+2} and u_{6i+5}. Now we assign the label 2 to the vertices u_{6i+3} and u_{6i+4} for all the values of i=0,1,2,....,$\lceil \frac{n}{6} \rceil - 1$.Next we move to the vertices v_i. Assign the label 3 to the vertices v_{6i+1} and v_{6i+5} for i=0,1,2,....,$\lceil \frac{n}{6} \rceil - 1$ and we assign the label 2 to the vertices v_{6i+2} for all the values of i=0,1,2,....,$\lceil \frac{n}{6} \rceil - 1$ .Then we assign the label 2 to the vertices v_{6i} for i=1,2,....,$\lceil \frac{n}{6} \rceil - 1$. For all the values of i=0,1,2,.....,$\lceil \frac{n}{6} \rceil - 1$, assign the label 1 to the vertices v_{6i+3} and v_{6i+4}.

Case 2. $n \equiv 1 \pmod 3$.

Assign the label 1 to the vertex u_1. Then we assign the label 1 to the vertices u_{6i+2} and u_{6i+5} for all the values of i=0,1,2,.....,$\lceil \frac{n}{6} \rceil - 1$ and we assign the label 1 to the vertices u_{6i} for all the values of i=1,2,....,$\lceil \frac{n}{6} \rceil - 1$. Now we assign the label 3 to the vertices u_{6i+3} for i=0,1,2,.... and assign the label 3 to the vertices u_{6i+1} for all the values of i=1,2,....,$\lceil \frac{n}{6} \rceil - 1$. Then assign the label 2 to the vertices u_{6i+4} for i=1,2,.....,$\lceil \frac{n}{6} \rceil - 1$. Next we move to the vertices v_i. Assign the label 2 to the vertex v_1. Then assign the label 1 to the vertices v_{6i+2} for i=0,1,2,....,$\lceil \frac{n}{6} \rceil - 1$ and assign the label 3 to the vertices v_{6i+3} and v_{6i+5} for all the values of i=0,1,2,....,$\lceil \frac{n}{6} \rceil - 1$. Now we assign the label 2 to the vertices v_{6i+4} for all the values of i=0,1,2,....,$\lceil \frac{n}{12} \rceil - 1$. For all the values of i=1,2,....,$\lceil \frac{n}{6} \rceil - 1$, assign the label 2 to the vertices v_{6i} and v_{6i+1}.

Case 3. $n \equiv 2 \pmod 3$.

First we consider the vertices u_i. Assign the label 1 to the vertices u_1 and u_2. Then assign the label 3 to the vertices u_3 and u_6 and we assign the label 2 to the vertices u_4 and u_7. Now we assign the label 1 to the vertices u_5 and u_8. Assign the label 3 to the vertices u_{6i+5} for all the values of i=1,2,....,$\lceil \frac{n}{6} \rceil - 1$ and we assign the label 1 to the vertices u_{6i+3} and u_{6i+4} for i=1,2,3.....,$\lceil \frac{n}{6} \rceil - 1$. For all the values of i=2,3,....,$\lceil \frac{n}{6} \rceil - 1$, assign the label 1 to the vertices u_{6i} and u_{6i+1}. Then assign the label 2 to the vertices u_{6i+2} for i=2,3.....,$\lceil \frac{n}{6} \rceil - 1$. Next we move to the vertices v_i. Assign the label 2 to the vertices v_1, v_5 and v_8 and we assign the label 3 to the vertices v_2, v_4 and v_7. Then we assign the label 1 to the vertices v_3 and v_6. For all the values of i=1,2,....,$\lceil \frac{n}{6} \rceil - 1$, assign the label 2 to the vertices v_{6i+3} and v_{6i+4}. Assign the label 2 to the vertices v_{6i} for all the values of i=2,3,....,$\lceil \frac{n}{6} \rceil - 1$ and we assign the label 3 to the vertices v_{6i+5} for i=1,2,3.....,x. Now we assign the label 3 to the vertices v_{6i+1} and v_{6i+2} for all the values of i=2,3,4,.....,$\lceil \frac{n}{6} \rceil - 1$. Clearly the vertex and edge conditions are given in table 7 and table 8.

Values of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 3$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 1 \pmod 3$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$
$n \equiv 2 \pmod 3$	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$

TABLE 7

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0, 1, 5 \pmod 6$	$n+1$	n
$n \equiv 2, 3, 4 \pmod 4$	n	$n+1$

TABLE 8

□

Example 2.3. A 3-difference cordial labeling of the dumbbell Db_5 is given in figure 3.

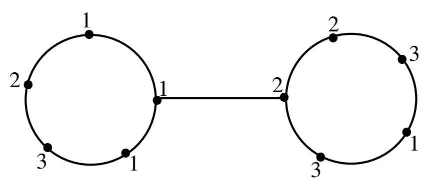

FIGURE 3

The graph $F_n = P_n + K_1$ is called a fan where $P_n : u_1u_2 \ldots u_n$ be a path and $V(K_1) = \{u\}$. The Umbrella $U_{n,m}$, $m > 1$ is obtained from a fan F_n by pasting the end vertex of the path $P_m : v_1v_2 \ldots v_m$ to the vertex of K_1 of the fan F_n.

Theorem 2.4. *The umbrella $U_{n,m}$, $m > 1$ is 3-difference cordial.*

Proof. It is clear that $U_{n,m}$ has n+m vertices and 2n+m-2 edges.

Case 1. $n \equiv 0 \pmod 6$ and $m \equiv 0 \pmod 3$.

First we consider the vertices u_i. Assign the labels 1,3,2,3,2,1 to the first six vertices $u_1, u_2, u_3, u_4, u_5, u_6$ of the path P_n. Then assign the labels 1,3,2,3,2,1 to the next six vertices $u_7, u_8, u_9, u_{10}, u_{11}, u_{12}$ respectively. Proceeding like this assign the label to the next six vertices and so on. Note that the vertex u_n received the label 1. Now our attention turn to the vertices v_i. Assign the label 1,3,2,2,3,1 to the first six vertices $v_1, v_2, v_3, v_4, v_5, v_6$ respectively. Then we assign the labels 1,3,2,2,3,1 to the next six vertices $v_7, v_8, v_9, v_{10}, v_{11}, v_{12}$ respectively. Continuing like this until we reach the vertex v_n. It is easy to verify that the last vertex v_n received the label 2 or 1 according as $n \equiv 3 \pmod 6$ or $n \equiv 0 \pmod 6$. Clearly the vertex condition is $v_f(1) = v_f(2) = v_f(3) = \frac{n+m}{3}$ and the edge condition is given in table 9.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 6$ & $m \equiv 0 \pmod 3$ & $n = 2m$	$\frac{2n+m-3}{2}$	$\frac{2n+m-1}{2}$
$n \equiv 0 \pmod 6$ & $m \equiv 0 \pmod 3$ & $n \neq 2m$	$\frac{2n+m-1}{2}$	$\frac{2n+m-3}{2}$

Table 9

Case 2. $n \equiv 0 \pmod 6$ and $m \equiv 1 \pmod 3$.

As in case 1, assign the label to the vertices u_i $(1 \leq i \leq n)$. Now we move to the vertices v_i. Fix the labels 1,3,1 to the first three vertices v_1, v_2, v_3 respectively. Assign the labels 2 to the vertices u_{12i+4} and u_{12i+7} for all the values of i=0,1,2,......,x and assign the label 2 to the vertices v_{12i} and v_{12i+3} for all the values of i=1,2,......,$\lceil \frac{n}{12} \rceil - 1$. For all the values of i=0,1,2,......,$\lceil \frac{n}{12} \rceil - 1$, assign the label 3 to the vertices v_{12i+5}, v_{12i+8} and v_{12i+11}. Now we assign the label 1 to the vertices v_{12i+6}, v_{12i+9} and v_{12i+10} for i=0,1,2,......,$\lceil \frac{n}{12} \rceil - 1$ and assign the label 1 to the vertices v_{12i+1} for all the values of i=1,2,......,$\lceil \frac{n}{12} \rceil - 1$. For all the values of i=1,2,......,$\lceil \frac{n}{12} \rceil - 1$, assign the label 3 to the vertices v_{12i+2}. Therefore the vertex condition is $v_f(1) = \frac{n+m+2}{3}$ and $v_f(2) = v_f(3) = \frac{n+m-1}{3}$ and the edge condition is given in table 10.

Nature of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 6$, $m = 6t_2 + 4$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$
$n = 12t_1$, $m = 6t_2 + 1$	$\frac{2n+m-3}{2}$	$\frac{2n+m-1}{2}$

TABLE 10

Case 3. $n \equiv 0 \pmod 6$ and $m \equiv 2 \pmod 3$.

Label to the vertices u_i $(1 \leq i \leq n)$ as in case 1. Now we consider to the vertices v_i. Assign the label 1 to the vertices v_1, v_5, v_6 and we assign the label 3 to the vertices v_2, v_3 and v_7. Then we assign the label 2 to the vertices v_4 and v_8. Assign the label 1 to the vertices v_{12i+9} for all the values of i=0,1,2,.......,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 1 to the vertices v_{12i+2}, v_{12i+5} and v_{12i+8} for i=1,2,...... Next we assign the label 3 to the vertices v_{12i+10} for i=0,1,2,......,$\lceil \frac{n}{12} \rceil - 1$ and assign the label 3 to the vertices v_{12i+1}, v_{12i+3} and v_{12i+7} for all the values of i=1,2,........,$\lceil \frac{n}{12} \rceil - 1$. Then assign the label 2 to the vertices v_{12i+11} for i=0,1,2,......,$\lceil \frac{n}{12} \rceil - 1$. For all the values of i=1,2,........,$\lceil \frac{n}{12} \rceil - 1$, assign the label 2 to the vertices v_{12i}, v_{12i+4} and v_{12i+6}. Clearly the vertex condition is $v_f(1) = v_f(3) = \frac{n+m+1}{3}$ and $v_f(2) = \frac{n+m-2}{3}$ and the edge condition is given in table 11.

Case 4. $n \equiv 1 \pmod 6$ and $m \equiv 0 \pmod 3$.

First we consider the vertices u_i. Fix the label 1 to the first path vertex u_1. Now we assign the labels 3,2,1,2,3,1 to the path vertices $u_2, u_3, u_4, u_5, u_6, u_7$ respectively. Then we assign the labels 3,2,1,2,3,1 to the next six path vertices $u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}$

Valuues of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 6,\ m = 6t_2 + 5$	$\frac{2n+m-3}{2}$	$\frac{2n+m-1}{2}$
$n = 12t_1,\ m = 6t_2 + 2$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$

Table 11

respectively. Continuing like this we assign the label to the next six vertices and so on. Note that the vertex u_n received the label 1. Now our attention turn to the vertices v_i. Assign the label 1 to the vertices v_{12i+1}, v_{12i+4} and v_{12i+9} for all the values of i=0,1,2,........,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 1 to the vertices v_{12i} for i=1,2,3,....,x. Then we assign the label 3 to the vertices $v_2, v_5, v_8, ...$ For all the values of i=0,1,2,3,...,$\lceil \frac{n}{12} \rceil - 1$ assign the label 2 to the vertices $v_{12i+3}, v_{12i+6}, v_{12i+7}$ and v_{12i+10}. Therefore the vertex condition is $v_f(1) = \frac{n+m+2}{3}$ and $v_f(2) = v_f(3) = \frac{n+m-1}{3}$ and the edge condition is given in table 12.

Nature of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 7,\ m = 6t_2 + 3$	$\frac{2n+m-1}{2}$	$\frac{2n+m-3}{2}$
$n = 12t_1 + 1,\ m = 6t_2$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$

Table 12

Case 5. $n \equiv 1 \pmod 6$ and $m \equiv 1 \pmod 3$.

Assign the label to the vertices u_i $(1 \le i \le n)$ as in case 4. Now we move to the vertices v_i. Fix the label 3 to the vertex v_1. Assign the labels 2 to the vertices v_{12i+2}, v_{12i+5} and v_{12i+10} for all the values of i=0,1,2,......,x and we assign the label 2 to the vertices v_{12i+1} for i=1,2,......,$\lceil \frac{n}{12} \rceil - 1$. Then we assign the label 3 to the vertices $v_3, v_6, v_9, ...$ For all the values of i=0,1,2,......,$\lceil \frac{n}{12} \rceil - 1$, assign the label 1 to the vertices $v_{12i+4}, v_{12i+7}, v_{12i+8}$ and v_{12i+11}. Clearly the vertex condition is $v_f(1) = v_f(3) = \frac{n+m+1}{3}$ and $v_f(2) = \frac{n+m-2}{3}$ and the edge condition is given in table 13.

Case 6. $n \equiv 1 \pmod 6$ and $m \equiv 2 \pmod 3$.

Vlaues of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 7,\ m = 6t_2 + 4$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$
$n = 12t_1 + 1,\ m = 6t_2 + 1$	$\frac{2n+m-3}{2}$	$\frac{2n+m-1}{2}$

Table 13

Label to the vertices u_i $(1 \leq i \leq n)$ as in case 4. Now our attention move to the vertices v_i. Assign the label 3,2 to the vertices v_1 and v_2 respectively. Then we assign the labels 2 to the vertices v_{12i+3}, v_{12i+6} and v_{12i+11} for all the values of i=0,1,2,......,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 2 to the vertices v_{12i+2} for i=1,2,......,$\lceil \frac{n}{12} \rceil - 1$. Next we assign the label 3 to the vertices $v_4, v_7, v_{10}, \ldots..$ For all the values of i=0,1,2,......,$\lceil \frac{n}{12} \rceil - 1$, assign the label 1 to the vertices $v_{12i+5}, v_{12i+8}, v_{12i+9}$ and v_{12i+11}. Finally we assign the label 1 to the vertices v_{12i} for i=1,2,3,...,$\lceil \frac{n}{12} \rceil - 1$. Therefore the vertex condition is $v_f(1) = v_f(2) = v_f(3) = \frac{n+m}{3}$ and the edge condition is given in table 14.

Nature of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 7,\ m = 6t_2 + 5$	$\frac{2n+m-1}{2}$	$\frac{2n+m-3}{2}$
$n = 12t_1 + 1,\ m = 6t_2 + 2$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$

Table 14

Case 7. $n \equiv 2 \pmod 6$ and $m \equiv 0 \pmod 3$.

Consider the path vertices u_i. Now we fix the labels 2,1 to the first path vertex u_1, u_2 respectively. Assign the labels 3,2,1,2,3,1 to the path vertices $u_3, u_4, u_5, u_6, u_7, u_8$ respectively. Then we assign the labels 3,2,1,2,3,1 to the next six path vertices $u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}$ respectively. Proceeding like this we assign the label to the next six vertices and so on. Therefore the last vertex u_n received the label 1. Next we move to the vertices v_i. Fix the labels 1,3,2 to the vertices v_1, v_2, v_3 respectively. Assign the label 2 to the vertices v_{12i+4} and v_{12i+7} for all the values of i=0,1,2,.........,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 2 to the vertices v_{12i} and v_{12i+3} for i=1,2,3,....$\lceil \frac{n}{12} \rceil - 1$. Then we assign the label 3 to the vertices $v_5, v_8, v_{11}, ...$ For all the values of i=0,1,2,3,...,$\lceil \frac{n}{12} \rceil - 1$, assign the label 1 to the vertices v_{12i+6}, v_{12i+9} and v_{12i+10}. Finally we assign the label 1 to the vertices v_{12i+1} for i=1,2,3,...,$\lceil \frac{n}{12} \rceil - 1$. Clearly the vertex condition is $v_f(1) = v_f(2) = \frac{n+m+1}{3}$ and $v_f(3) = \frac{n+m-2}{3}$. Also the edge condition is given in table 15.

Values of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 8,\ m = 6t_2 + 3$	$\frac{2n+m-3}{2}$	$\frac{2n+m-1}{2}$
$n = 12t_1 + 2,\ m = 6t_2$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$

Table 15

Case 8. $n \equiv 2 \pmod 6$ and $m \equiv 1 \pmod 3$.

First we consider the path vertices u_i. Assign the label to the vertices u_i $(1 \le i \le n)$ as in case 6. Next our attention move to the vertices v_i. Fix the label 3 to the vertex v_1. Assign the label 1 to the vertices v_{12i+2}, v_{12i+5} and v_{12i+10} for all the values of i=0,1,2,.........,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 1 to the vertices v_{12i+1} for i=1,2,3,....,$\lceil \frac{n}{12} \rceil - 1$. Then we assign the label 3 to the vertices $v_3, v_6, v_9, ...$ For all the values of i=0,1,2,3,...,$\lceil \frac{n}{12} \rceil - 1$, assign the label 2 to the vertices $v_{12i+4}, v_{12i+7}, v_{12i+8}$ and v_{12i+11}. Therefore the vertex condition of this case is $v_f(1) = v_f(2) = v_f(3) = \frac{n+m}{3}$ and the edge condition is given in table 16.

Nature of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 8,\ m = 6t_2 + 4$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$
$n = 12t_1 + 2,\ m = 6t_2 + 3$	$\frac{2n+m-3}{2}$	$\frac{2n+m-1}{2}$

Table 16

Case 9. $n \equiv 2 \pmod 6$ and $m \equiv 2 \pmod 3$.

Label the vertices u_i $(1 \le i \le n)$ as in case 7. Now we consider the vertices v_i. Fix the labels 1,3 to the vertices v_1, v_2 respectively. Assign the label 1 to the vertices

v_{12i+3}, v_{12i+6} and v_{12i+11} for all the values of i=0,1,2,........., $\lceil \frac{n}{12} \rceil - 1$ and we assign the label 1 to the vertices v_{12i+2} for i=1,2,3,...., $\lceil \frac{n}{12} \rceil - 1$. Then we assign the label 3 to the vertices v_4, v_7, v_{13} respectively. For all the values of i=0,1,2,3,..., $\lceil \frac{n}{12} \rceil - 1$, assign the label 2 to the vertices v_{12i+5}, v_{12i+8} and v_{12i+9}. Finally we assign the label 2 to the vertices v_{12i} for i=1,2,3,..., $\lceil \frac{n}{12} \rceil - 1$. Clearly the vertex condition is $v_f(1) = \frac{n+m+2}{3}$ and $v_f(2) = v_f(3) = \frac{n+m-1}{3}$. Also the edge condition is given in table 17.

Values of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 8,\ m = 6t_2 + 5$	$\frac{2n+m-1}{2}$	$\frac{2n+m-3}{2}$
$n = 12t_1 + 2,\ m = 6t_2 + 2$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$

TABLE 17

Case 10. $n \equiv 3 \pmod 6$ and $m \equiv 0 \pmod 3$.

First we consider the path vertices u_i. Assign the labels 2,3,1,3,2,1 to the path vertices $u_1, u_2, u_3, u_4, u_5, u_6$ respectively. Then we assign the labels 2,3,1,3,2,1 to the next six path vertices $u_7, u_8, u_9, u_{10}, u_{11}, u_{12}$ respectively. Continuing this way assign the label to the next six vertices and so on. Therefore the last vertex u_n received the label by the integer 1. Now our attention turn to the vertices v_i. Fix the labels 1,2,3 to the vertices v_1, v_2, v_3 respectively. Assign the label 1 to the vertices v_{12i+4} and v_{12i+7} for all the values of i=0,1,2,........., $\lceil \frac{n}{12} \rceil - 1$ and we assign the label 1 to the vertices v_{12i} and v_{12i+3} for i=1,2,3,...., $\lceil \frac{n}{12} \rceil - 1$. Then we assign the label 3 to the vertices $v_5, v_8, v_{11}, ...$ For all the values of i=0,1,2,3,..., $\lceil \frac{n}{12} \rceil - 1$, assign the label 2 to the vertices v_{12i+6}, v_{12i+9} and v_{12i+10}. Finally we assign the label 2 to the vertices v_{12i+1} for i=1,2,3,..., $\lceil \frac{n}{12} \rceil - 1$. Therefore the vertex condition is $v_f(1) = v_f(2) = v_f(3) = \frac{n+m}{3}$ and the edge condition is given in table 18.

Case 11. $n \equiv 3 \pmod 6$ and $m \equiv 1 \pmod 3$.

Assign the label to the vertices u_i $(1 \leq i \leq n)$ as in case 10. Now we consider the vertices v_i. Assign the labels 1,2,3,1 to the vertices v_1, v_2, v_3, v_4 respectively.

Nature of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 9,\ m = 6t_2 + 3$	$\frac{2n+m-3}{2}$	$\frac{2n+m-1}{2}$
$n = 12t_1 + 3,\ m = 6t_2$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$

TABLE 18

Now we assign the label 1 to the vertices v_{12i+5} and v_{12i+8} for all the values of i=0,1,2,.........,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 1 to the vertices v_{12i+1} and v_{12i+4} for i=1,2,3,....,$\lceil \frac{n}{12} \rceil - 1$. Then we assign the label 3 to the vertices $v_6, v_9, v_{12}, ...$ For all the values of i=0,1,2,3,...,$\lceil \frac{n}{12} \rceil - 1$. assign the label 2 to the vertices v_{12i+7}, v_{12i+10} and v_{12i+11}. Finally we assign the label 2 to the vertices v_{12i+2} for i=1,2,3,...,$\lceil \frac{n}{12} \rceil - 1$. Clearly the vertex condition is $v_f(1) = \frac{n+m+2}{3}$ and $v_f(2) = v_f(3) = \frac{n+m-1}{3}$. Also the edge condition is given in table 19.

Values of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 9,\ m = 6t_2 + 4$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$
$n = 12t_1 + 3,\ m = 6t_2 + 1$	$\frac{2n+m-3}{2}$	$\frac{2n+m-1}{2}$

TABLE 19

Case 12. $n \equiv 3 \pmod 6$ and $m \equiv 2 \pmod 3$.

First we consider the path vertices u_i. Label the vertices u_i $(1 \le i \le n)$ as in case 10. Now our attention move to the vertices v_i. Assign the labels 1,2,1,3,2 to the vertices v_1, v_2, v_3, v_4, v_5 respectively. Now we assign the label 2 to the vertices v_{12i+6} and v_{12i+9} for all the values of i=0,1,2,.........,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 2 to the vertices v_{12i+2} and v_{12i+5} for i=1,2,3,....,x. Then we assign the label 3 to the vertices $v_7, v_{10}, v_{13}, ...$ For all the values of i=0,1,2,3,...,$\lceil \frac{n}{12} \rceil - 1$, assign the label 1 to the vertices v_{12i+8} and v_{12i+11}. Finally we assign the label 1 to the vertices v_{12i} and v_{12i+3} for i=1,2,3,...,$\lceil \frac{n}{12} \rceil - 1$. Therefore the vertex condition is $v_f(1) = v_f(2) = \frac{n+m+1}{3}$ and $v_f(3) = \frac{n+m-2}{3}$. Also the edge condition is given in table 20.

Nature of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 9$, $m = 6t_2 + 5$	$\frac{2n+m-3}{2}$	$\frac{2n+m-1}{2}$
$n = 12t_1 + 3$, $m = 6t_2 + 2$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$

TABLE 20

Case 13. $n \equiv 4 \pmod 6$ and $m \equiv 0 \pmod 3$.

First we consider the path vertices u_i. Fix the label 2 to the first path vertex u_1. Assign the labels 2,3,1,3,2,1 to the path vertices $u_2, u_3, u_4, u_5, u_6, u_7$ respectively. Then we assign the labels 2,3,1,3,2,1 to the next six path vertices $u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}$ respectively. Proceeding like this assign the label to the next six vertices and so on. Therefore the last vertex u_n received the label 1. Now our attention turn to the vertices v_i. Assign the label to the vertices v_i $(1 \leq i \leq m)$ as in case 4. Clearly the vertex condition is $v_f(1) = v_f(3) = \frac{n+m-1}{3}$ and $v_f(2) = \frac{n+m+2}{3}$ and the edge condition is given in table 21.

Values of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 10$, $m = 6t_2 + 3$	$\frac{2n+m-1}{2}$	$\frac{2n+m-3}{2}$
$n = 12t_1 + 4$, $m = 6t_2$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$

TABLE 21

Case 14. $n \equiv 4 \pmod 6$ and $m \equiv 1 \pmod 3$.

Assign the label to the vertices u_i $(1 \leq i \leq n)$ as in case 13. Next we move to the vertices v_i. Fix the label 1 to the vertex v_1. Assign the label 2 to the vertices v_{12i+2}, v_{12i+5} and v_{12i+10} for all the values of i=0,1,2,.........,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 2 to the vertices v_{12i+1} for i=1,2,3,....,$\lceil \frac{n}{12} \rceil - 1$. Then we assign the label 3 to the vertices $v_3, v_6, v_9, ...$ For all the values of i=0,1,2,3,...,$\lceil \frac{n}{12} \rceil - 1$, assign the label 1 to the vertices $v_{12i+4}, v_{12i+7}, v_{12i+8}$ and v_{12i+11}. Therefore the vertex condition is

$v_f(1) = v_f(2) = \frac{n+m+1}{3}$ and $v_f(3) = \frac{n+m-2}{3}$ and the edge condition is given in table 22.

Nature of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 10$, $m = 6t_2 + 4$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$
$n = 12t_1 + 4$, $m = 6t_2 + 1$	$\frac{2n+m-3}{2}$	$\frac{2n+m-1}{2}$

TABLE 22

Case 15. $n \equiv 4 \pmod 6$ and $m \equiv 2 \pmod 3$.

Consider the path vertices u_i. Label the vertices u_i $(1 \leq i \leq n)$ as in case 13. Next our attention move to the vertices v_i. Fix the labels 1 and 3 to the vertex v_1 and v_2 respectively. Assign the label 2 to the vertices v_{12i+3}, v_{12i+6} and v_{12i+11} for all the values of i=0,1,2,........,$\lceil \frac{n}{12} \rceil - 1$ and we assign the label 2 to the vertices v_{12i+2} for i=1,2,3,....,$\lceil \frac{n}{12} \rceil - 1$. Then we assign the label 3 to the vertices $v_4, v_7, v_{10}, ...$ For all the values of i=0,1,2,3,...,$\lceil \frac{n}{12} \rceil - 1$, assign the label 1 to the vertices v_{12i+5}, v_{12i+8} and v_{12i+9}. Finally we assign the label 1 to the vertices v_{12i} for i=1,2,3,...,$\lceil \frac{n}{12} \rceil - 1$. Clearly the vertex condition is $v_f(1) = v_f(2) = v_f(3) = \frac{n+m}{3}$ and the edge condition is given in table 23.

Nature of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 10$, $m = 6t_2 + 5$	$\frac{2n+m-1}{2}$	$\frac{2n+m-3}{2}$
$n = 12t_1 + 4$, $m = 6t_2 + 2$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$

TABLE 23

Case 16. $n \equiv 5 \pmod 6$ and $m \equiv 0 \pmod 3$.

Fix the labels 1,2 to the first two path vertices u_1, u_2 respectively. Assign the labels 2,3,1,3,2,1 to the path vertices $u_3, u_4, u_5, u_6, u_7, u_8$ respectively. Then we assign the labels 2,3,1,3,2,1 to the next six path vertices $u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}$ respectively. Proceeding like this assign the label to the next six vertices and so on. Therefore the last vertex u_n received the label 1. Now our attention turn to the vertices v_i. Assign the label to the vertices v_i $(1 \leq i \leq m)$ as in case 4. Therefore the vertex condition is $v_f(1) = v_f(2) = \frac{n+m+1}{3}$ and $v_f(3) = \frac{n+m-2}{3}$. Also the edge condition is given in table 24.

Values of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 11$, $m = 6t_2 + 3$	$\frac{2n+m-1}{2}$	$\frac{2n+m-3}{2}$
$n = 12t_1 + 4$, $m = 6t_2$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$

Table 24

Case 17. $n \equiv 5 \pmod 6$ and $m \equiv 1 \pmod 3$.

Assign the labels to the vertices u_i $(1 \leq i \leq n)$ as in case 16. Then label the vertices v_i $(1 \leq i \leq m)$ as in case 5. Clearly the vertex condition is $v_f(1) = v_f(2) = v_f(3) = \frac{n+m}{3}$. Also the edge condition is given in table 25.

Values of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 11$, $m = 6t_2 + 4$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$
$n = 12t_1 + 5$, $m = 6t_2 + 1$	$\frac{2n+m-3}{2}$	$\frac{2n+m-1}{2}$

Table 25

Case 18. $n \equiv 5 \pmod 6$ and $m \equiv 2 \pmod 3$.

Label the vertices u_i $(1 \leq i \leq n)$ as in case 16. Also assign the label to the vertices v_i $(1 \leq i \leq m)$ as in case 6. Therefore the vertex condition is $v_f(1) = v_f(3) = \frac{n+m-1}{3}$ and $v_f(2) = \frac{n+m+2}{3}$. Also the edge condition is given in table 26.

Nature of n	$e_f(0)$	$e_f(1)$
$n = 12t_1 + 11$, $m = 6t_2 + 5$	$\frac{2n+m-1}{2}$	$\frac{2n+m-3}{2}$
$n = 12t_1 + 5$, $m = 6t_2 + 2$	$\frac{2n+m-2}{2}$	$\frac{2n+m-2}{2}$

Table 26

□

Example 2.4. A 3-difference cordial labeling of the dumbbell $U_{6,5}$ is given in figure 4.

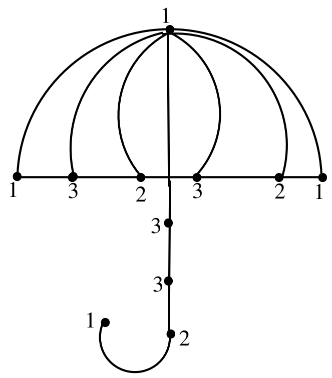

FIGURE 4

Acknowledgement. The authors thanks to the both referees for their valuable comments and suggestions.

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