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NEW SCRAMBLING RANDOMIZED RESPONSE MODELS

TANVEER AHMAD TARRAY [(1)] AND HOUSILA PRASAD SINGH [(2)]

Abstract. In this article, a new randomized response model has been proposed. It is shown that Gupta and Thorntons (2002) and Hussains (2012) randomized response models are particular member of the proposed model. The proposed model is found to be more efficient than the randomized response models studied by Gupta and Thornton (2002) and Hussain (2012) under a realistic condition. The relative efficiency of the proposed model has been studied with respect to the Gupta and Thorntons (2002) and Hussains (2012) models. Numerical illustrations are also given in support of the present study.

1. Introduction

One of the leading cogs for obtaining data pertaining to human populations is the social survey. To measure opinions, attitudes, and behaviors that cover a wide band of interests, the social survey has been established as being tremendously practical. The surveys are conducted due to many reasons, non availability of certain facts / information in the archives being the most understandable and apparent. For instance, if one is interested in knowing crime rate, information about unseen crimes or unreported victimization experience is not available in formal records on crimes. Sometimes the facts about the individuals (in a population) are inaccessible to the investigators for legal reasons. Questionnaires, in particular social surveys, generally

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consist of many items. Some of the items may be about sensitive / high risk behavior, due to the social stigma carried by them. One problem with research on high risk behavior is that respondents may consciously or unconsciously provide incorrect information. In psychological surveys, a social desirability bias has been observed as a major cause of distortion in standardized personality measures. Survey researchers have similar concerns about the truth of survey results/ findings about such topics as drunk driving, use of marijuana, tax evasion, illicit drug use, induced abortion, shop lifting, child abuse, family disturbances, cheating in exams, HIV/AIDS, and sexual behavior. Warner (1965) introduced a randomized response model to estimate a population proportion for sensitive attribute. A detailed review and applications of such technique can be had from Chaudhuri and Mukerjee (1988), Ryu et al. (1993), Singh (2003), Mahajan (2005-2006), Mahajan et al. (2007), Ryu et al. (2005-2006), Hong (2005-2006), Javed and Grewal (2005-2006), Grewal et al. (2005-2006), Sidhu and Bansal (2008), Perri (2008), Zaizai et al. (2008), Chaudhuri (2011), Singh and Tarray (2013, 2014, 2015), Hussain et al (2015) and Tarray and Singh (2015 a,b) etc. Eichorn and Hayre (1983) suggested a multiplicative model to collect information on sensitive quantitative variables like income, tax evasion, amount of drug used etc. According to them, each respondent in the sample is requested to report the scrambled response $Z_i = SY_i$, where Y_i is the real value of the sensitive quantitative variable, and S is the scrambling variable whose distribution is assumed to be known.
In other words $E_R(S) = \theta$ and$V_R(S) = \gamma^2$ are assumed to be known and positive.Then an estimator of the population mean μ_Y under the simple random sampling with replacement (SRSWR) due to Eichhorn and Hayre (1983) is given by:

(1.1) $$\hat{\mu}_Y = \frac{1}{n}\sum_{i=1}^{n}\frac{Z_i}{\theta}$$

with variance

(1.2) $$V(\hat{\mu}_Y) = \frac{1}{n}[\sigma_Y^2 + C_\gamma^2(\sigma_Y^2 + \mu_Y^2)],$$

where $C_{\gamma}^{2} = \frac{\gamma^{2}}{\theta^{2}}$, we shall now discuss randomized response models studied by Gupta and Thornton (2002) and Hussain (2012).

1.1. **Gupta and Thornton (2002) randomized response model.** Let X be the sensitive variable of interest and Y be an unrelated non sensitive variable. The population mean μ_X is the parameter of interest. The distribution of variable Y, say $f(Y)$, is completely known mean$\mu_Y(-\infty < \mu_Y < -\infty)$ and variance $\sigma_Y^2(> 0)$.To estimate the mean μ_X , Gupta and Thornton (2002) described a partial quantitative randomized response model. In their technique, some known proportion of respondents responds truthfully while the remaining proportion of respondents reports scrambled responses. The scrambling is done in an additive way. The i-th respondents is first requested to generate a value Y, from $f(Y)$ and then provided a randomization device consisting of two statements: (i) Report your true response on sensitive variable X, and (ii) Report the scrambled response as $(X_i + Y_i)$, represented with probability T and $(1 - T)$, respectively. LetZ_{1i} be the reported response of the i-th respondent then it can be written as:

$$Z_{1i} = \alpha_i X_i + (1 - \alpha_i)(X_i + Y_i), \tag{1.3}$$

where α_i is a Bernoulli random variable with mean T . The expected response from the i-th respondent is given by

$$E(Z_{1i}) = E(\alpha_i)E(X_i) + E((1 - \alpha_i))E((X_i + Y_i)),$$

$$= \mu_X + (1 - T)\mu_Y. \tag{1.4}$$

An unbiased estimator of μ_X due to Gupta and Thornton (2002) is:

$$\hat{\mu}_{1X} = \bar{Z}_1 - (1-T)\mu_Y. \tag{1.5}$$

The variance of the estimator $\hat{\mu}_{1X}$ is given by

$$V(\hat{\mu}_{1X}) = V(\bar{Z}_1) = \frac{\sigma_X^2}{n} + \frac{(1-T)(\sigma_Y^2 + T\mu_Y^2)}{n} \tag{1.6}$$

1.2. **Hussian's (2012) randomized response model.** The model envisaged by Gupta and Thornton (2002) is improved by talking two responses from each respondent defining two dependent estimators with equal variances. To obtain the second response, Hussain (2012) used subtractive scrambling. In the Hussain (2012) procedure, Let Z_{2i} be the second response from the i-th respondent taken as

$$Z_{2i} = \alpha_i X_i + (1-\alpha_i)(X_i - Y_i), \tag{1.7}$$

where α_i is a Bernoulli random variable defined as above. The second expected response from the i-th respondent is given by

$$E(Z_{2i}) = E(\alpha_i)E(X_i) + E((1-\alpha_i))E((X_i - Y_i)),$$
$$= \mu_X - (1-T)\mu_Y.$$

This led Hussain (2012) to define another unbiased estimator, based on the second set of responses, of μ_X as

$$\hat{\mu}_{2X} = \bar{Z}_2 + (1-T)\mu_Y. \tag{1.8}$$

whose variance is given by

$$V(\hat{\mu}_{2X}) = V(\bar{Z}_2) = \frac{\sigma_X^2}{n} + \frac{(1-T)(\sigma_Y^2 + T\mu_Y^2)}{n} \tag{1.9}$$

which is same as obtained by Gupta and Thornton (2002).
Further Hussain (2012) derived the optimum estimator of μ_X as

$$\hat{\mu}_{3X} = \frac{\hat{\mu}_{1X} + \hat{\mu}_{2X}}{2}, \tag{1.10}$$

whose variance is given by

$$V(\hat{\mu}_{3X}) = V(\bar{Z}_3) = \frac{\sigma_X^2}{n} \tag{1.11}$$

Hussain (2012) claimed that (i) $V(\hat{\mu}_{3X}) = V(\bar{Z}_3) = \dfrac{\sigma_X^2}{n}$ is the lower bound on the variance of an estimator based on SRSWR and utilizing randomized responses (ii) scrambling variance is eliminated and no further reduction of scrambling is possible i.e. scrambling effect is removed by taking two responses from each respondent and using additive and subtracting scrambling simultaneously. Hence it should be mentioned that the above claim made by Hussain (2012) is very artificial, because the optimum estimator μ_{3X} in (1.10) obtained by him does not depend on the scrambling response it only depends on the true response x. Proof of the statement is given below:

Proof. We have

$$\hat{\mu}_{3X} = \frac{\hat{\mu}_{1X} + \hat{\mu}_{2X}}{2}.$$

Putting the values of $\hat{\mu}_{1X}$, $\hat{\mu}_{2X}$ and $\hat{\mu}_{3X}$ in we have

$$\hat{\mu}_{3X} = \frac{1}{2}[\bar{Z}_1 - (1-T)\mu_Y + \bar{Z}_2 + (1-T)\mu_Y],$$

$$= \frac{1}{2}[\bar{Z}_1 + \bar{Z}_2]. \tag{1.12}$$

Substituting $\bar{Z}_1$ and $\bar{Z}_2$ in (1.12) we have

$$\hat{\mu}_{3X} = \frac{1}{2}[\frac{1}{n}\sum_{i=1}^{n}(\alpha_i X_i + (1-\alpha_i)(X_i + Y_i)) + \frac{1}{n}\sum_{i=1}^{n}(\alpha_i X_i + (1-\alpha_i)(X_i - Y_i))],$$

$$= \bar{X}.$$

Thus the variance of $\hat{\mu}_{3X}$ is

$$V(\hat{\mu}_{3X}) = \frac{\sigma_X^2}{n},$$

which is the variance of the unbiased estimator $\bar{X}$ based on true responses. So the claim made by the Hussain (2012) is not logically and theoretically correct.

In this paper we have suggested a randomized response additive model which generalizes the randomized response models earlier considered by Gupta and Thornton (2002) and Hussain (2012). We have also shown that the proposed model is better than both the randomized response models due to Gupta and Thornton (2002) and Hussain (2012) under a realistic condition. Numerical illustration is given in support of the present study.

2. Proposed Randomized Response Model

The model envisaged by Gupta and Thornton (2002) is improved by taking two responses from each respondent and defining two dependent estimators with equal variances while to obtain the second response, Hussain (2012) used the additive scrambling. Here to obtain the second response, introducing a known real constant α a more general additive scrambling is used. In this way, the two responses from each respondent are correlated. Let Z_i be the second response from i-th respondent taken as

$$Z_i = \alpha_i X_i + (1-\alpha_i)(X_i + \alpha Y_i), \tag{2.1}$$

where α_i is a Bernoulli random variable with mean T. The expected response from the i-th respondent is given by

$$E(Z_i) = E(\alpha_i)E(X_i) + E((1-\alpha_i))E((X_i + \alpha Y_i)),$$

$$= \mu_X + \alpha(1-T)\mu_Y. \tag{2.2}$$

This yields an unbiased estimator based on the second set of responses of μ_X is given by

$$\hat{\mu}_X = \bar{Z} - \alpha(1-T)\mu_Y. \tag{2.3}$$

We note that for $\alpha = 1$, the proposed estimator $\hat{\mu}_X$ reduces to the estimator $\hat{\mu}_{1X}$ in (1.5) reported Gupta and Thornton (2002) while for $\alpha = -1$ it reduces to the estimator $\hat{\mu}_{2X}$ in (1.8) obtained by Hussain (2012). If we set $\alpha = 0$, then $\hat{\mu}_X = \bar{X}$ which is the sample mean of true response only. The variance of the estimator $\hat{\mu}_X$ is given by

$$V(\hat{\mu}_X) = V(\bar{Z} - \alpha(1-T)\mu_Y) = V(\bar{Z}) = \frac{1}{n}V(Z_i). \tag{2.4}$$

Consider

$$V(Z_i) = E(Z_i^2) - (E(Z_i))^2,$$

$$V(Z_i) = E(\alpha_i^2 X_i^2 + (1-\alpha_i)^2(X_i + \alpha Y_i)^2 + 2\alpha_i(1-\alpha_i)X_i(X_i + \alpha Y_i)) - (E(Z_i)^2),$$

$$= \sigma_X^2 + \alpha^2(1-T)(T\mu_Y^2 + \sigma_Y^2), \tag{2.5}$$

Substituting (2.5) in (2.4), we have

$$V(\hat{\mu}_X) = \frac{\sigma_X^2}{n} + \frac{\alpha^2(1-T)(\sigma_Y^2 + T\mu_Y^2)}{n} \tag{2.6}$$

For $\alpha = +1(or - 1)$, (2.6) reduces to

$$V(\hat{\mu}_{1X}) = V(\hat{\mu}_{2X}) = \frac{\sigma_X^2}{n} + \frac{(1-T)(\sigma_Y^2 + T\mu_Y^2)}{n} \tag{2.7}$$

which is same as obtained by Gupta and Thornton (2002) and Hussain (2012). If we set $\alpha = 0$ in (2.6); then it reduces to

$$V(\hat{\mu}_X) = V(\bar{Z}) = \frac{\sigma_X^2}{n} \tag{2.8}$$

which is the variance of the sample mean of true responses.

3. Efficiency Comparison

From (1,6), (1.9) and (2.6), we have

$$V(\hat{\mu}_{iX}) - V(\hat{\mu}_X) = \frac{(1-T)(\sigma_Y^2 + T\mu_Y^2)(1-\alpha^2)}{n}, for i = 1, 2$$

which is always positive if $1-\alpha^2 > 0$ i.e. if $\alpha^2 < 1$ i.e. if $-1 < \alpha < 1$.

$$i.e. if |\alpha| < 1 \tag{3.1}$$

Thus we established the following theorem.

Theorem 3.1. *proposed unbiased estimator $\hat{\mu}_X$ is better than Gupta and Thornton (2002) estimator $\hat{\mu}_{1X}$ and Hussain (2012) estimator $\hat{\mu}_{2X}$ if $|\alpha| < 1$.*

4. Numerical illustration

To have the tangible idea about the performance of the proposed estimator $\hat{\mu}_X$ over the estimator Gupta and Thornton (2002) estimator $\hat{\mu}_{1X}$ and Hussain (2012) estimator $\hat{\mu}_{2X}$ we have computed the percent relative efficiency (PRE) by using the following formula:

$$PRE(\hat{\mu}_X, \hat{\mu}_{iX}) = \frac{[\sigma_X^2 + (1-T)(\sigma_Y^2 + T\mu_Y^2)]}{[\sigma_X^2 + \alpha^2(1-T)(\sigma_Y^2 + T\mu_Y^2)]} \times 100, for i = 1, 2 \tag{4.1}$$

for different values of $T = 0.1(0.1)0.9$, $\sigma_X = 0.5(0.5)2.0$, $\mu_Y = 1, 2, 3$, $\sigma_Y = 1, 1.5, 2$, $\alpha = -1(.25)1$. Findings are displayed in Tables 1-3.

It is observed from Tables 1- 3 that the percent relative efficiency are greater than 100 which follows that the proposed estimator $\hat{\mu}_X$ is more efficient than Gupta and Thornton (2002) estimator $\hat{\mu}_{1X}$ and Hussain (2012) estimator $\hat{\mu}_{2X}$ with considerable gain in efficiency. Thus our recommendation is to prefer the proposed study over Gupta and Thornton and Hussain (2012).

Table 1. The $PRE(\hat{\mu}_X, \hat{\mu}_{iX}), i = 1, 2 for \mu_Y = 1, \sigma_Y = 1$

						T				
σ_X	α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	-1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.75	153.68	153.16	152.26	150.87	148.84	145.90	141.56	134.81	123.29
	-0.50	249.25	246.94	242.93	236.96	228.57	217.07	201.32	179.41	147.90
	-0.25	397.60	390.32	378.00	360.33	336.84	306.90	269.62	223.85	168.02
0.5	0.00	496.00	484.00	464.00	436.00	400.00	356.00	304.00	244.00	176.00
	0.25	397.60	390.32	378.00	360.33	336.84	306.90	269.62	223.85	168.02
	0.50	249.25	246.94	242.93	236.96	228.57	217.07	201.32	179.41	147.90
	0.75	153.68	153.16	152.26	150.87	148.84	145.90	141.56	134.81	123.29
	1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.75	127.82	127.27	126.33	124.96	123.08	120.59	117.34	113.10	107.51
	-0.50	159.52	158.06	155.60	152.07	147.37	141.38	133.92	124.77	113.60
	-0.25	187.40	184.91	180.72	174.82	167.16	157.69	146.34	133.01	117.60
1.0	0.00	199.00	196.00	191.00	184.00	175.00	164.00	151.00	136.00	119.00
	0.25	187.40	184.91	180.72	174.82	167.16	157.69	146.34	133.01	117.60
	0.50	159.52	158.06	155.60	152.07	147.37	141.38	133.92	124.77	113.60
	0.75	127.82	127.27	126.33	124.96	123.08	120.59	117.34	113.10	107.51
	1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.75	115.43	115.05	114.42	113.50	112.28	110.73	108.80	106.42	103.53
	-0.50	129.73	128.92	127.55	125.61	123.08	119.92	116.09	111.54	106.20
	-0.25	140.15	138.96	136.98	134.20	130.61	126.20	120.95	114.85	107.88
1.5	0.00	144.00	142.67	140.44	137.33	133.33	128.44	122.67	116.00	108.44
	0.25	140.15	138.96	136.98	134.20	130.61	126.20	120.95	114.85	107.88
	0.50	129.73	128.92	127.55	125.61	123.08	119.92	116.09	111.54	106.20
	0.75	115.43	115.05	114.42	113.50	112.28	110.73	108.80	106.42	103.53
	1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.75	109.50	109.25	108.82	108.22	107.42	106.42	105.20	103.75	102.02
	-0.50	117.48	116.98	116.14	114.96	113.43	111.54	109.27	106.60	103.52
	-0.25	122.85	122.17	121.03	119.43	117.37	114.85	111.86	108.39	104.44
2.0	0.00	124.75	124.00	122.75	121.00	118.75	116.00	112.75	109.00	104.75
	0.25	122.85	122.17	121.03	119.43	117.37	114.85	111.86	108.39	104.44
	0.50	117.48	116.98	116.14	114.96	113.43	111.54	109.27	106.60	103.52
	0.75	109.50	109.25	108.82	108.22	107.42	106.42	105.20	103.75	102.02
	1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

TABLE 2. The $PRE(\hat{\mu}_X, \hat{\mu}_{iX}), i = 1, 2 for \mu_Y = 2, \sigma_Y = 1.5$

						T				
σ_X	α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	-1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.75	165.56	165.79	165.69	165.23	164.32	162.78	160.14	155.25	144.20
	-0.50	311.37	312.79	312.15	309.37	304.00	295.10	280.72	256.46	210.73
	-0.25	660.30	668.32	664.69	649.13	620.41	576.11	512.06	421.22	291.38
0.5	0.00	1054.00	1076.00	1066.00	1024.00	950.00	844.00	706.00	536.00	334.00
	0.25	660.30	668.32	664.69	649.13	620.41	576.11	512.06	421.22	291.38
	0.50	311.37	312.79	312.15	309.37	304.00	295.10	280.72	256.46	210.73
	0.75	165.56	165.79	165.69	165.23	164.32	162.78	160.14	155.25	144.20
	1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.75	144.56	144.99	144.80	143.95	142.35	139.77	135.79	129.56	119.26
	-0.50	212.06	213.66	212.94	209.83	204.08	195.22	182.41	164.24	138.28
	-0.25	294.59	298.48	296.71	289.24	275.86	256.22	229.75	195.67	152.91
1.0	0.00	338.50	344.00	341.50	331.00	312.50	286.00	251.50	209.00	158.50
	0.25	294.59	298.48	296.71	289.24	275.86	256.22	229.75	195.67	152.91
	0.50	212.06	213.66	212.94	209.83	204.08	195.22	182.41	164.24	138.28
	0.75	144.56	144.99	144.80	143.95	142.35	139.77	135.79	129.56	119.26
	1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.75	129.05	129.47	129.28	128.47	126.98	124.69	121.37	116.66	109.92
	-0.50	162.85	163.99	163.47	161.27	157.30	151.38	143.22	132.41	118.31
	-0.25	193.20	195.21	194.30	190.45	183.61	173.69	160.58	144.08	123.99
1.50	0.00	206.00	208.44	207.33	202.67	194.44	182.67	167.33	148.44	126.00
	0.25	193.20	195.21	194.30	190.45	183.61	173.69	160.58	144.08	123.99
	0.50	162.85	163.99	163.47	161.27	157.30	151.38	143.22	132.41	118.31
	0.75	129.05	129.47	129.28	128.47	126.98	124.69	121.37	116.66	109.92
	1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.75	119.53	119.87	119.72	119.07	117.89	116.13	113.66	110.34	105.91
	-0.50	138.92	139.70	139.34	137.85	135.17	131.24	125.95	119.13	110.58
	-0.25	153.89	155.09	154.54	152.25	148.20	142.36	134.69	125.12	113.59
2.0	0.00	159.63	161.00	160.38	157.75	153.13	146.50	137.88	127.25	114.63
	0.25	153.89	155.09	154.54	152.25	148.20	142.36	134.69	125.12	113.59
	0.50	138.92	139.70	139.34	137.85	135.17	131.24	125.95	119.13	110.58
	0.75	119.53	119.87	119.72	119.07	117.89	116.13	113.66	110.34	105.91
	1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Table 3. The $PRE(\hat{\mu}_X, \hat{\mu}_{iX}), i = 1, 2 for \mu_Y = 3, \sigma_Y = 2$

						T				
σ_X	α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	-1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.75	170.66	170.98	171.05	170.87	170.41	169.56	168.00	164.90	156.88
	-0.50	344.55	346.81	347.28	346.04	342.86	336.97	326.65	307.41	264.25
	-0.25	886.56	905.56	909.55	899.07	872.73	826.80	753.74	638.46	448.37
0.5	0.00	1864.00	1956.00	1976.00	1924.00	1800.00	1604.00	1336.00	996.00	584.00
	0.25	886.56	905.56	909.55	899.07	872.73	826.80	753.74	638.46	448.37
	0.50	344.55	346.81	347.28	346.04	342.86	336.97	326.65	307.41	264.25
	0.75	170.66	170.98	171.05	170.87	170.41	169.56	168.00	164.90	156.88
	1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.75	155.43	156.23	156.40	155.96	154.84	152.81	149.37	143.36	131.50
	-0.50	257.31	261.11	261.91	259.81	254.55	245.36	230.75	207.69	169.67
	-0.25	424.11	437.21	440.02	432.68	414.81	385.43	342.80	284.21	205.46
1.0	0.00	541.00	564.00	569.00	556.00	525.00	476.00	409.00	324.00	221.00
	0.25	424.11	437.21	440.02	432.68	414.81	385.43	342.80	284.21	205.46
	0.50	257.31	261.11	261.91	259.81	254.55	245.36	230.75	207.69	169.67
	0.75	155.43	156.23	156.40	155.96	154.84	152.81	149.37	143.36	131.50
	1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.75	140.78	141.77	141.98	141.43	140.07	137.69	133.90	127.92	118.06
	-0.50	198.66	202.05	202.78	200.88	196.23	188.40	176.67	159.79	135.55
	-0.25	263.70	271.26	272.89	268.64	258.39	241.85	218.57	187.87	148.78
1.5	0.00	296.00	306.22	308.44	302.67	288.89	267.11	237.33	199.56	153.78
	0.25	263.70	271.26	272.89	268.64	258.39	241.85	218.57	187.87	148.78
	0.50	198.66	202.05	202.78	200.88	196.23	188.40	176.67	159.79	135.55
	0.75	140.78	141.77	141.98	141.43	140.07	137.69	133.90	127.92	118.06
	1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	-0.75	129.77	130.71	130.91	130.39	129.10	126.90	123.56	118.63	111.31
	-0.50	164.82	167.44	168.00	166.54	162.96	157.09	148.56	136.84	121.09
	-0.25	196.70	201.40	202.42	199.77	193.41	183.23	169.09	150.72	127.83
2.0	0.00	210.25	216.00	217.25	214.00	206.25	194.00	177.25	156.00	130.25
	0.25	196.70	201.40	202.42	199.77	193.41	183.23	169.09	150.72	127.83
	0.50	164.82	167.44	168.00	166.54	162.96	157.09	148.56	136.84	121.09
	0.75	129.77	130.71	130.91	130.39	129.10	126.90	123.56	118.63	111.31
	1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

5. Discussion

Utilizing the idea of obtaining two responses from each respondent, a new class of unbiased estimators $\hat{\mu}_X$ has been proposed. It is shown that the estimators $\hat{\mu}_{1X}$ and $\hat{\mu}_{2X}$ due to Gupta and Thorntons (2002) and Hussains (2012) respectively are members of the proposed class of estimators. We have obtained the variance of the proposed class of unbiased estimators $\hat{\mu}_X$ and compared with Gupta and Thorntons (2002) estimator $\hat{\mu}_{1X}$ and Hussains (2012) estimator $\hat{\mu}_{2X}$. It has been found that the proposed class of unbiased estimators $\hat{\mu}_X$ is more efficient than $\hat{\mu}_{1X}$ and $\hat{\mu}_{2X}$ under very realistic condition. We have also shown numerically that the proposed class of estimators $\hat{\mu}_X$ is also better than $\hat{\mu}_{1X}$ and $\hat{\mu}_{2X}$. Thus our recommendation is to use the proposed class of unbiased estimators $\hat{\mu}_X$ instead of $\hat{\mu}_{1X}$ and $\hat{\mu}_{2X}$ in practice.

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(1) Department of Computer Science and Engineering, Islamic University of Science and Technology Awantipora Pulwama Kashmir India 192122

E-mail address: tanveerstat@gmail.com

(2) School of Studies in Statistics, Vikram University Ujjain , M.P. India-456010