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A SLACKS-BASED MEASURE OF EFFICIENCY FOR PARALLEL AND SERIES PRODUCTION SYSTEMS

ALI ASHRAFI (1) AND MOZHGAN MANSOURI KALEIBAR (2)

Abstract. Data envelopment analysis (DEA) is a mathematical approach for evaluating the efficiency of decision making units (DMUs) that convert multiple inputs into multiple outputs. In many cases, production systems may have internal or network structures, such as series and parallel structures, which are composed of several processes interacting with each other. In the last two decades, several authors have proposed DEA models which usually utilize the radial measure to calculate the efficiency of the DMUs with internal structure. In this paper, we first introduce a non-radial DEA model in the slacks-based measure (SBM) framework for evaluating the overall efficiency of parallel production systems by considering the parallel relationship between sub-processes. We will, then extend the methodology to series production systems. A numerical experiment is used to demonstrate and compare the results with obtained using new methods and show the application of the proposed approach.

1. Introduction

As a non-parametric technique, Data envelopment analysis (DEA) was first introduced by Charnes et al. (1978) for evaluating relative efficiency and performance of a set of production systems, or Decision Making Units (DMUs), in changing inputs

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into outputs. Standard DEA models make no assumptions regarding the internal operations of a DMU. Rather, the DEA models treat each DMU as a black box, in which only initial inputs and final outputs of the DMU are considered in evaluating the efficiency. In many cases, DMUs may have internal or network structures, such as series and parallel structures, which are composed of several processes interacting with each other. In the last two decades, several authors have proposed DEA models that consider the internal structure of the DMUs. For a classification of these models see Castelli *et al.* [2]. In particular, there are several models for evaluating the efficiency of network systems of interrelated processes; see for example, Fre and Grosskopf [8], Lewis and sexton [18], Prieto and Zofio [24], Yu and Lin [28], Tone and Tsutsui [26], Kao [16] and Cook *et al.* [7].

Special attention has been allocated, due to their basic structures, to the parallel systems [10, 18, 28] and also to the series systems [6, 11, 16, 20, 21, 23, 24].

The above DEA models usually utilize the radial measure to calculate the efficiency of the DMUs with internal structure. Radial models, e.g. the CCR [4] or the BCC [1] models, assume proportional change of inputs or outputs and usually disregard the existence of slacks in the efficiency scores. Non-radial models, e.g. the slacks-based measure (SBM) [25], on the other hand, regard the slacks of each input or output, and the variations of inputs and outputs are not proportional; in other words in non-radial models the inputs/outputs are allowed to decrease/increase at different rates.

This paper introduces an alternative method for measuring the efficiency of parallel and series production systems that considers the parallel and series relationships between sub-processes. Here, we first provide Production Possibility Set (PPS) of the parallel production systems because PPS is practically used by DEA models for measuring the efficiency of DMUs. Later, according to this PPS, we introduce a non-radial DEA model in the SBM framework [25] for parallel production systems. We'll then extend the methodology to series production systems. Two data sets from the literature account for the clarification of the new approaches and compare them with recent radial DEA models [8, 15, 17]. In this work, the non-radial DEA models based on SBM model have been used. This model evaluates both radial and non-radial inefficiencies.

The section contents are as follows: Section 2 develops a non-radial DEA model for evaluating the efficiency of parallel production system. In Section 3, we extend this model to include series production systems. Numerical examples are given for illustration in Section 4. The feedbacks are compared with the radial models proposed by Kao [17] and Kao and Hwang [15]. The conclusion is meant to appear in Section 5.

2. A SBM MODEL FOR PARALLEL PROCESSES

In this section, we introduce a non-radial DEA model based on the slacks-based measure (SBM) framework for measuring the efficiency of parallel processes by considering the parallel relationship between sub-processes.

Consider a parallel production process as shown in Figure 1. Suppose we have n DMUs, of which each DMU$_j$ ($j = 1, \ldots, n$) is made up of t_j units (sub-DMUs) related in parallel form. Working independently, each sub-DMU utilizes the same inputs to generate the same outputs. Sub-DMU$_p$ ($p = 1, \ldots, t_j$) includes m inputs x_{ij}^p (i =1, …, m) and s outputs y_{rj}^p ($r = 1, ..., s$). The sum of all x_{ij}^p over p, namely $\sum_{p=1}^{t_j} x_{ij}^p$, and the sum of all y_{rj}^p over p, namely $\sum_{p=1}^{t_j} y_{rj}^p$, are the ith input and rth output of the system DMU$_j$, respectively. This is shown in the following equation:

$$\sum_{p=1}^{t_j} x_j^p = x_j,$$

$$\sum_{p=1}^{t_j} y_j^p = y_j. \tag{2.1}$$

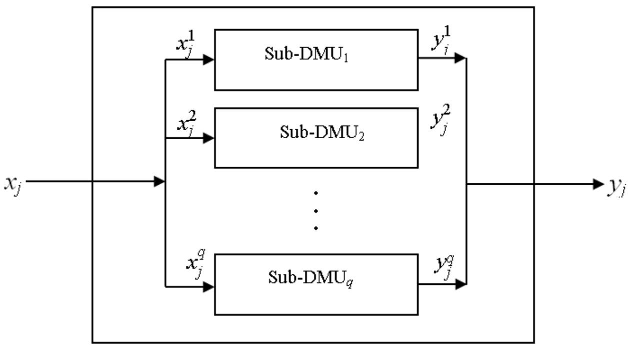

FIG. 1. The parallel production system

Following the conventional DEA approach, the PPS of Sub-DMU$_p$ $(p = 1, \dots, t_j)$ under the constant returns to scale (CRS) assumption is defined as:

$$T_C^p = \left\{ (x^p, y^p) \,\middle|\, x^p \geq \sum_{j=1}^{n} \lambda_j^p x_j^p, y^p \leq \sum_{j=1}^{n} \lambda_j^p y_j^p, \quad \lambda_j^p \geq 0, \quad j = 1, ..., n \right\}.$$

Based on the PPS T_C^p of individual sub-DMUs, the PPS of the whole parallel production system can be defined as the composition of its sub-DMUs, i.e.

$$T_C^{\text{parallel}} = \left\{ (x, y) \,\middle|\, x \geq \sum_{j=1}^{n} \sum_{p=1}^{t_j} \lambda_j^p x_j^p, y \leq \sum_{j=1}^{n} \sum_{p=1}^{t_j} \lambda_j^p y_j^p, \quad \lambda_j^p \geq 0, \quad j = 1, ..., n, p = 1, ..., t_j \right\}.$$

From (2.1), it can be realized that each DMU$_j$ $(j = 1, \dots, n)$ belongs to $T_C^{parallel}$. If all $\lambda_j^p, (p = 1, ..., t_j)$, associated with the sub-DMUs within the DMU$_j$ are the same, then T_C^{parallel} converts to the conventional PPS.

Suppose, DMU_o $(o \in \{1, ..., n\})$ be the DMU under evaluation. Based on T_C^{parallel}, for measuring the overall efficiency score of DMU_o, we formulate the following model:

$$
\begin{aligned}
& \rho_o^* = \min \rho_o = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^-}{y_{ro}}} \\
\text{s.t.} \qquad & x_o = \sum_{p=1}^{t_o} x_o^p = \sum_{j=1}^{n} \sum_{p=1}^{t_j} \lambda_j^p x_j^p + S^-, \\
& y_o = \sum_{p=1}^{t_o} y_o^p = \sum_{j=1}^{n} \sum_{p=1}^{t_j} \lambda_j^p y_j^p - S^+, \\
& \lambda_j^p, S^-, S^+ \geq 0, \quad p = 1, ..., t_j \quad j = 1, ..., n.
\end{aligned}
\tag{2.2}
$$

Model (2.2) takes advantage of the operation of the all sub-DMUs to measure the overall efficiency of DMU_o.

Note that model (2.2) is a fractional programming problem that can be switched into linear programming problem by using the Charnes-Cooper transformation [3, 25].

The following theorem explains the relationship between the efficiency of a parallel production system and its production units:

Theorem 2.1. *If* $DMU_k = (x_k, y_k)$ *is CRS-efficient, then each sub-*$DMU_p = (x_k^p, y_k^p)$ $(p = 1, \ldots, t_k)$ *of* DMU_k *is CRS-efficient.*

Proof. We prove this by contradiction.

Suppose any of sub-$\text{DMU}_p = (x_k^p, y_k^p)$ $(p = 1, \ldots, t_k)$ to be CRS-inefficient. We will show that there is $(\bar{x}, \bar{y}) \in T_C^{\text{parallel}}$ such that $\text{DMU}_k = (x_k, y_k)$ is dominated by $(\bar{x}, \bar{y})$.

Without loss of generality, we assume that sub-$\text{DMU}_1 = (x_k^1, y_k^1)$ is CRS-inefficient. Then, the following system has a solution $\{\lambda_j^{p*}, p = 1..., t_j, j = 1, ..., n; S^{-*}; S^{+*}\}$ with

$(S^{-*}, S^{+*}) \geq (0,0)$ and $(S^{-*}, S^{+*}) \neq (0,0)$.

$$x_k^1 = \sum_{j=1}^{n}\sum_{p=1}^{t_j} \lambda_j^{p*} x_j^p + S^{-*},$$

$$y_k^1 = \sum_{j=1}^{n}\sum_{p=1}^{t_j} \lambda_j^{p*} y_j^p - S^{+*}. \tag{2.3}$$

We set

$$\bar{x}_k^1 = x_k^1 - S^{-*} = \sum_{j=1}^{n}\sum_{p=1}^{t_j} \lambda_j^{p*} x_j^p,$$

$$\bar{y}_k^1 = y_k^1 + S^{+*} = \sum_{j=1}^{n}\sum_{p=1}^{t_j} \lambda_j^{p*} y_j^p. \tag{2.4}$$

From (2.3) and (2.4), we will have

$$x_k = \bar{x}_k^1 + \sum_{p=2}^{t_k} x_k^{p*} + S^{-*},$$

$$y_k = \bar{y}_k^1 + \sum_{p=2}^{t_k} y_k^{p*} - S^{+*}. \tag{2.5}$$

Now we define

$$\bar{x}_k = \bar{x}_k^1 + \sum_{p=2}^{t_k} x_k^{p*},$$

$$\bar{y}_k = \bar{y}_k^1 + \sum_{p=2}^{t_k} y_k^{p*}. \tag{2.6}$$

Since $(\bar{x}_k^1, \bar{y}_k^1) \in T_C^{parallel}$, thus $(\bar{x}_k, \bar{y}_k) \in T_C^{parallel}$. Hence we have

$$x_k = \bar{x}_k + S^{-*},$$

$$y_k = \bar{y}_k - S^{+*}. \tag{2.7}$$

Thus, $\mathrm{DMU}_k = (x_k, y_k)$ has non-zero slacks (S^{-*}, S^{+*}) versus $(\bar{x}_k, \bar{y}_k)$. Therefore, $\mathrm{DMU}_k = (x_k, y_k)$ is dominated by $(\bar{x}, \bar{y})$. Hence, $\mathrm{DMU}_k = (x_k, y_k)$ is CRS-inefficient which is in conflict with the assumption. □

Regard that the reverse of this theorem is not always true. By this we mean that it is possible for a parallel production system that all its sub-DMUs are efficient but the whole system is not.

It is worth noting that although this theorem has already been expressed by Kao [16, 17], it has not been proved theoretically. Here, for the first time, we proved this theorem theoretically. Moreover, unlike Kao's opinion, we showed that the reverse of this theorem is not always true.

As a contraposition to Theorem 1, we have

Theorem 2.2. *If any sub-DMU*$_p = (x_k^p, y_k^p)$ $(p = 1, \dots, t_k)$ *of DMU*$_k$ *is CRS-inefficient, then DMU*$_k = (x_k, y_k)$ *is CRS-inefficient.*

3. Extension to series processes

The proposed model to the series production systems is extended as the following. Consider a general series system as shown in Figure 2. Suppose, DMU$_j$ $(j = 1, \dots, n)$ is made up of t_j units (sub-DMUs) linked by intermediate products. Sub-DMU$_p$ $(p = 1, \dots, t_j)$ includes m inputs x_{ij}^p $(i = 1, \dots, m)$ and s outputs y_{rj}^p $(r = 1, ..., s)$. Denote z_{dj}^p $(d = 1, ..., D)$as the dth linking intermediate product from sub-DMU$_p$ to sub-DMU$_{(p+1)}$ of DMU$_j$. The total amount of input i used by DMU$_j$ is the sum of those used by all of its sub-DMUs. Also, the total amount of output r produced by DMU$_j$ is the sum of those produced by all of its sub-DMUs. Comparing Figure 1 with Figure 2, we find out that the series production system is the parallel production system with linking intermediate product.

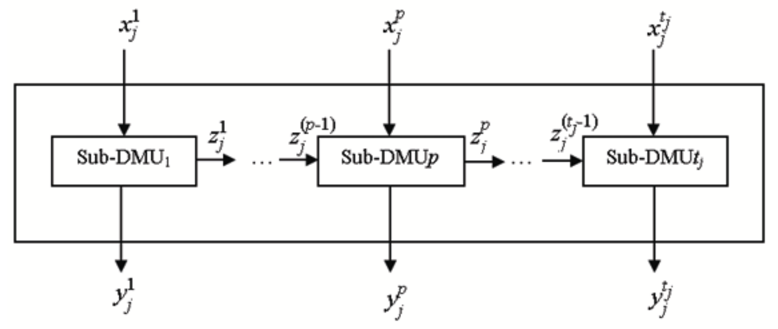

FIG. 2. The series production system

Similar to parallel production system, the PPS of series production systems can be defined as follows:

$$T_C^{\text{Series}} = \left\{ (x,y) \middle| x \geq \sum_{j=1}^{n} \sum_{p=1}^{t_j} \lambda_j^p x_j^p, y \leq \sum_{j=1}^{n} \sum_{p=1}^{t_j} \lambda_j^p y_j^p, \mathrm{z} \leq \sum_{j=1}^{n} \lambda_j^p z_j^p, z \geq \sum_{j=1}^{n} \lambda_j^{(p+1)} z_j^p,\ \lambda_j^p \geq 0,\ \ \forall j,p \right\},$$

where $z_j^p = (z_{1j}^p, ..., z_{Dj}^p)$ is the intermediate product vector of sub-DMU$_p$ of DMU$_j$.

Note that in T_C^{Series}, term $\sum_{j=1}^{n} \lambda_j^p z_j^p$ represents the level taken as the output of sub-DMU$_p$ and term $\sum_{j=1}^{n} \lambda_j^{(p+1)} z_j^p$ represents the level taken as the input of sub-DMU$(_{p+1})$.

According to the PPS T_C^{Series}, we evaluate the overall efficiency of DMU$_o$ by solving the following model:

$$\begin{aligned}
& \rho_o^* = \min \rho_o = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^-}{y_{ro}}} \\
s.t. \qquad & x_o = \sum_{p=1}^{t_o} x_o^p = \sum_{j=1}^{n} \sum_{p=1}^{t_j} \lambda_j^p x_j^p + S^-, \\
& y_o = \sum_{p=1}^{t_o} y_o^p = \sum_{j=1}^{n} \sum_{p=1}^{t_j} \lambda_j^p y_j^p - S^+, \\
& \sum_{j=1}^{n} \lambda_j^p z_j^p = \sum_{j=1}^{n} \lambda_j^{(p+1)} z_j^p, \quad p = 1, ..., (t_j - 1), \\
& \lambda_j^p, S^-, S^+ \geq 0, \quad p = 1, ..., t_j, \quad j = 1, ..., n, \qquad (3.1)
\end{aligned}$$

where, the continuity of intermediate products between sub-DMU$_p$ and sub-DMU$_{(p}$+1) of DMU$_j$is guaranteed by the following constraints:

$$\sum_{j=1}^{n} \lambda_j^p z_j^p = \sum_{j=1}^{n} \lambda_j^{(p+1)} z_j^p, \quad p = 1, ..., (t_j - 1). \tag{3.2}$$

On optimality, the condition $\sum_{j=1}^{n} \lambda_j^{p*} z_{dj}^p = \sum_{j=1}^{n} \lambda_j^{(p+1)*} z_{dj}^p$ $(d = 1, ..., D)$, means that the target value of intermediate product d when considered as output in sub-DMU$_p$ is equal to that considered as the input in sub-DMU$_{(p+1)}$.

In the efficiency measurement of DMUs, here are two differences point of view:

(1) The whole system is regarded as a black box and the connection between sub-DMUs neglected. Therefore only the first inputs and last outputs are considered.

(2) The system is a network of sub-DMUs where each sub-DMU has its own inputs and outputs.

4. Numerical examples

In this part, the proposed models are used to evaluate the efficiency of DMUs in two data sets. First, we apply the parallel SBM model to the data set of the national forests in Taiwan [13, 14, 17]. In Taiwan, the forest lands are divided into eight regions, each of which is divided into four or five sub-regions called working circles (WCs). These WCs are the basic parts in the management of the forest industry. The forest production process is a characteristic parallel production process, in that each region has various inferior WCs operating independently. Each WC is correlated to four inputs: Land (area in thousand hectares), Labor (number of employees in persons), Expenditures (money spent each year in ten thousand new Taiwan dollars) and Initial stocks (volume of forest stock before the period of evaluation in 10000 m^3) and three outputs: Timber production (timber produced each year in cubic meters), Soil conservation (volume of forest stock in 10000 m^3, as higher stock level leads to less soil erosion) and Recreation (visitors serviced by forests every year in thousands of visits). The data are provided in Table 1. For each input or output the quantity of a region is the sum of its sub-regions.

In order to compare the results with the input-oriented CCR model proposed by Kao [17], we apply the input-oriented parallel SBM model to evaluate the overall efficiencies of the eight forest regions with the 34 sub-regions. The overall efficiency scores for eight forest regions are calculated by applying input-oriented of model (2.2).

The results are reported in Table 2 under the heading parallel SBM model. If the operations of sub-DMUs are not taken into account, the efficiency scores of eight DMUs calculated by conventional SBM model are shown in Table 2 under the heading conventional SBM model. It is obvious that, six DMUs are efficient under the conventional SBM model, while under the parallel SBM model none of them performs efficiently. Actually, this occurs because in the efficiency measurement of a system by the conventional SBM model, the operations of individual processes are neglected.

The efficiency scores of the eight forest regions, based on Kao's approach, are reported in the last column of Table 2. It can be seen that none of the DMUs are efficient based on Kao's approach. Also, each sub-DMU is SBM-efficient if and only if it is CCR-efficient. However, a contradiction is observed in the Table 2 in the case of Sub-DMU 14, which shows that it is not CCR-efficient based on Kao's paper. However, we recalculated the efficiency score of this sub-DMU using CCR model by Lingo 9.0 programming and it was found that this sub-DMU is CCR-efficient and there was a mistake in reporting results of Kao's paper. Besides, as it was shown by Tone [25], the efficiency scores of DMUs calculated by parallel SBM model are not greater than the efficiency scores calculated by parallel CCR model. Moreover, a DMU is SBM-efficient if and only if it is CCR-efficient.

Finally, we apply model (3.1) to the data set of non-life insurance companies in Taiwan as studied in Kao and Hwang [15]. They divide the production process of the non-life insurance industry into two stages connected in series. The inputs of the system that are used only in stage 1 are Operating expenses and Insurance expenses. The intermediate products are direct written premiums and Reinsurance premiums. The outputs of the systems that are produced only in stage 2 are underwriting profit and Investment profit. The data are provided in Table 3. The overall efficiency score of 24 non-life insurance companies calculated by the input-oriented version of models (3.1) are displayed in second column of Table 4. Also, the last column of Table 4 shows the overall efficiency scores reported by Kao and Hwang [15]. The results indicate that none of the insurance companies performs efficiently. As expected, the efficiency scores of DMUs calculated by series SBM model are not greater than the efficiency scores calculated by series CCR model.

Table 1. Taiwan forests data

Working circles	**Input**					**Output**		
	Land	**labor**	**Expenditures**	**Initial stocks**		**Timber**	**Soil cons.**	**Recreation**
Lotung Region	**175.73**	**248.33**	**1581.60**	**1604.38**		**746.04**	**1604.01**	**207.59**
1. Taipei	18.23	45.33	608.32	125.46		19.59	125.46	0.00
2. Tai-ping-shan	55.49	98.00	336.33	584.85		17.70	584.85	207.59
3. Chao-chi	31.44	51.00	263.99	147.76		0.00	147.39	0.00
4. Nan-au	28.94	27.33	166.78	263.02		38.00	263.02	0.00
5. Ho-ping	41.63	26.67	206.18	483.29		670.75	483.29	0.00
Hsinchu Region	**162.81**	**316.67**	**850.05**	**2609.79**		**16823.42**	**2603.99**	**308.97**
6. Guay-shan	41.48	86.33	158.49	386.03		26.37	386.03	114.16
7. Ta-chi	29.72	58.00	260.02	638.87		42.53	638.87	181.01
8. Chu-tung	59.28	77.67	220.97	1218.07		1350.65	1214.48	13.80
9. Ta-hu	32.33	94.67	210.57	366.82		15403.87	364.61	0.00
Tungshi Region	**138.42**	**310.34**	**864.42**	**2348.03**		**4778.32**	**2819.48**	**264.92**
10. Shan-chi	10.40	50.67	218.55	103.86		2842.34	165.63	0.00
11. An-ma-shan	33.64	111.33	153.07	731.43		0.00	728.19	38.98
12. Li-yang	38.01	97.67	272.32	421.41		1935.98	558.17	111.26
13. Li-shan	56.37	50.67	220.48	1091.33		0.00	1367.49	114.68
Nantou Region	**211.82**	**287.32**	**1835.20**	**2352.10**		**11429.54**	**2343.86**	**0.00**
14. Tai-chung	10.57	64.33	319.51	39.12		3330.16	39.12	0.00
15. Tan-ta	52.69	49.00	340.54	688.60		1242.50	688.60	0.00
16. Pu-li	77.22	68.33	652.53	966.44		4134.43	966.44	0.00
17. Shui-li	54.29	59.33	348.33	602.24		2574.87	602.24	0.00
18. Chu-shan	17.05	46.33	174.29	55.70		147.58	47.46	0.00
Chiayi Region	**139.65**	**203.00**	**215.77**	**1316.48**		**1086.00**	**1330.10**	**845.05**
19. A-li-shan	42.81	69.33	62.51	527.44		0.00	527.40	845.05
20. Fan-chi-hu	19.28	35.33	54.71	96.00		1086.00	95.97	0.00
21. Ta-pu	32.86	44.67	60.41	196.30		0.00	195.85	0.00
22. Tai-nan	44.70	53.67	38.14	496.74		0.00	510.88	0.00
Pingtung Region	**196.06**	**250.33**	**1230.56**	**1588.02**		**7236.45**	**1588.02**	**939.69**
23. Chih-shan	35.64	61.33	37.92	150.90		1405.76	150.90	0.00
24. Chao-chou	70.19	62.00	188.12	624.80		1802.85	624.80	0.00
25. Liu-guay	70.96	55.67	461.42	722.46		4027.84	722.46	8.08
26. Heng-chun	19.27	71.33	543.10	89.86		0.00	89.86	931.61
Taitung Region	**226.54**	**141.67**	**755.20**	**2679.98**		**8086.47**	**2679.98**	**161.38**
27. Kuan-shan	113.42	54.67	272.35	1607.90		7669.57	1607.90	57.87
28. Chi-ben	44.54	41.00	184.65	552.13		416.90	552.13	103.51
29. Ta-wu	44.03	20.33	100.70	394.03		0.00	394.03	0.00
30. Chan- kong	24.55	25.67	197.50	125.92		0.00	125.92	0.00
Hualien Region	**320.43**	**284.00**	**1092.92**	**4001.21**		**2263.01**	**4410.58**	**53.19**
31. Shin-chan	85.95	64.00	314.71	1074.86		17.77	1085.88	0.00
32. Nan-hua	51.60	76.00	228.40	886.07		110.28	882.20	16.50
33. Wan-yong	59.53	74.00	282.01	829.11		339.91	819.16	0.00
34. Yu-li	123.35	70.00	267.80	1611.17		1795.05	1623.34	36.69

Table 2. Efficiency scores

Working circles	**Parallel SBM model**	**Conventional SBM model**	**Parallel CCR model**
Lotung Region	**0.410**	**0.589**	**0.752**
1. Taipei	0.305		0.667
2. Tai-ping-shan	0.484		0.760
3. Chao-chi	0.296		0.670
4. Nan-au	0.449		0.766
5. Ho-ping	0.618		0.789
Hsinchu Region	**0.731**	**1.000**	**0.823**
6. Guay-shan	0.467		0.781
7. Ta-chi	0.690		0.784
8. Chu-tung	0.822		0.799
9. Ta-hu	1.000		1.000
Tungshi Region	**0.715**	**1.000**	**0.937**
10. Shan-chi	1.000		1.000
11. An-ma-shan	0.674		0.791
12. Li-yang	1.000		1.000
13. Li-shan	1.000		1.000
Nantou Region	**0.529**	**0.732**	**0.773**
14. Tai-chung	1.000		1.000
15. Tan-ta	0.597		0.793
16. Pu-li	0.635		0.821
17. Shui-li	0.571		0.792
18. Chu-shan	0.231		0.341
Chiayi Region	**0.717**	**1.000**	**0.901**
19. A-li-shan	1.000		1.000
20. Fan-chi-hu	0.466		0.648
21. Ta-pu	0.432		0.686
22. Tai-nan	1.000		1.000
Pingtung Region	**0.553**	**1.000**	**0.799**
23. Chih-shan	0.571		0.760
24. Chao-chou	0.588		0.782
25. Liu-guay	0.626		0.769
26. Heng-chun	1.000		1.000
Taitung Region	**0.724**	**1.000**	**0.860**
27. Kuan-shan	1.000		1.000
28. Chi-ben	0.627		0.722
29. Ta-wu	0.629		0.778
30. Chan- kong	0.323		0.451
Hualien Region	**0.684**	**1.000**	**0.794**
31. Shin-chan	0.628		0.796
32. Nan-hua	0.610		0.788
33. Wan-yong	0.570		0.773
34. Yu-li	0.828		0.808

Finally, we apply model (3.1) to the data set of non-life insurance companies in Taiwan as studied in Kao and Hwang [15]. They divide the production process of the non-life insurance industry into two stages connected in series. The inputs of the system that are used only in stage 1 are Operating expenses and Insurance expenses. The intermediate products are direct written premiums and Reinsurance premiums. The outputs of the systems that are produced only in stage 2 are underwriting profit and Investment profit. The data are provided in Table 3. The overall efficiency score of 24 non-life insurance companies calculated by the input-oriented version of models (3.1) are displayed in second column of Table 4. Also, the last column of Table 4 shows the overall efficiency scores reported by Kao and Hwang. The results indicate that none of the insurance companies performs efficiently. As expected, the efficiency scores of DMUs calculated by series SBM model are not greater than the efficiency scores calculated by series CCR model. Some DMUs in traditional models are efficient but in this models are inefficient.

The optimal solution of SBM is not greater than the optimal solution of CCR. SBM model is non-radial approach and it neglected the radial models.

Table 3. Taiwan non-life insurance companies data

Company	Operation expenses(x_1)	Insurance expenses(x_2)	Direct written premiums(z_1)	Reinsurance premiums(z_2)	Underwriting profit(y_1)	Investment profit(y_2)
1 Taiwan Fire	1,178,744	673,512	7,451,757	856,735	984,143	681,687
2 Chung Kuo	1,381,822	1,352,755	10,020,274	1,812,894	1,228,502	834,754
3 Tai Ping	1,177,494	592,790	4,776,548	560,244	293,613	658,428
4 China Mariners	601,320	594,259	3,174,851	371,863	248,709	177,331
5 Fubon	6,627,707	3,531,614	37,392,862	1,753,794	7,851,229	3,925,272
6 Zurich	2,627,707	668,363	9,747,908	952,326	1,713,598	415,058
7 Taian	1,942,833	1,443,100	10,685,457	643,412	2,239,593	439,039
8 Ming Tai	3,789,001	1,873,530	17,267,266	1,134,600	3,899,530	622,868
9 Central	1,567,746	950,432	11,473,162	546,337	1,043,778	264,098
10 The First	1,303,249	1,298,470	8,210,389	504,528	1,697,941	554,806
11 Kuo Hua	1,962,448	672,414	7,222,378	643,178	1,486,014	18,259
12 Union	2,592,790	650,952	9,434,406	1,118,489	1,574,191	909,295
13 Shingkong	2,609,941	1,368,802	13,921,464	811,343	3,609,236	223,047
14 South China	1,396,002	988,888	7,396,396	465,509	1,401,200	332,283
15 Cathay Century	2,184,944	651,063	10,422,297	749,893	3,355,197	555,482
16 Allianz president	211,716	415,071	5,606,013	402,881	854,054	197,947
17 Newa	1,453,797	1,085,019	7,695,461	342,489	3,144,484	371,984
18 AIU	757,515	547,997	3,631,484	995,620	692,731	163,927
19 North America	159,422	182,338	1,141,950	483,291	519,121	46,857
20 Federal	145,442	53,518	316,829	131,920	355,624	26,537
21 Royal Sunalliance	84,171	26,224	225,888	40,542	51,950	6491
22 Asia	15,993	10,502	52,063	14,574	82,141	4181
23 AXA	54,693	28,408	245,910	49,864	0.1	18,980
24 Mitsui Sumitomo	163,297	235,094	476,419	644,816	142,370	16,976

TABLE 4. Efficiency scores of companies

Company	**Series SBM model**	**Series CCR model**
1 Taiwan Fire	0.679	0.699
2 Chung Kuo	0.545	0.625
3 Tai Ping	0.669	0.690
4 China Mariners	0.264	0.304
5 Fubon	0.742	0.767
6 Zurich	0.350	0.390
7 Taian	0.273	0.277
8 Ming Tai	0.263	0.275
9 Central	0.216	0.223
10 The First	0.410	0.466
11 Kuo Hua	0.124	0.164
12 Union	0.687	0.760
13 Shingkong	0.187	0.208
14 South China	0.284	0.289
15 Cathay Century	0.559	0.614
16 Allianz president	0.305	0.320
17 Newa	0.353	0.360
18 AIU	0.256	0.259
19 North America	0.361	0.411
20 Federal	0.464	0.547
21 Royal Sunalliance	0.179	0.201
22 Asia	0.548	0.590
23 AXA	0.408	0.420
24 Mitsui Sumitomo	0.106	0.135

5. CONCLUSIONS

Earlier papers have introduced two radial DEA models offered by Kao [15] and Kao and Hwang [14] for measuring the efficiency of parallel and series production systems.

In this paper, by defining the production possibility set (PPS) for the parallel and series production systems, an alternative methodology based on slacks-based measure (SBM) framework is adopted in order to evaluate the overall efficiency of the systems by considering the operation of its units. Unlike Kao's model and Kao and Hwang's model, the proposed models are non-radial and enable us to deal with inputs and outputs individually while being changeable non-proportionally.

The proposed models are based on the assumption of constant returns to scale (CRS). Having added the convexity constraint into the PPS of parallel and series systems, the discussion can be expanded to variable returns to scale (VRS) assumption.

It is noteworthy that real systems are generally more complex than the parallel and series systems discussed in this paper. Since the parallel and series structure are two basic structures of a network system, we can transform a network system into a combination of series and parallel structures to evaluate the overall efficiency of the whole system by taking into account the operations of the processes within the system.

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(1) Faculty of Mathematics, Statistics and Computer Science, Semnan University,
Semnan, Iran

E-mail address: `a_ashrafi@semnan.ac.ir`

(2) Faculty of Mathematics, Statistics and Computer Science, Semnan University,
Semnan, Iran

E-mail address: `Mozhganmansouri953@gmail.com`