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# ON AN OPENNESS WHICH IS PLACED BETWEEN TOPOLOGY AND LEVINE'S OPENNESS

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Abstract. The goal of this present paper is to investigate new relationships of $I_g^*$-openness. The notion of $I_g^*$-open sets was introduced by Ekici in 2011. $I_g^*$-openness is placed between topology and Levine's openness. New investigations among $I_g^*$-openness, topology and maps are introduced.

## 1. Introduction

In all brances of mathematics, we can observe the applications of maps or sets (e.g. [1, 13]). In many of these applications, we observe sets or maps and so topology in background of these studies. There exist the examples and applications of these studies in the literature, e.g. in physics, computer science, engineering, computer graphics etc. (e.g. [8, 9, 10, 14]). Meanwhile, the applications of maps and sets have been occured in ideal spaces with topology (e.g. [2, 5, 6]). In 2011, Ekici introduced the notion of $I_g^*$-open sets. $I_g^*$-openness is placed between topology and Levine's openness. In this present paper, the notion of $O^\star$-open maps is introduced and new relationships of $I_g^*$-openness are investigated. New properties among $I_g^*$-openness, topology and maps are introduced.

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$\Im$ denotes a topological space with the topology $\vartheta$ in this paper. For a set $G$ in $\Im$, the interior of $G$ is denoted by $\hat{\imath}(G)$ and the closure of $G$ is denoted by $\hat{\mathbf{c}}(G)$.

An ideal $\ell$ on a nonempty set $\Im$ is a nonempty class of sets in $\Im$ such that the following two conditions are satisfied

(1) If $G \in \ell$ and $V \subseteq G$, then $V \in \ell$,

(2) If $G$, $V \in \ell$, then $G \cup V \in \ell$ [11].

For a topological space $\Im$ and an ideal $\ell$ on $\Im$, the operator $(\ )^* : P(\Im) \longrightarrow P(\Im)$, $G^*(\ell, \vartheta) = \{x \in \Im \mid V \cap G \notin \ell \text{ for each } V \in \vartheta \text{ with } x \in V\}$ for $G \subseteq \Im$, is called local function [11] of $G$ where $P(\Im)$ is the power set of $\Im$.

For a topological space $\Im$ and an ideal $\ell$ on $\Im$, $\hat{\mathbf{c}}^*(G) = G \cup G^*(\ell, \vartheta)$ is a Kuratowski closure operator for a topology $\vartheta^*(\ell, \vartheta)$ and $\vartheta^*$ is called $\star$-topology [7]. The elements of $\vartheta^*$ are called $\star$-open sets and the complement of a $\star$-open set is called $\star$-closed [7]. For any set $G$ in a topological space $\Im$, $G$ is called $g$-closed [12] if $\hat{\mathbf{c}}(G) \subseteq H$ whenever $G \subseteq H$ and $H$ is open in $\Im$ and $G$ is called $g$-open [12] if $\Im \setminus G$ is $g$-closed.

**Definition 1.1.** ([3]) Let $\Im$ be a topological space and $\ell$ be an ideal on $\Im$. For a set $G$ in $\Im$, $G$ is called

(1) $I_g^*$-closed if $\hat{\mathbf{c}}(G) \subseteq H$ for each $\star$-open set $H$ in $\Im$ such that $G \subseteq H$.

(2) $I_g^*$-open if $\Im \setminus G$ is an $I_g^*$-closed set in $\Im$.

Remark 1. ([3]) Let $\Im$ be a topological space and $\ell$ be an ideal on $\Im$. For a set $G$ in $\Im$, the below implications hold and reverses of these implications are not true in general as shown in [3]:

$$\begin{array}{ccc} I_g^*\text{-open} & \longrightarrow & g\text{-open} \\ \uparrow & & \\ \text{open} & & \\ \downarrow & & \\ \star\text{-open} & & \end{array}$$

## 2. $I_g^*$-OPENNESS, TOPOLOGY AND MAPS

**Definition 2.1.** Let $\Im_1$ and $\Im_2$ be topological spaces and $\ell$, $\ell'$ be ideals on $\Im_1$ and $\Im_2$, respectively. For a function $\hbar : \Im_1 \longrightarrow \Im_2$, $\hbar$ is called $O^\star$-open if $\hat{\mathbf{c}}(V) \subseteq \hbar(G)$ for each $\star$-open set $G$ in $\Im_1$ and each $I_g^*$-open and $I_g^*$-closed set $V$ in $\Im_2$ such that $V \subseteq \hbar(G)$.

**Definition 2.2.** Let $\Im$ be a topological space with an ideal $\ell$. Then $\Im$ is said to be a $T_{I_g^* \leftrightarrows OC}$ ideal space if each $I_g^*$-open and $I_g^*$-closed set in $\Im$ is open and closed.

**Theorem 2.1.** *If a function $\hbar : \Im_1 \longrightarrow \Im_2$ is $O^\star$-open for every topological space $\Im_1$ with any ideal $\ell$, then the topological space $\Im_2$ with an ideal $\ell'$ is a $T_{I_g^* \leftrightarrows OC}$ space.*

*Proof.* For a topological space $\Im_2$ with an ideal $\ell'$, let $G \subseteq \Im_2$ be an $I_g^*$-open and $I_g^*$-closed set. Take the identity function $\hbar : \Im_2 \longrightarrow \Im_2$ with the same ideal where $\{\Im_2, \varnothing, G, \Im_2 \setminus G\}$ is the topology on the domain $\Im_2$.

By the hypothesis, $\hbar : \Im_2 \longrightarrow \Im_2$ is an $O^\star$-open function. For $I_g^*$-open and $I_g^*$-closed set $G \subseteq \Im_2$ and $\star$-open set $G \subseteq \Im_2$, since $G \subseteq \hbar(G)$, we have

$$\hat{\mathbf{c}}(G) \subseteq \hbar(G) = G.$$

Consequently, $G$ is a closed set in $\Im_2$.

On the other hand, for $I_g^*$-open and $I_g^*$-closed set $\Im_2 \setminus G \subseteq \Im_2$ and $\star$-open set $\Im_1 \setminus G \subseteq \Im_1$, since

$$\Im_2 \setminus G \subseteq \hbar(\Im_1 \setminus G),$$

we have

$$\begin{aligned} & \hat{\mathbf{c}}(\Im_2 \setminus G) \\ \subseteq \; & \hbar(\Im_1 \setminus G) = \Im_2 \setminus G. \end{aligned}$$

Consequently, $\Im_2 \setminus G$ is a closed set in $\Im_2$.

Hence, the topological space $\Im_2$ with an ideal $\ell'$ is a $T_{I_g^* \leftrightarrows OC}$ space. □

**Theorem 2.2.** *Suppose that a topological space $\Im_2$ with an ideal $\ell'$ is a $T_{I_g^* \leftrightarrows OC}$ space. Then a function $\hbar : \Im_1 \longrightarrow \Im_2$ is $O^\star$-open for every topological space $\Im_1$ with any ideal $\ell$.*

*Proof.* Assume that a topological space $\Im_2$ with an ideal $\ell'$ is a $T_{I_g^* \leftrightarrows OC}$ space. Let $W$ be a $\star$-open set in $\Im_1$ and $V \subseteq \Im_2$ be an $I_g^*$-open and $I_g^*$-closed set such that $V \subseteq \hbar(W)$. Since $\Im_2$ is an $T_{I_g^* \leftrightarrows OC}$ space, then $V$ is closed and open. Thus, $\hat{\mathbf{c}}(V) \subseteq \hbar(W)$. Consequently, $\hbar$ is $O^\star$-open. □

**Corollary 2.1.** *A function $\hbar : \Im_1 \longrightarrow \Im_2$ is $O^\star$-open for every topological space $\Im_1$ with any ideal $\ell$ if and only if topological space $\Im_2$ with an ideal $\ell'$ is a $T_{I_g^* \leftrightarrows OC}$ space.*

*Proof.* This follows from Theorem 2.1 and 2.2. □

**Theorem 2.3.** ([3]) *Let $\Im$ be a topological space with an ideal $\ell$ and $G \subseteq \Im$. Then $G$ is $I_g^*$-open if and only if $H \subseteq \hat{\imath}(G)$ for each $\star$-closed set $H$ in $\Im$ such that $H \subseteq G$.*

**Theorem 2.4.** *Let $\hbar : \Im_1 \longrightarrow \Im_2$ be an $O^\star$-open continuous and bijective function where $\Im_1$ and $\Im_2$ are topological spaces and $\ell$, $\ell'$ be ideals on $\Im_1$ and $\Im_2$, respectively. Then $\hbar^{-1}(G)$ is an $I_g^*$-open set in $\Im_1$ for any $I_g^*$-open and $I_g^*$-closed set $G$ in $\Im_2$.*

*Proof.* Let $V$ be a $\star$-closed set in $\Im_1$ and $G$ be an $I_g^*$-open and $I_g^*$-closed set in $\Im_2$ such that $V \subseteq \hbar^{-1}(G)$. Since $V \subseteq \hbar^{-1}(G)$, we have

$$\begin{aligned} & \Im_1 \setminus \hbar^{-1}(G) \\ = \; & \hbar^{-1}(\Im_2 \setminus G) \subseteq \Im_1 \setminus V. \end{aligned}$$

Furthermore, we have

$$\Im_2 \setminus G \subseteq \hbar(\Im_1 \setminus V).$$

This implies that $\hat{\mathbf{c}}(\Im_2 \setminus G) \subseteq \hbar(\Im_1 \setminus V)$ and then $\Im_2 \setminus \hat{\imath}(G) \subseteq \hbar(\Im_1 \setminus V)$. We have

$$\begin{aligned} & \hbar^{-1}(\Im_2 \setminus \hat{\imath}(G)) \\ = \; & \Im_1 \setminus \hbar^{-1}(\hat{\imath}(G)) \subseteq \Im_1 \setminus V. \end{aligned}$$

Then $V \subseteq \hbar^{-1}(\hat{\imath}(G))$. Furthermore, we have

$$\begin{aligned} V &\subseteq \hbar^{-1}(\hat{\imath}(G)) \\ &= \hat{\imath}(\hbar^{-1}(\hat{\imath}(G))) \\ &\subseteq \hat{\imath}(\hbar^{-1}(G)). \end{aligned}$$

Consequently, $V \subseteq \hat{\imath}(\hbar^{-1}(G))$ and hence, $\hbar^{-1}(G)$ is $I_g^*$-open. □

Remark 2. Example 2.1 enable us to realize that Theorem 2.4 is not true without continuity and bijectiveness.

**Example 2.1.** *Assume $\Im = \{t_1, t_2, t_3, t_4\}$, $\vartheta = \{\Im, \varnothing, \{t_1\}, \{t_1, t_2\}, \{t_3, t_4\}, \{t_1, t_3, t_4\}\}$ and the ideal $\ell = \{\varnothing, \{t_1\}, \{t_4\}, \{t_1, t_4\}\}$. Take $\hbar : \Im \longrightarrow \Im$, $\hbar(t_1) = t_4$, $\hbar(t_2) = t_1$,*

*$\hbar(t_3) = t_1$, $\hbar(t_4) = t_4$. Then $\hbar$ is $O^\star$-open but it is not continuous. Also, for $I_g^*$-open and $I_g^*$-closed set $\{t_1, t_2\}$, $\hbar^{-1}(\{t_1, t_2\})$ is not $I_g^*$-open.*

**Corollary 2.2.** *Let $\hbar : \Im_1 \longrightarrow \Im_2$ be an $O^\star$-open continuous and bijective function where $\Im_1$ and $\Im_2$ are topological spaces and $\ell$, $\ell'$ be ideals on $\Im_1$ and $\Im_2$, respectively. Then $\hbar^{-1}(G)$ is an $I_g^*$-open and $I_g^*$-closed set in $\Im_1$ for any $I_g^*$-open and $I_g^*$-closed set $G$ in $\Im_2$.*

*Proof.* This follows from Theorem 2.4. □

**Theorem 2.5.** *Let $\Im_1$ and $\Im_2$ be topological spaces and $\ell$, $\ell'$ be ideals on $\Im_1$ and $\Im_2$, respectively. Suppose $\hbar : \Im_1 \longrightarrow \Im_2$ is an $O^\star$-open function. Then $V^* \subseteq \hbar(G)$ for each $\star$-open set $G$ in $\Im_1$ and each $I_g^*$-open and $I_g^*$-closed set $V$ in $\Im_2$ such that $V \subseteq \hbar(G)$.*

*Proof.* Suppose that $\hbar : \Im_1 \longrightarrow \Im_2$ is an $O^\star$-open function. For a $\star$-open set $G$ in $\Im_1$ and an $I_g^*$-open and $I_g^*$-closed set $V$ in $\Im_2$ such that $V \subseteq \hbar(G)$, since $\hbar$ is $O^\star$-open, then $\hat{\mathbf{c}}(V) \subseteq \hbar(G)$. We have $V^* \subseteq \hat{\mathbf{c}}(V) \subseteq \hbar(G)$. Consequently, we have $V^* \subseteq \hbar(G)$. □

## 3. The relationships

**Theorem 3.1.** *Let $\hbar : \Im_1 \longrightarrow \Im_2$ be a function where $\Im_1$ and $\Im_2$ are topological spaces and $\ell$, $\ell'$ be ideals on $\Im_1$ and $\Im_2$, respectively. If $\hbar(G)$ is closed in $\Im_2$ for each $\star$-open set $G$ in $\Im_1$, then $\hbar$ is $O^\star$-open.*

*Proof.* For any $\star$-open set $G$ in $\Im_1$ and any $I_g^*$-open and $I_g^*$-closed set $V$ in $\Im_2$ such that $V \subseteq \hbar(G)$, since $V \subseteq \hbar(G)$ and $\hbar(G)$ is closed in $\Im_2$ for $\star$-open set $G$ in $\Im_1$, then

$$\begin{aligned} \hat{\mathbf{c}}(V) &\subseteq \hat{\mathbf{c}}(\hbar(G)) \\ &= \hbar(G). \end{aligned}$$

Consequently, $\hbar$ is $O^\star$-open. □

Remark 3. The converse of Theorem 3.1 is generally false. The below example shows this situation:

**Example 3.1.** *Assume* $\Im = \{t_1, t_2, t_3, t_4\}$, $\vartheta = \{\Im, \varnothing, \{t_1\}, \{t_1, t_2\}, \{t_3, t_4\}, \{t_1, t_3, t_4\}\}$ *and the ideal* $\ell = \{\varnothing, \{t_1\}, \{t_4\}, \{t_1, t_4\}\}$. *Take* $\hbar : \Im \longrightarrow \Im$, $\hbar(t_1) = t_3$, $\hbar(t_2) = t_3$, $\hbar(t_3) = t_1$, $\hbar(t_4) = t_1$. *Then* $\hbar$ *is* $O^\star$*-open but* $\hbar(\{t_1\}) = \{t_3\}$ *is not closed.*

**Theorem 3.2.** *Let* $\hbar : \Im_1 \longrightarrow \Im_2$ *be a function where* $\Im_1$ *and* $\Im_2$ *are topological spaces and* $\ell$, $\ell'$ *be ideals on* $\Im_1$ *and* $\Im_2$*, respectively. If* $\hbar$ *is* $O^\star$*-open and each set in* $\Im_2$ *is* $I_g^*$*-closed, then* $\hbar(G)$ *is closed in* $\Im_2$ *for each* $\star$*-open set* $G$ *in* $\Im_1$*.*

*Proof.* Let $G$ be a $\star$-open set in $\Im_1$. Since each set in $\Im_2$ is $I_g^*$-closed, $\hbar(G)$ is an $I_g^*$-open and $I_g^*$-closed set in $\Im_2$. Since $\hbar$ is an $O^\star$-open function, then we have $\hat{\mathbf{c}}(\hbar(G)) \subseteq \hbar(G)$. Consequently, $\hbar(G)$ is closed in $\Im_2$. □

**Corollary 3.1.** *Let* $\hbar : \Im_1 \longrightarrow \Im_2$ *be a function where* $\Im_1$ *and* $\Im_2$ *are topological spaces and* $\ell$, $\ell'$ *be ideals on* $\Im_1$ *and* $\Im_2$*, respectively. If each set in* $\Im_2$ *is* $I_g^*$*-closed, then* $\hbar$ *is* $O^\star$*-open if and only if* $\hbar(G)$ *is closed in* $\Im_2$ *for each* $\star$*-open set* $G$ *in* $\Im_1$*.*

*Proof.* This follows from Theorem 3.1 and 3.2. □

**Definition 3.1.** Let $\hbar : \Im_1 \longrightarrow \Im_2$ be a function where $\Im_1$ and $\Im_2$ are topological spaces and $\ell$, $\ell'$ be ideals on $\Im_1$ and $\Im_2$, respectively. $\hbar$ is said to be

(1) $\star$-open [4] if $\hbar(G)$ is $\star$-open for every $\star$-open subset $G \subseteq \Im_1$.

(2) $\star$-closed [3] if $\hbar(G)$ is $\star$-closed for every $\star$-closed subset $G \subseteq \Im_1$.

Remark 4. Let $\hbar : \Im_1 \longrightarrow \Im_2$ be a function where $\Im_1$ and $\Im_2$ are topological spaces and $\ell$, $\ell'$ be ideals on $\Im_1$ and $\Im_2$, respectively. If $\hbar$ is $\star$-open, then $\hbar$ is $O^\star$-open. The converse of this implication is generally false. The below example shows this situation:

**Example 3.2.** *Assume* $\Im = \{t_1,\ t_2,\ t_3,\ t_4\}$, $\vartheta = \{\Im,\ \varnothing,\ \{t_1\},\ \{t_2, t_3\},\ \{t_1, t_2, t_3\}\}$ *and the ideal* $\ell = \{\varnothing,\ \{t_1\},\ \{t_4\},\ \{t_1, t_4\}\}$. *Take* $\hbar : \Im \longrightarrow \Im$, $\hbar(t_1) = t_4$, $\hbar(t_2) = t_4$, $\hbar(t_3) = t_1$, $\hbar(t_4) = t_1$. *Then* $\hbar$ *is* $O^\star$*-open but* $\hbar$ *is not* $\star$*-open.*

Remark 5. Let $\hbar : \Im_1 \longrightarrow \Im_2$ be a function where $\Im_1$ and $\Im_2$ are topological spaces and $\ell$, $\ell'$ be ideals on $\Im_1$ and $\Im_2$, respectively. By Theorem 3.1 , Remarks 3 and 4, the below imlications hold:

$$\begin{array}{ccc} \hbar(G) \text{ is closed in } \Im_2 \text{ for every} \star\text{-open set } G \text{ in } \Im_1 & \longrightarrow & O^\star\text{-open} \\ & & \uparrow \\ & & \star\text{-open} \end{array}$$

**Corollary 3.2.** *Let* $\hbar : \Im_1 \longrightarrow \Im_2$ *be a continuous bijective function where* $\Im_1$ *and* $\Im_2$ *are topological spaces and* $\ell$, $\ell'$ *be ideals on* $\Im_1$ *and* $\Im_2$, *respectively.*

*(1) If* $\hbar$ *is* $\star$*-open, then* $\hbar^{-1}(G)$ *is an* $I_g^*$*-open and* $I_g^*$*-closed set in* $\Im_1$ *for any* $I_g^*$*-open and* $I_g^*$*-closed set* $G$ *in* $\Im_2$.

*(2) If* $\hbar$ *is* $\star$*-closed, then* $\hbar^{-1}(G)$ *is an* $I_g^*$*-open and* $I_g^*$*-closed set in* $\Im_1$ *for any* $I_g^*$*-open and* $I_g^*$*-closed set* $G$ *in* $\Im_2$.

*Proof.* This follows from Theorem 2.4 and Remark 5. □

Remark 6. The following example shows that the composition of two $O^\star$-open maps need not be $O^\star$-open.

**Example 3.3.** *Assume* $\Im_1 = \{t_1, t_2, t_3, t_4, t_5\}$, $\vartheta_1 = \{\Im_1,\ \varnothing,\ \{t_1\},\ \{t_4\},\ \{t_1, t_2\},\ \{t_1, t_4\},\ \{t_1, t_2, t_4\}\}$, *the ideal* $\ell_1 = \{\varnothing,\ \{t_2\}\}$ *and* $\Im_2 = \{t_1, t_2, t_3, t_4\}$, $\vartheta_2 = \{\Im_2,$

*$\varnothing$, $\{t_1\}$, $\{t_1, t_2\}$, $\{t_3, t_4\}$, $\{t_1, t_3, t_4\}\}$, the ideal $\ell_2 = \{\varnothing, \{t_1\}, \{t_4\}, \{t_1, t_4\}\}$. Take $\hbar_1 : \Im_1 \longrightarrow \Im_1$, $\hbar_1(t_1) = t_1$, $\hbar_1(t_2) = t_1$, $\hbar_1(t_3) = t_4$, $\hbar_1(t_4) = t_4$, $\hbar_1(t_5) = t_4$ and $\hbar_2 : \Im_1 \longrightarrow \Im_2$, $\hbar_2(t_1) = t_3$, $\hbar_2(t_2) = t_3$, $\hbar_2(t_3) = t_3$, $\hbar_2(t_4) = t_3$, $\hbar_2(t_5) = t_3$. Then $\hbar_1$ and $\hbar_2$ are $O^\star$-open but $\hbar_2 o \hbar_1$ is not $O^\star$-open.*

**Definition 3.2.** Let $\hbar : \Im_1 \longrightarrow \Im_2$ be a function where $\Im_1$ and $\Im_2$ are topological spaces and $\ell$, $\ell'$ be ideals on $\Im_1$ and $\Im_2$, respectively. $\hbar$ is called $I_g^*$-irresolute if $\hbar^{-1}(G)$ is $I_g^*$-closed for each $I_g^*$-closed subset $G \subseteq \Im_2$.

**Theorem 3.3.** *Let $\hbar_1 : \Im_1 \longrightarrow \Im_2$, $\hbar_2 : \Im_2 \longrightarrow \Im_3$ be functions where $\Im_1$, $\Im_2$ and $\Im_3$ are topological spaces and $\ell$, $\ell'$ and $\ell''$ be ideals on $\Im_1$, $\Im_2$ and $\Im_3$, respectively. Suppose that $\hbar_1$ is $O^\star$-open and $\hbar_2$ is closed, bijective $I_g^*$-irresolute. Then $\hbar_2 o \hbar_1$ is $O^\star$-open.*

*Proof.* For any $I_g^*$-open and $I_g^*$-closed set $V$ in $\Im_3$ and any $\star$-open set $G$ in $\Im_1$ such that $V \subseteq (\hbar_2 o \hbar_1)(G)$, we have $\hbar_2^{-1}(V) \subseteq \hbar_1(G)$. Since $\hbar_2^{-1}(V)$ is $I_g^*$-open and $I_g^*$-closed and also $\hbar_1$ is $O^\star$-open, then $\hat{\mathbf{c}}(\hbar_2^{-1}(V)) \subseteq \hbar_1(G)$. Furthermore, we have $\hbar_2(\hat{\mathbf{c}}(\hbar_2^{-1}(V))) \subseteq \hbar_2(\hbar_1(G)) = (\hbar_2 o \hbar_1)(G)$. This implies that

$$\begin{aligned} \hat{\mathbf{c}}(V) &\subseteq \hat{\mathbf{c}}(\hbar_2(\hbar_2^{-1}(V)) \\ &\subseteq \hbar_2(\hat{\mathbf{c}}(\hbar_2^{-1}(V))). \end{aligned}$$

We have $\hat{\mathbf{c}}(V) \subseteq (\hbar_2 o \hbar_1)(G)$. Consequently, $\hbar_2 o \hbar_1$ is $O^\star$-open. □

**Corollary 3.3.** *Let $\hbar_1 : \Im_1 \longrightarrow \Im_2$, $\hbar_2 : \Im_2 \longrightarrow \Im_3$ be functions where $\Im_1$, $\Im_2$ and $\Im_3$ are topological spaces and $\ell$, $\ell'$ and $\ell''$ be ideals on $\Im_1$, $\Im_2$ and $\Im_3$, respectively. Suppose that $\hbar_1$ is $O^\star$-open and $\hbar_2$ is open, bijective $I_g^*$-irresolute. Then $\hbar_2 o \hbar_1$ is $O^\star$-open.*

*Proof.* This follows from Theorem 3.3. □

**Theorem 3.4.** *Let* $\hbar_1 : \Im_1 \longrightarrow \Im_2$, $\hbar_2 : \Im_2 \longrightarrow \Im_3$ *be functions where* $\Im_1$, $\Im_2$ *and* $\Im_3$ *are topological spaces and* $\ell$, $\ell'$ *and* $\ell''$ *be ideals on* $\Im_1$, $\Im_2$ *and* $\Im_3$, *respectively. Suppose that* $\hbar_1$ *is* $\star$*-open and* $\hbar_2$ *is* $O^\star$*-open. Then* $\hbar_2 o \hbar_1$ *is* $O^\star$*-open.*

*Proof.* For any $I_g^*$-open and $I_g^*$-closed set $V$ in $\Im_3$ and any $\star$-open set $G$ in $\Im_1$ such that $V \subseteq (\hbar_2 o \hbar_1)(G)$, since $\hbar_1$ is $\star$-open, then $\hbar_1(G)$ is $\star$-open. Also since $\hbar_2$ is $O^\star$-open, we have

$$\begin{aligned} \hat{\mathbf{c}}(V) &\subseteq \hbar_2(\hbar_1(G)) \\ &= (\hbar_2 o \hbar_1)(G). \end{aligned}$$

This implies that $\hat{\mathbf{c}}(V) \subseteq (\hbar_2 o \hbar_1)(G)$. Consequently, $\hbar_2 o \hbar_1$ is $O^\star$-open. □

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