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Communication

Reflectance Enhancement Factor Associated with Coherent Interference of Light in an Unbacked or Embedded Quarter-Wave Layer

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Abstract: If a semi-infinite transparent substrate is replaced by a quarter-wave layer (QWL) of the same material, which is embedded in the same ambient, the intensity reflectance is increased by a significant factor. A simple expression is derived for this reflectance enhancement factor (REF) η that results from coherent multiple-beam interference of monochromatic light within the QWL. The REF η is a monotonically decreasing function of the Fresnel intensity reflectance at the ambient-layer interface with maximum and minimum values of 4 and 1, respectively. The expression for η is applicable for the p and s linear polarizations, at any angle of incidence and for any wavelength within the common transparency bandwidth of both layer and ambient. The results are particularly relevant to light reflection by index-near-one materials and Brewster-angle reflection polarizers that use high-index, IR-transparent semiconductors such as Ge and Si, in bulk and pellicle form.

Keywords: Interference, Reflection, Polarization, Thin films.

Introduction

Thin film optics is a well-established field with many important applications (see, e.g. [1 – 5]). For example, it is well known that the reflectance of a surface can be deliberately increased or decreased by exploiting the coherent interference of light in transparent thin films.

In this paper, an expression is derived for the reflectance enhancement factor (REF) that results when a semi-infinite transparent substrate is replaced by an unbacked quarter-wave layer (QWL) of the same material. The functional expression of the REF, associated with coherent multiple-beam interference in the QWL, is independent of incident light polarization (p or s), wavelength and angle of incidence. The significance of the REF is demonstrated by two examples: (a) the reflection of light by index-near-one (INO) skeletal or porous materials [6] and (b) Brewster-angle reflection polarizers that use high-index, IR-transparent semiconductor materials both in bulk and thin-film (pellicle) form.

Reflectance Enhancement Factor

The reflection of p- and s-polarized monochromatic light at the planar interface between two transparent media (e.g. air and glass, denoted by 0 and 1) at an angle of incidence ϕ is governed by the well-known Fresnel reflection coefficients $r_{01\nu}, \nu = p,s$ [7]. If the semi-infinite substrate is truncated to a thin film of uniform thickness d and the exit medium below the dashed line in Fig. 1 is the same as the ambient medium 0, the complex-amplitude reflection coefficients that account for coherent multiple reflections within the unbacked layer are given by [8]:

$$R_\nu = \frac{r_{01\nu}(1-X)}{(1-r_{01\nu}{}^2 X)}, \nu = p,s. \quad (1)$$

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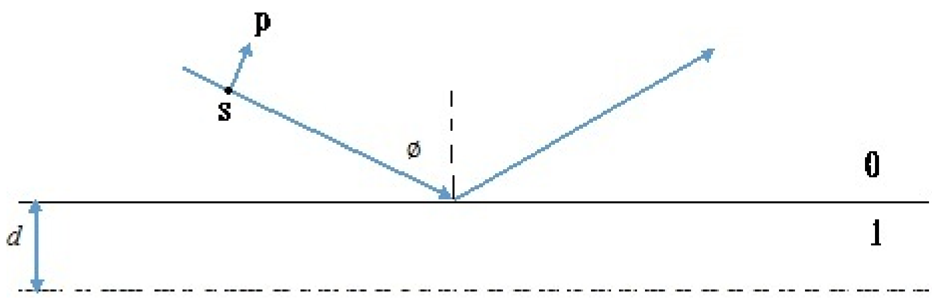

FIG. 1. Reflection of p- and s-polarized light at the interface between two transparent media 0 and 1. Reflectance enhancement due to the coherent interference of light within a truncated slab of thickness d (and exit medium 0) is the subject of this paper. In this figure, the s polarization is normal to the page; i.e., the plane of incidence.

In Eq. (1), X is a complex periodic function of film thickness d:

$$X = \exp(-i2\pi d / D_\phi), \quad (2)$$

with a period given by:

$$D_\phi = (\lambda/2)(n_1^2 - n_0^2 \sin^2\phi)^{-1/2}. \quad (3)$$

In Eq. (3), λ is the wavelength of light, while n_0 and n_1 are the refractive indices of the transparent media of incidence and refraction, respectively.

For a layer of half-wave optical thickness [$d = D_\phi$ and $X = 1$ from Eq. (2)], destructive interference leads to zero reflectance, $R_\nu = 0$, $\nu = p, s$, according to Eq. (1).

On the other hand, the highest reflectance is achieved with a quarter-wave layer (QWL) of thickness $d = D_\phi / 2$ and $X = -1$ [from Eq. (2)]. Substitution of $X = -1$ in Eq. (1) leads to the desired expression:

$$R_\nu = \frac{2r_{01\nu}}{(1 + r_{01\nu}^2)}, \nu = p, s. \quad (4)$$

The relation between the intensity reflectance of a QWL and that of a semi-infinite substrate $(d \to \infty)$ of the same refractive index is obtained by squaring both sides of Eq. (4):

$$R_\nu^2 = \frac{4r_{01\nu}^2}{(1 + r_{01\nu}^2)^2}, \nu = p, s. \quad (5)$$

The reflectance enhancement factor (REF) η is defined by:

$$\eta = R_\nu^2 / r_{01\nu}^2 = 4/(1 + r_{01\nu}^2)^2, \nu = p, s. \quad (6)$$

For simplicity, the intensity reflectance of the 01 interface for the ν polarization is denoted by:

$$x = r_{01\nu}^2, \quad (7)$$

so that the REF η becomes:

$$\eta = 4/(1 + x)^2. \quad (8)$$

The REF of Eq. (8) is applicable for: (a) the p and s orthogonal linear polarizations, parallel and perpendicular to the plane of incidence, respectively; (b) any angle of incidence ϕ of external reflection $(n_1 > n_0)$ or partial internal reflection below the critical angle in the case of an embedded low-index layer $[n_1 < n_0, \phi < \arcsin(n_1/n_0)]$; and (c) any wavelength of incident light or refractive indices of the two transparent media.

For a given interface between two transparent media $r_{01p}^2 \leq r_{01s}^2$ at all angles of incidence [7], hence from Eqs. (7) and (8) one obtains:

$$\eta_p \geq \eta_s. \quad (9)$$

Fig. 2 shows a graph of the REF η over the full range of $x = r_{01\nu}^2$ $(0 \leq x \leq 1)$. In Fig. 2, the maximum value $\eta = 4$ is attained in the limit as $x \to 0$ and the minimum value $\eta = 1$ in the limit as $x \to 1$ and the initial and final slopes of the η-versus-x curve in Fig. 2 are -8 and -1, respectively.

Two examples are given in the next section that illustrate the significance of the REF.

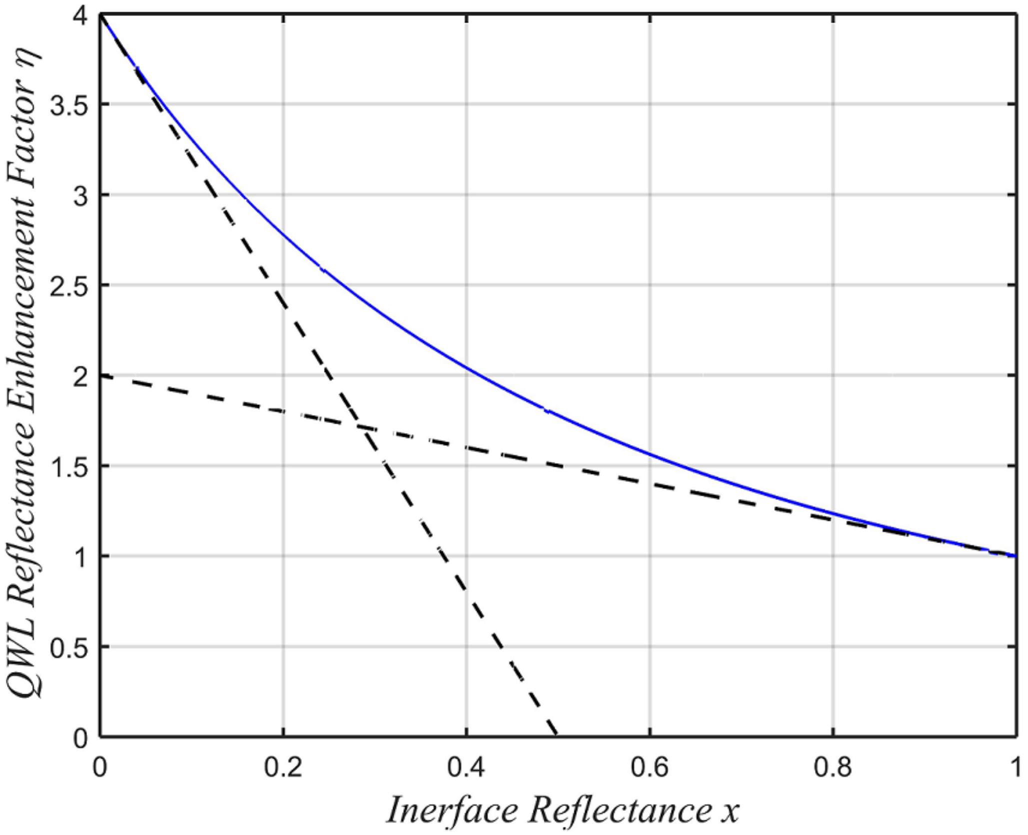

FIG. 2. Reflectance enhancement factor η [Eq. (8)] due to coherent interference of light in a QWL is plotted as a function of the intensity reflectance of the ambient-layer interface $x = r_{01\nu}{}^2$.

Reflection by Index-Near-One Materials and Bulk-versus-Pellicle Brewster-Angle Reflection Polarizers

Except near grazing incidence, the reflection of light by a skeletal or porous index-near-one (INO) material [6] is very weak. For example, the intensity reflectance of *s*-polarized light at the interface between air and a bulk INO material with n = 1.1 at 60° angle of incidence is only $r_{01s}{}^2 = 0.02288$. On the other hand, if the theoretically semi-infinite INO substrate is replaced by a QWL of the same refractive index (n = 1.1), the intensity reflectance is boosted by a significant factor of $\eta_s = 3.911$ or nearly 6 dB.

As another example, consider the simplest method for generating linearly polarized light by reflection at the Brewster angle (BA) $\phi_B = \arctan(n); n = n_1 / n_0$ of an air-dielectric plane boundary. At the BA, the *p* polarization is suppressed on reflection ($r_{01p} = 0$) and the reflected light is *s* polarized. The intensity reflectance for the *s* polarization at the BA is given by:

$$r_{01s}{}^2 = \cos^2(2\phi_B) = (n^2 - 1)^2 / (n^2 + 1)^2. \qquad (10)$$

For an *efficient* BA polarizer, the *s* reflectance of Eq. (10) should be as high as possible. This is feasible only with high-index, IR-transparent semiconductors [9, 10] such as Si and Ge with $n \approx 3.4$ and 4, respectively.

For bulk Ge BA reflection polarizer (n = 4), Eq. (10) gives $r_{01s}{}^2 = 0.7785$, which may be considered sufficiently high. However, the *s* reflectance of the Ge substrate at the BA (75.964°) can be enhanced by replacing bulk Ge by an unbacked Ge pellicle of QWL thickness. (At the BA, a transparent layer or parallel slab of *any* thickness is, of course, *non-reflecting* for the *p* polarization.) Substitution of $r_{01s}{}^2 = 0.7785$ in Eq. (6) gives a REF for the *s* polarization of $\eta_s = 1.2645$. Therefore, the enhanced *s* reflectance of the QWL Ge pellicle BA polarizer is $\eta_s r_{01s}{}^2 = 1.2645 \times 0.7785$ $= 0.9844$. This 20.6% increase in efficiency of the BA polarizer is clearly traceable to coherent multiple-beam interference of *s*-polarized light within the Ge QWL.

Conclusion

In this communication, a reflectance enhancement factor η [Eq. (8)] is introduced to quantify the role of coherent multiple-beam interference in augmenting the intensity reflectance of an unbacked or embedded QWL, when compared to the intensity reflectance of a semi-infinite substrate of the same material as the QWL. The result is valid for the p or s linear polarizations, at any angle of incidence or wavelength within the common transparency spectral range of the media of incidence and refraction. The results are particularly relevant to light reflection by index-near-one (INO) materials and Brewster-angle reflection polarizers that use high-index, IR-transparent semiconductors, such as Ge and Si, in bulk and pellicle form.

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