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Fuel Ignition Conditions in Thermonuclear Fusion

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Abstract: The isobaric and isochoric models of inertial confinement fusion (ICF) are compared in hot spot concept. Heating and cooling mechanisms of fuel are theoretically investigated. Some corrections are suggested to improve the Bremsstrahlung emission calculation at ultra-relativistic regime and super high temperature plasma situation. An admissible region of values is determined which satisfy hot spot spark-ignition condition to start a self-sustaining fusion burn. An optimized point of this region is specified to achieve a maximum fuel gain. The density and radius of this optimum point are determined applying a hydrodynamic model. The results show that a fuel gain maximum will be achieved with the required minimum laser energy supply through the optimization process.
Keywords: Isobaric, Isochoric, Spark-ignition, Self-sustaining burn, Fuel gain.

Introduction

For a typical inertial confinement fusion, ICF, target implosion will undergo four phases: ablation, compression, ignition and burn. The energy delivering to the target is based on direct or indirect irradiation. The ignition starts in conventional manner or isobaric model at the consequence of high compression and hot spot formation in center of pellet, but during the implosion stage, some accompanying hydrodynamic instabilities such as; *Rayleigh - Taylor* and *Richtmyer - Meshkov* instabilities, set an upper limit on the implosion velocity and then tend at first to destroy the imploding shell and later hinder the formation of the central hot spot [1, 2]. There is an alternative approach to ICF; namely isochoric model in which these instabilities have no important role. In isochoric model, compression and ignition stages are distinct [3]. In isochoric fast ignition model, the energy is delivered to the target at three stages: at first the compression with usual laser, second, hole boring with a short pulse laser beam (usually $10^{18}\,w/cm^2$and$100\,ps$) drilling a hole through the under dense plasma surrounding the dense fuel core [4] (this hole acts as an open channel which is relatively free of plasma for the ignition pulse to reach the pre-compressed fuel with minimum energy loss). In the third phase, an ultra-short pulse with a power in excess of a petawatt is used to ignite the fuel [5]. The pellet consists of three regions known as: the ablator, a layer of solid-ice fuel occupying most of the volume and central region of Deuterium-Tritium ($D-T$) gas[6]. During implosion, the layer of fuel has a velocity in the order of $3-4\times 10^7\,cm/s$ at the stagnation time (the end of implosion phase). The main fuel conversion ratio is: $\frac{R_0}{R_c}=20-40$ and the ratio of outer shell to hot spot radius is typically $\frac{R_c}{R_s}\approx 10.$ The hot spot life time is assumed approximately $(100-200)ps$ [1, 2, 4]. To achieve a maximum gain from an optimized condition, it is necessary to consider the ignition and spark formation conditions simultaneously.

This paper is organized as follows: in section 2, the heating and cooling mechanisms in hot spot region are discussed. Section 3 investigates the conditions to achieve an optimal point of hot spot areal density and temperature leading to

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maximum fuel gain. In section 4, the parameters of optimized point versus laser energy driver are specified by applying hydrodynamic model. In section 5, the fuel gain calculation corresponding to optimal point values in isochoric fuel is performed. Finally, section 6 presents a conclusion about optimization process in ICF context.

Heating and Cooling Mechanisms in Hot Spot Region

In Deuterium-Tritium $(D-T)$ fusion reaction, ${}_1^2D + {}_1^3T \rightarrow {}_2^4\alpha(3.5MeV) + {}_0^1n(14.1MeV)$, the energy contributions are related to masses ratio as: $\frac{E_\alpha}{E_n} = \frac{M_n}{M_\alpha} \approx \frac{1}{4}$. The neutron mean free path is $l_n = \frac{1}{\sigma n_i}$, where n_i and σ are the ion density and the cross section averaged over the plasma ions, respectively. For $D-T$ plasma, $\rho_n l_n = 4.7\ g/cm^2$. This is much larger than $\rho_s R_s$ of a typical igniting hot spot ($\approx 0.3\ g/cm^2$). Therefore, the neutron energy deposition can be neglected for central ignition. So, when a $D-T$ fusion reaction is started in ignition region, 20 percent of the energy deposition of α particles is considered as a heating mechanism. On the other hand, if the plasma to be considered is optically thin, the cooling mechanisms will be mainly Bremsstrahlung emission, electron thermal conduction for isobaric model and in addition, mechanical work due to fuel expansion in isochoric model. To generate a spark in hot spot region, firstly the cooling time should be greater than the time of mechanical wave propagation (sound) which corresponds to disassembly time or confinement time of spark region (spark formation condition). Secondly, the thermonuclear heating rate should be greater than the cooling rate [7]. The volumetric power generation from α particle energy deposition can be determined by [1, 8]:

$$P_\alpha = 1/5 P_{(\alpha+n)} = A_\alpha < \sigma v > {\rho_s}^2 f_\alpha \quad (1)$$

where ρ_s and f_α are hot spot density and the fraction of alpha particles that remain in the spark region and deposit their energy, respectively. $A_\alpha = 8 \times 10^{40} erg/g^2$ and $< \sigma v >$ is reactivity which is given by[9]:

$$< \sigma v >= \exp\left[A_1 + A_2 \left|ln \frac{T_s}{A_3}\right|^{A_4}\right]\ (cm^3/s) \quad (2)$$

where $A_1 = -34.629731, A_2 = -0.57164663$, $A_3 = 64.221524\ and\ A_4 = 2.1373239$. The plasma has non-degenerate state with a Maxwellian energy distribution function; therefore, the volumetric power loss of Bremsstrahlung from a hydrogen isotope plasma by same temperature assumption for electrons and ions in spark region $T_s = T_e = T_i$ can be given by [1,7]:

$$P_{br}(erg, s^{-1}, cm^{-3}) = 3.36 \times 10^{-24} n_e \times (n_T + n_D) {T_s}^{\frac{1}{2}} \quad (3)$$

where n_e, $n_T\ and\ n_D$ are electron, tritium and deuterium densities in cm^{-3}, respectively and T_s is the hot spot temperature. For equimolar fuel, $n_T = n_D = n_e/2$; so, we have:

$$P_{br}(erg, s^{-1}, cm^{-3}) = 3.36 \times 10^{-24} {n_s}^2 {T_s}^{1/2}. \quad (4)$$

The above equation is only valid in non-relativistic regime with Maxwellian distribution function. In this regime, the e-i collisions are only important and the quantum relativistic corrections are not included. Some quantum relativistic corrections are needed at the ultra-relativistic regimes. Firstly, non-relativistic Maxwellian distribution function (for non-degenerate plasma) should be replaced by relativistic distribution function. Secondly, the quantum relativistic and the screening effect corrections are strongly necessary to apply in differential cross-section equations. Also, the Bremsstrahlung emission due to e-e collisions should be taken into account. The relativistic Maxwellian energy distribution function is given by:

$$f(v) = [2\left(\frac{T_e}{mc^2}\right)^2 K_1\left(\frac{mc^2}{T\_e}\right) + \left(\frac{T_e}{mc^2}\right) K_0\left(\frac{mc^2}{T_e}\right)]^{-1} c^{-3} v^2 \gamma^5 exp\left(-\frac{\gamma mc^2}{T_e}\right) \quad (5)$$

where m is the electron rest mass, c is light velocity, K_1 and K_0 are the modified Bessel functions of the second kind.

Applying the quantum relativistic and the screening effect corrections indicated as Gaunt factor $G(E_k, hv)$ on $Kramer's$ cross-section, the improved differential cross-section is read as:

$$\frac{d\sigma(E_k,hv)}{d(hv)} = G(E_k, hv) \frac{d\sigma_{kr}(E_k,hv)}{d(hv)}. \quad (6)$$

Here, $G(E_k, hv)$ represents corrections including quantum, relativistic and screening effects. Although $G(E_k, h\nu)$ is a complex function of E_k and $(h\nu)$, in our discussion we consider a simple approximation for $G(E_k, h\nu)$ as $G \approx \frac{2\sqrt{3}}{\pi} \approx 1.10$ [1]. Kramer's cross-section is given as:

$$\frac{d\sigma_{kr}(E_k, h\nu)}{d(h\nu)} = \frac{Z^2}{h\nu\left(\frac{v}{c}\right)^2} S_{kr} \tag{7}$$

$$S_{kr} = \frac{16\pi}{3^{1/3}} \alpha_{fs}{}^3 \left(\frac{\hbar}{mc}\right)^2 \tag{8}$$

where v, m and α_{fs} are the relative velocity of particles (which simply can be related to energy parameters), the emitter rest mass and fine structure constant, respectively. For electron, we have: $S_{kr} = 5.61 \times 10^{-31}$. The emitted *Volumetric power (Specific power)* of electrons with distribution function $f(v)$ in plasma medium is obtained by:

$$P_{br}{}^{e-i,e-e}(h\nu) = \int \frac{d\sigma^{e-i.e-e}(E_k, h\nu)}{d(h\nu)} n_{i.e} n_e v f(v) dv\ . \tag{9}$$

Fig.1 shows Bremsstrahlung specific power versus electron energy by comparing relativistic and non-relativistic Maxwellian energy distribution functions while the improved formula on differential cross-section is applied too. It is found that the considerable difference in results is only formed at high temperature far from ICF region ($T \gg 5 - 10\ keV$). So, these corrections are only necessary for fuels with ultra-temperatures. It should be noticed that in degenerate plasma practically, there is no considerable Bremsstrahlung emission.

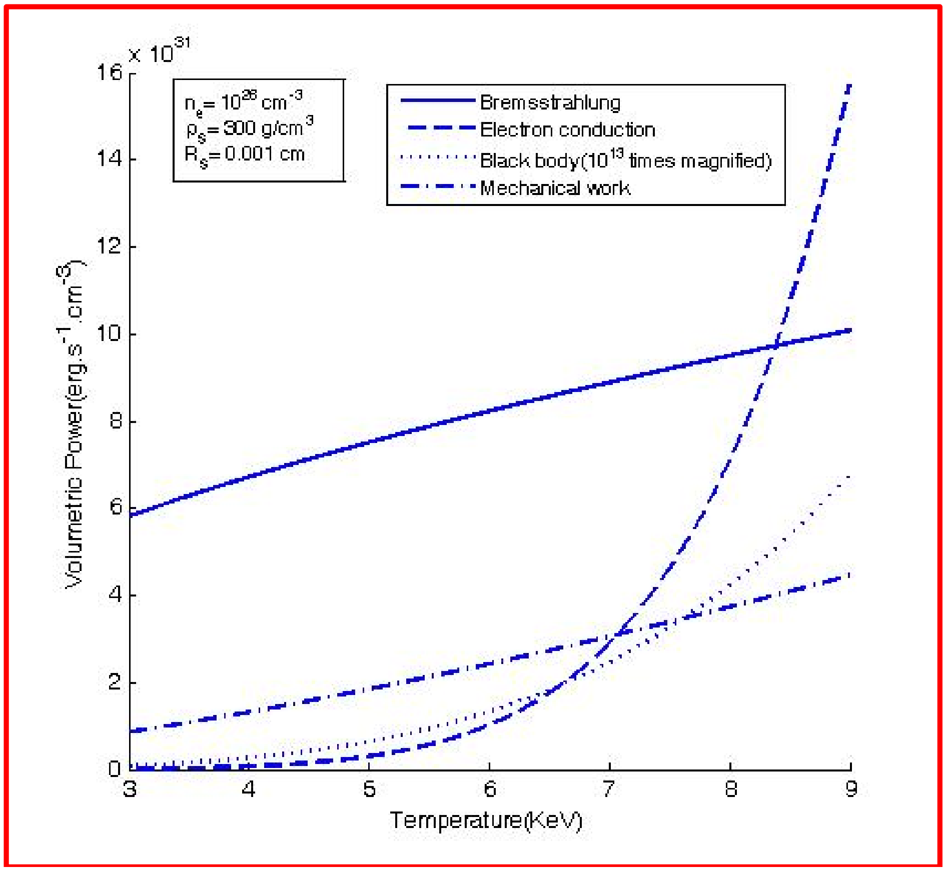

FIG. 1. The variation of Bremsstrahlung (solid line), electron conduction (dashed line), black body (dotted line, 10^{13} times magnified) and mechanical work (dash-dotted line) versus plasma temperature.

Volumetric power loss of electrons' thermal conduction in hot spot is determined by [1]:

$$P_{ec}(erg, s^{-1}, cm^{-3}) = -\frac{\chi_e \nabla T_s S}{V} \tag{10}$$

where S and V denote the surface and volume of hot spot region, $A_e = 9.5 \times 10^{19} \left(erg, s^{-1}, cm^{-3} keV^{-\frac{7}{2}}\right)$, $\chi_e = A_e T_e{}^{5/2} / ln\Lambda$, where $ln\Lambda$ is Coulomb logarithm which is given by[7]:

$$ln\Lambda = \ln\left(60 T_s \sqrt{\frac{2.5}{\rho_S}}\right). \tag{11}$$

So, finally we will have:

$$P_{ec}(erg,s^{-1},cm^{-3}) \simeq \frac{3C_eA_e}{ln\Lambda}\frac{T_s^{7/2}}{R_s^{2}} \qquad (12)$$

where C_e is a numerical coefficient close to unity. The other cooling mechanism in hot spot is mechanical work due to pressure imbalance between hot spot P_s and surrounding fuel P_c. When the igniting fuel is perfectly isobaric (P_s=P_c), then there is no mechanical work, but when the fuel is isochoric, there is a large pressure gradient between hot spot and surrounding fuel $P_s \gg P_c$, so that a shock is driven into the lower pressure cold fuel. The volumetric mechanical work power is given by [1]:

$$P_{m(}erg,s^{-1},cm^{-3}) = A_m\rho_sR_s^{-1}T_s^{3/2} \qquad (13)$$

with $A_m = 0$ for isobaric ignition and $A_m = 5.5\times10^{22}cm^3s^{-3}keV^{-3/2}$ for isochoric ignition and $\rho_s, R_s\ and\ T_s$ are density, radius and temperature of hot spot, respectively. When the radius of hot spot is much smaller than Planck mean free path of photons, $l_{ph} = 14.4T_s^{7/2}/\rho_s^{2}$, ($l_{ph} = 14.4T_s^{7/2}/\rho_s^{2}$), the plasma will be optically thin and black body radiation as a cooling mechanism is negligible (for example, $\rho_s = 300$, $R_s = 0.001\ cm$, $T_s = 7.5keV$, $l_{ph} = 0.1849\ cm$, $R_s/l_{ph} = 0.0054$). The volumetric power of black body radiation is determined by $P_{bb(}erg,s^{-1},cm^{-3}) = \sigma_BT_s^{4}R_s^{-1}$, where $\sigma_B = 1.03\times10^{24}erg,s^{-1},cm^{-2},keV^{-4}$. On the other hand, the mechanisms of heating through reabsorption such as Compton scattering and inverse Bremsstrahlung are also negligible due to thinness condition. Fig. 2 shows the contribution of each different cooling mechanism versus temperature of overdense plasma.

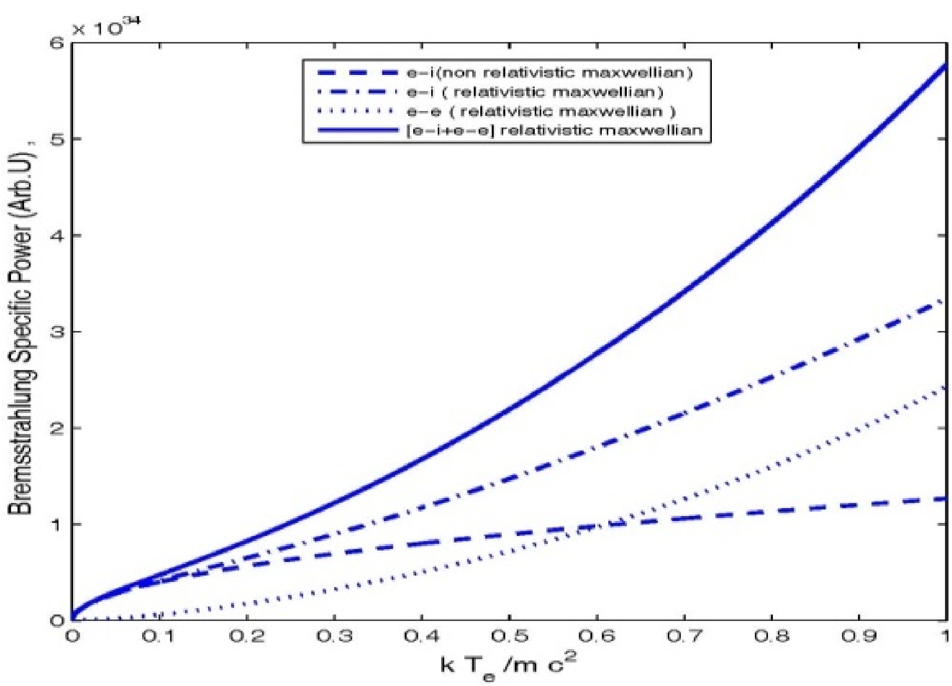

FIG. 2. Bremsstrahlung specific power versus electron energy normalized to electron rest mass with relativistic and non-relativistic Maxwellian energy distribution functions.

Optimization Condition to Achieve Maximum Fuel Gain

To achieve a self-heating fusion burn condition, it is needed to determine the admissible region inside the enclosed area between the spark formation and ignition condition curves through crossing them in $H_s - T_s$ plane, where H_s is the areal density of hot spot. Optimal point which leads to a maximum gain is actually the minimum point of this admissible region.

The ignition condition for isobaric fuel is written as:

$$P_\alpha + P_n + P_{Re} \geq P_{br} + P_{ec} + P_{bb}\ . \qquad (14)$$

The plasma is assumed optically thin, so that the reabsorption mechanisms P_{Re} and black body radiation P_{bb} contributions are neglected [9]. Because the value of neutron mean free path is greater than hot spot areal density, the neutron heating contribution P_n is also neglected. Finally, the above relation leads to:

$$\left(A_\alpha < \sigma v > f_\alpha - A_{br} T_s^{\frac{1}{2}}\right) H_s^{\,2} - \frac{3C_e A_e T_s^{\,7/2}}{ln\Lambda} \geq 0. \quad (15)$$

On the other hand, condition for a thin isochoric fuel is written as:

$$P_\alpha \geq P_{br} + P_{ec} + P_m. \quad (16)$$

This leads to:

$$\left(A_\alpha < \sigma v > f_\alpha - A_{br} T_s^{\frac{1}{2}}\right) H_s^{\,2} - A_m T_s^{\frac{3}{2}} H_s - \frac{3C_e A_e T_s^{\frac{7}{2}}}{ln\Lambda} \geq 0. \quad (17)$$

The spark formation condition can be written as [10]:

$$t_{co} \leq t_c \quad (18)$$

where t_{co} and t_c are confinement time and cooling time scale, respectively. Confinement time is given by:

$$t_{co} = \frac{R_s}{c_s} \quad (19)$$

where $c_s = 3.5 \times 10^7\, T_s^{\,1/2}\ (cm/s)$ with T_s in keV is sound propagation velocity in $D - T$ homogenous plasma. The cooling time scale is also given by [7]:

$$t_c = \frac{3kn_s T_s}{P_{br} + P_{ec}} \quad (20)$$

where $k = 1.6 \times 10^{-9} erg/ke$V. Finally, by substituting Eqs. (19 and 20) in Eq.18 we shall have:

$$(7.6)\frac{H_s}{T_s} + (7.1 \times 10^{-3})\frac{T_s^{\,2}}{H_s ln\Lambda} \leq 1\ . \quad (21)$$

According to Fig. 3, this optimized point is $H_s = 0.25 g/cm^2$ and $T_s = 5keV$ for isobaric fuel and $H_s = 0.3\ g/cm^2$ and $T_s = 7.5keV$ for isochoric fuel.

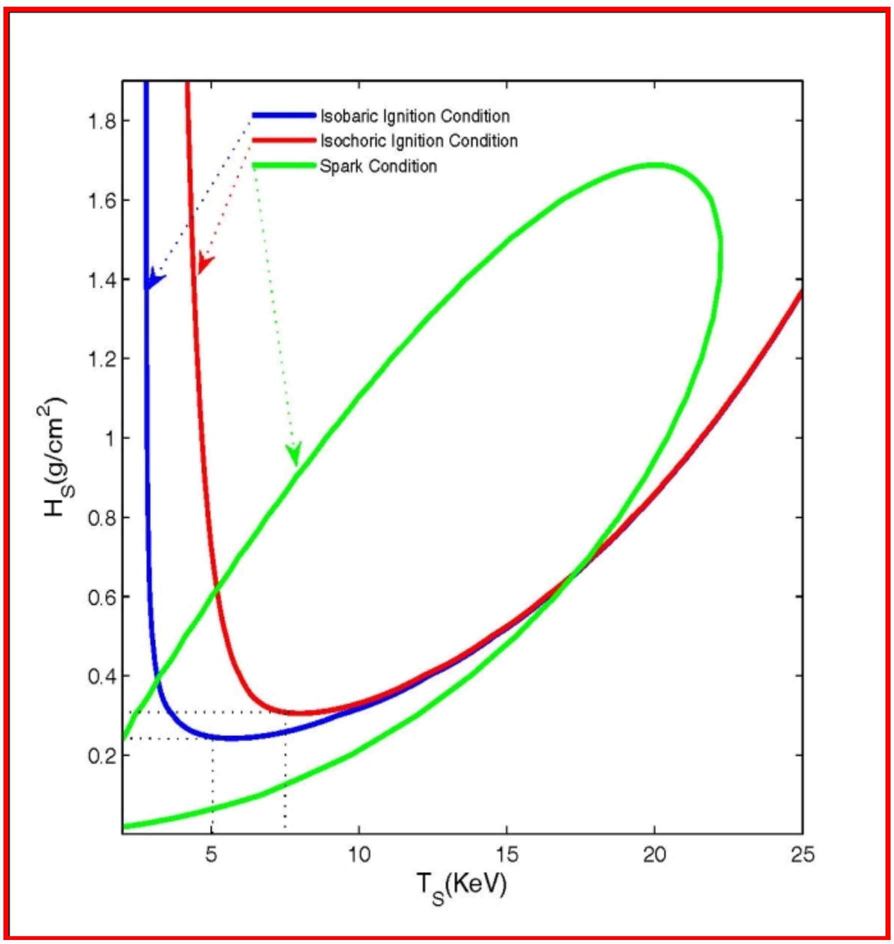

FIG. 3. Crossing of spark formation and ignition condition curves in $(H_s - T_s)$ plane to obtain optimal point.

Hot Spot Parameters According to Optimal Point

The parameters of optimized point in self-heating condition can be determined versus implosion parameters by hydrodynamic model of shock- ignition. The characteristics of hot spot and the surrounding shell versus implosion parameters are listed in Table. 1 [11]. While $\alpha = \frac{P_c}{P_{FD}}$ is isentropic parameter with P_{FD} as Fermi degenerate pressure, $\epsilon = \frac{P_c}{P_s}$ indicates the dropping pressure factor and $\beta = \frac{H_s}{0.4}$. Considering $\eta E_L = E_c + E_s$ and the fuel mass as $M_F = M_c + M_s$, by substituting the related formulae, the fuel mass can be obtained as:

$$M_F = \frac{\eta E_L - 2.4\times10^3(\gamma\beta)^3\epsilon^2 {P_c}^{-2}}{3.9\alpha^{3/5}{P_c}^{2/5}} + 4.3\frac{\beta^3(\gamma\epsilon)^2}{{P_c}^2}. \quad (22)$$

In ***isobaric*** fuel, the shell pressure is equal with hot spot region, but in ***isochoric*** fuel, the density of shell is equal with the hot spot. The laser energy versus hot spot radius R_s is shown in Fig. 4 in optimized point, $H_s = 0.25 g/cm^2$, $\beta = \frac{0.25}{0.4}$, $\gamma = 5/3$, $\epsilon = 1$ and assuming $\eta = 0.1$. $M_F = 0.3\ (mg)$ for isobaric fuel. It is shown that the minimum laser energy corresponds to hot spot radius $R_s = 8.33 \times 10^{-4} cm$ and density $\rho_s = 300 g/cm^3$. Repeating this route to determine the values for isochoric fuel, according to Fig. 5, $R_s = 10^{-3} cm$, $\rho_s = 300 g/cm^3$ and $\epsilon = 0.03$ correspond to minimum required laser energy. In addition, Figs. 4 and 5 show the pellet conversion ratio and the variation of shell radius too. The conversion ratio in isobaric fuel is greater than the isochoric one (about 30/20); therefore, more compression energy is needed with an increase in hydrodynamic instabilities.

Fuel Gain in Isochoric Fuel

The fuel gain in isochoric fuel is: $\frac{E_F}{E_c + E_{ig}}$, where E_F, E_c and E_{ig} indicate the fusion released energy, compression energy and the ignition energy required to supply the fuel driver to start fusion reaction, respectively. The fusion energy is determined by:

$$E_F = \varepsilon_{DT} f_b M_F \quad (23)$$

where $f_b = \frac{\rho R}{\rho R + 7}$, $\rho R = H_F$ is fuel areal density and $\varepsilon_{DT} = 17.6/5AMU$ is specific fusion energy. Then, the fusion released energy is rewritten as:

$$E_F(MJ) = 3.37 \times 10^5 \left(\frac{\rho R}{\rho R + 7}\right) M_F\ . \quad (24)$$

According to implosion parameters, the fuel areal density is $\rho R = 0.4\beta + \rho_c(R_c - R_s)$ [11]. The compression energy required to compress the fuel to η times more than liquid hydrogen density in isentropic model is given by:

$$E_c(MJ) = 0.12\alpha\eta^{2/3} M_F \quad (25)$$

where $\eta^{2/3} = \frac{H_F}{H_0}$. $H_0 = \left(\frac{3M_F\rho_0}{4\pi}\right)^{1/3}$ and ρ_0 is liquid hydrogen density ($67.8 \times 10^{-3} g/cm^3$). The ignition energy is given by:

$$E_{ig} = 3 f_s T_s \left(\frac{M_F}{M_i}\right) \quad (26)$$

where $f_s = \left(\frac{M_s}{M_F}\right) = \left(\frac{H_s}{H_F}\right)^3$. Submitting $M_i = 25g$ as ion average mass and the value of f_s in Eq. 26, the ignition energy is rewritten as:

$$E_{ig} = 0.00324\ T_s\left(\frac{M_F}{H_F^3}\right). \quad (27)$$

The fuel gain can be determined by applying the optimal point parameters to Eqs. (24, 25 and 27). Fig. 6 shows that the fuel gain is maximized just in optimized point ($H_s = 0.3\ g/cm^2$).

TABLE 1. Some useful relations which are driven from hydrodynamics shock-ignition model [11].

Quantity	Hot spot	Main fuel
$(Density: g/cm^3)$	$\rho_s = \frac{0.4\beta}{R_s}$	$\rho_c = 51\left(\frac{\beta\gamma\epsilon}{\alpha R_s}\right)^{3/5}$
$(Energy: MJ)$	$E_s = 10^3\beta\gamma {R_s}^2$	$E_c = 0.32\alpha {\rho_c}^{2/3} M_c$
$(Mass: g)$	$M_s = \frac{4\pi}{3}\rho_S {R_s}^3$	$M_c = 0.21\left(\frac{R_s}{\beta\gamma\epsilon}\right)^{\frac{2}{5}} \alpha^{-\frac{3}{5}} \times (\eta E_L - E_s)$
$(Radius: cm)$	$R_s = \frac{0.4\beta}{\rho_s}$	$R_c = \left(\frac{3M_c}{4\pi\rho_c} + {R_s}^3\right)^{1/3}$
$(Pressure: MBar)$	$P_s = \frac{1.6\beta\gamma}{R_s}$	$P_c = 2.2 \times 10^{-3}\alpha {\rho_c}^{5/3}$

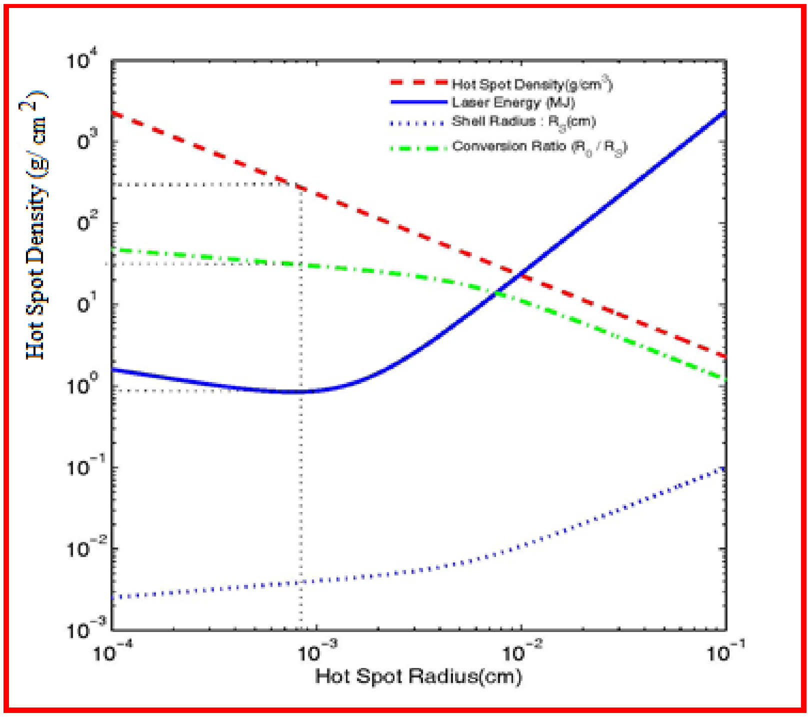

FIG. 4. The variation of hot spot density (g/cm^2, dashed line), laser driver energy with coupling efficiency $\eta_L = 0.1$ (MJ, solid line), surrounding shell radius (cm, dotted line) and fuel conversion ratio (R_0/R_S, dot-dashed line) with considering optimal point ($H_S = 0.25\, g/cm^2, T_S = 5\, keV$) of isobaric fuel versus hot spot radius.

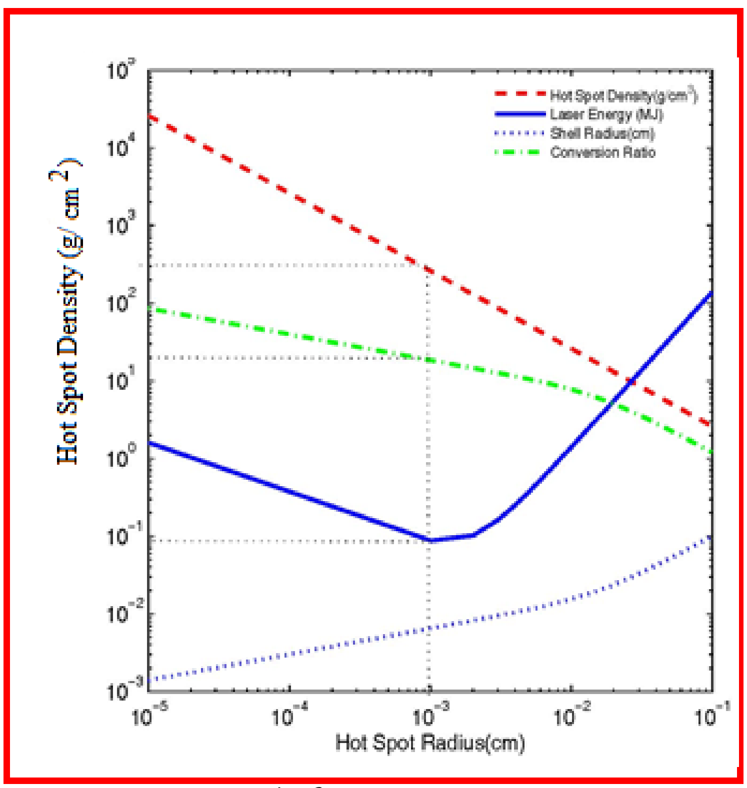

FIG. 5. The variation of hot spot density (g/cm^2, dashed line), laser driver energy with coupling efficiency $\eta_L = 0.1$ (MJ, solid line), surrounding shell radius (cm, dotted line) and fuel conversion ratio (R_0/R_S, dot-dashed line) with considering optimal point ($H_S = 0.3\, g/cm^2, T_S = 7.5\, keV$) of isochoric fuel versus hot spot radius.

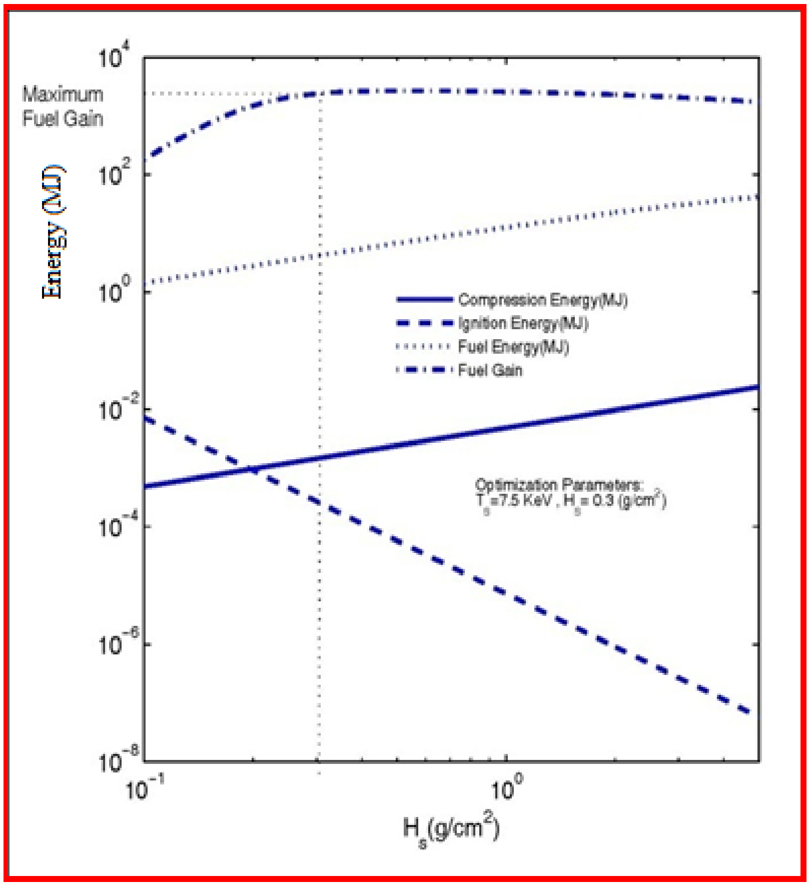

FIG. 6. The variation of compression energy (MJ,solid line), ignition energy (MJ, dashed line), fuel energy (MJ, dotted line) and fuel gain (dot-dashed line) versus hot spot areal density (g/cm^2) for isochoric fuel.

Conclusion

To obtain the required conditions to start a self-sustaining fusion burn, it is necessary to apply spark formation and ignition conditions simultaneously. Crossing the related curves gives an admissible region of hot spot areal density and temperature values which satisfy a self-sustaining fusion burn ($H_S = 0.3\, g/cm^2$, $T_S = 7.5\, keV$ (isochoric fuel)) (Fig. 3). The special values of radius and density ($R_S = 0.001\, cm$, $\rho_S = 300\, g/cm^3$ (isochoric fuel)) which correspond to required minimum laser energy describe the real optimized point of hot spot determined through applying the hydrodynamic model. It is shown that to calculate bremsstrahlung emission at high temperature ($T_e \gg 5 - 10\, keV$), some important corrections must be included, such as; relativistic Maxwellian distribution function and quantum relativistic effects (Gaunt factor) on differential cross-sections. Also, it is not necessary to include the black body radiation (as cooling mechanism) and the reabsorption mechanism (as heating mechanism) in energy balance equation for optically thin plasma. The isochoric fuel needs less energy than the isobaric fuel in optimal point. In isobaric model, the shell with density about 8 times greater than hot spot should be compressed with higher pellet conversion ratio; therefore, higher compression energy is necessary. So, a higher fuel gain is obtained from the isochoric model (about 8000), while it is needed to generate a hot spot with higher areal density and temperature in the isobaric model.

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