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ARTICLE

Compton Scattering of Twisted Light

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Abstract: The variation of photonic orbital angular momentum in Compton scattering is analytically analyzed. We determine the scattering matrix of twisted light based on the fundamental conservation of orbital angular momenta. Numerical values for two different twisted light modes: Laguerre-Gaussian and Bessel-Gaussian, are generated and illustrated. Our analysis indicates that states of photonic orbital angular momentum are highly changeable at wide angle scattering but more consistent at small angle scattering.
Keywords: Compton scattering, Twisted light, Laguerre-Gaussian mode, Bessel-Gaussian mode.

Introduction

Twisted light carries the angular form of an electromagnetic momentum. It is separable and decomposes into an orbital momentum part as well as a spin momentum part. The orbital angular momentum associates with the helical structure of the wave front, while the spin angular part associates with the polarization state [1].

Twisted light carries a well-defined photonic orbital angular momentum (POAM). Its wave front characterizes a specific azimuthal phase according to the POAM state [2]. Both photonic spin angular momentum and POAM are orthonormal components of the total light angular momentum.

Compton scattering describes the change in linear momentum at elastic collisions between a photon and an electron. The well-known shift in a scatter wave number reports a certain change in the linear momentum as well as in the energy of scattering photons [3, 4]. However, a variation in POAM has never been mentioned.

Several studies have recently been conducted to describe changes in POAM in Compton scattering in ultra-relativistic considerations [5, 6]. A non-relativistic framework has been implemented in the density matrix theory to inform about the variation of POAM in Compton scattering [7].

Our study briefly analyzes a change of POAM of twisted light in Compton scattering by evaluating the associated scattering matrix in a semi-classical framework. It illustrates the possibility for POAM to vary through scattering at free electrons and emphasizes the conservation of total angular momentum *via* exchange of POAM between photons and massive electrons.

The next section determines a particular analytical expression of the scattering matrix of Compton scattering for a twisted light. The expression is valid for axi-symmetric light beam that carries a well-defined POAM. In the following section, two different beams are examined. Our numerical calculations are then analyzed and discussed. The conclusion is then presented at the end of this study.

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Conservation of OAM in Compton Scattering

The schematic diagram of Compton scattering in Fig. 1 identifies the scattered photonic wave number (k') as well as POAM by primed parameters (l'), while the associated wave number and orbital angular momentum of the recoil electron are indicated by k_e and m_e, respectively. The scattering angle (θ) is defined as the angle between the incident beam and the scattering direction.

Our analysis is based on evaluating, semi-classically, the scattering matrix of Compton scattering to illustrate the fundamentals of an exchange of orbital angular momentum between the twisted light wave and a massive particle.

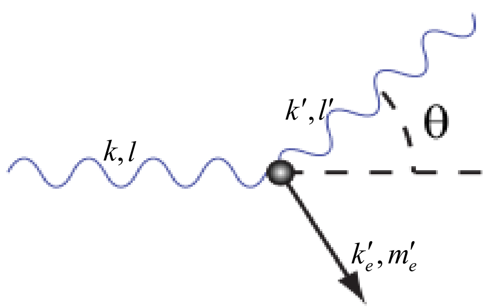

FIG.1. Schematic of Compton scattering with associated parameters.

The Compton scattering matrix is defined as [8]: $S=\left\langle f'\left|e^{i(\vec{k}'-\vec{k}).\vec{r}}\right|f\right\rangle$, where f', f represent the scattered and the initial state of the system, $\vec{k}$, $\vec{k}'$ are the incident and scattered wave vectors and $\vec{r}$ is a position vector. The cylindrical coordinates are used in such a way the optical path is considered along the z-axis.

Both, spatial electronic and photonic wave functions, Φ and Ψ, respectively, are considered. Their conjugates are indicated by superscript stars. The Compton scattering matrix can be presented as:

$$S=\left\langle \Phi^*(\vec{r}')\Psi^*(\vec{r}')\left|e^{i(\vec{k}'-\vec{k}).\vec{r}}\right|\Phi(\vec{r})\Psi(\vec{r})\right\rangle \quad (1)$$

Normalized wave functions in cylindrical coordinates are used due to an azimuthal symmetry of twisted light. Generally, electronic and photonic wave functions could be expressed as $\Phi(\vec{r})=P_m(\rho,z)e^{-im\phi}$ and $\Psi(\vec{r})=R_l(\rho,z)e^{-il\phi}$, where m,l are parameters indicating the electronic and POAM state, respectively. Therefore, Eq. (1) is written explicitly as:

$$S=\left\langle P^*_{m'}(\rho',z')e^{i(m'+l')\phi}R^*_{l'}(\rho',z')\left|e^{i(\vec{k}'-\vec{k}).\vec{r}}\right|P_m(\rho,z)e^{-i(m+l)\phi}R_l(\rho,z)\right\rangle \quad (2)$$

Due to the azimuthal symmetry of the spatial wave profiles, we can separate the exponential of azimuthal parameters out as follows:

$$S=\left\langle P^*_{m'}(\rho',z')R^*_{l'}(\rho',z')\left|e^{i(\vec{k}'-\vec{k}).\vec{r}}\right|P_m(\rho,z)R_l(\rho,z)\right\rangle\left\langle e^{i(\Delta m+\Delta l)\phi}\right\rangle \quad (3)$$

where $\Delta m=m'-m$ and $\Delta l=l'-l$. The last angle bracket in Eq. (3) is evaluated directly using the Dirac delta function:

$$\left\langle e^{i(\Delta m+\Delta l)\phi}\right\rangle=\delta(\Delta m+\Delta l) \quad (4)$$

Eq. (4) reads that a decrease in POAM must be equivalent to an increase of electronic one: $\Delta m=-\Delta l$. Consequently, the Compton scattering matrix is not terminated as long as the orbital angular momentum exchanges between the photon and the electron. It must be conserved through scattering.

Evaluating Compton scattering matrix requires resolving the wave functions as well as the middle exponential term. The electronic radial wave function $P_m(\rho,z)$ is a separable function. It is generally given by:

$$P_m(\rho,z)=J_m(\kappa_e\rho)e^{ik_ez} \quad (5)$$

where J_m is Bessel function and κ_e and k_e are transverse and longitudinal electronic wavenumbers, respectively. On the other hand, the wave vector of collimated twisted light in general is composed of two components: orbital and longitudinal [9]. It can be basically presented as:

$$\vec{k} = \kappa\hat{\phi} + k\hat{z} . \quad (6)$$

Hence, the middle term in the scattering matrix, the exponential term in Eq. (3), is simplified as follows:

$$\exp[(\vec{k}' - \vec{k}).\vec{r}] = \exp[(k' - k)z] . \quad (7)$$

Taking Eqs. (4, 5 and 7) together, we evaluate the scattering matrix as follows:

$$S = \left\langle J^*_{m'}(\kappa'_e\rho')R^*_l(\rho',z') \middle| J_m(\kappa_e\rho)R_l(\rho,z) \right\rangle$$

$$\delta(\Delta k_e + \Delta k). \quad (8)$$

The Dirac delta term indicates that the longitudinal components of the wave vector, which represent the linear momentum, are also exchanged between the electron and the photon. An increase in the linear momentum of the electron is equivalent to a decrease in the linear momentum of the photon. It is definitely consistent with the well-known Compton formula [3]. Conservation of both linear and orbital angular momentum emphasizes the conservation of total momentum for twisted photons in Compton scattering.

Our objective is to focus on the change of photonic orbital angular momentum in Compton scattering. It requires the evaluation of the scattering matrix regardless the initial OAM state. It is computed for a relative OAM exchange in the zero-order state of the "untwisted mode"; i.e.:

$$S = \left\langle J^*_{\Delta l}(\kappa'\rho)R^*_{\Delta l}(\rho,z') \middle| J_0(\kappa\rho)R_0(\rho,z) \right\rangle . \quad (9)$$

The conditions of Eqs. (3) and (7) have been used. Indeed, it has been assumed that there is no change in the radial parameter (ρ) through scattering and on the other hand, axial parameter (z) depends only on the scattering angle (θ).Thus, the scattering matrix is evaluated at a certain axial distance.

Results and Discussion

The scattering matrix in Eq. (9) is computed for two particular twisted light beams: *Bessel-Gaussian* and *Laguerre-Gaussian (LG).* The spatial wave function of LG is given by [10]:

$$R_l^{LG}(\rho,z) = \frac{C_p^l}{w(z)}\left[\frac{\sqrt{2}\rho}{w(z)}\right]^{|l|} \exp\left(-\frac{\rho^2}{w(z)^2}\right) L_p^{|l|}\left(\frac{2\rho^2}{w(z)^2}\right) \exp\left[\frac{-ik\rho^2 z}{2(z^2+z_R^2)} + iG_q^{|l|}(z)\right]\exp(-il\phi) \quad (10)$$

where C_p^l is a normalization constant, p is a radial index, $w(z)$ is the beam width at distance z and given by $w(z) = w_o\sqrt{1+(z/z_R)^2}$, z_R is the Rayleigh range defined as: $z_R = \pi w_o^2/\lambda$, $L_P^{|l|}(\bullet)$ is the associated Legendre polynomial and $G_p^{|l|}(z) = -i(2p+|l|+1)\tan^{-1}(z/z_R)$ is the Gouy phase[10]. It should be noted that the scattered axial parameter, z\`, varies only through the scattering angle (θ). Our computations are based on fixed z values.

Eqs. (9) and (10) are used to evaluate the scattering matrix of an x-ray of 1 nm wavelength and 1μm waist width through a wide scattering in the range $-\pi/2 < \theta < \pi/2$. It is assumed that there is no change in Gouy phase through scattering.

The computed elements that associate for a certain change (Δl) in photonic OAM are determined and plotted. Fig. 2 illustrates Compton scattering matrix *versus* scattering angle for several changes in POAM. The curves are generated by computing the normalized scattering matrix in a wide angle range for four successive state changes in POAM.

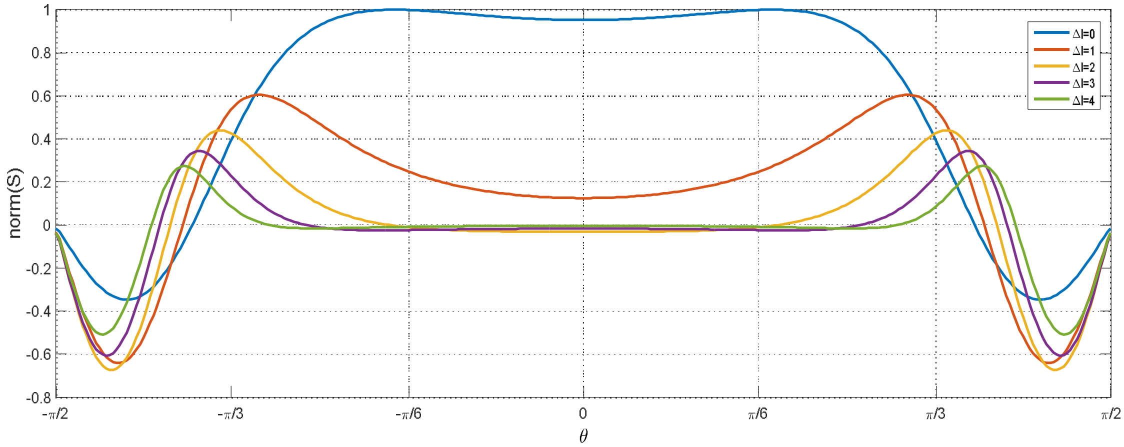

FIG. 2. Normalized Compton matrices *versus* scattering angle for LG beam at certain changes in POAM.

The normalized scattering elements shown in Fig. 2 are very small for narrow range scattering. The normalized scattering provides the probability of exchanging POAM, which is minimum at small scattering angle. However, a change by just one order state could happen with low possibility as the curve of $\Delta l=1$ shows.

High possibilities of changing POAM states through scattering are represented by the peaks shown in Fig. 2. Scattering matrix elements are relatively of small values for higher change in POAM. It could be interpreted as a low possibility for a big change in POAM at Compton scattering.

Negative-value elements in Fig. 2 indicate flipping up of OAM states. They inform that scattering in wide angles causes flipping up of the azimuthal phase. Consequently, POAM states are twisted "oppositely" at wide angles of Compton-scattering. However, scattering matrices are terminated at right angle scattering due to the minimized differential cross-section of Compton scattering [11].

Scattering matrix is also computed for large changes in photonic OAM states and is plotted in Fig. 3.

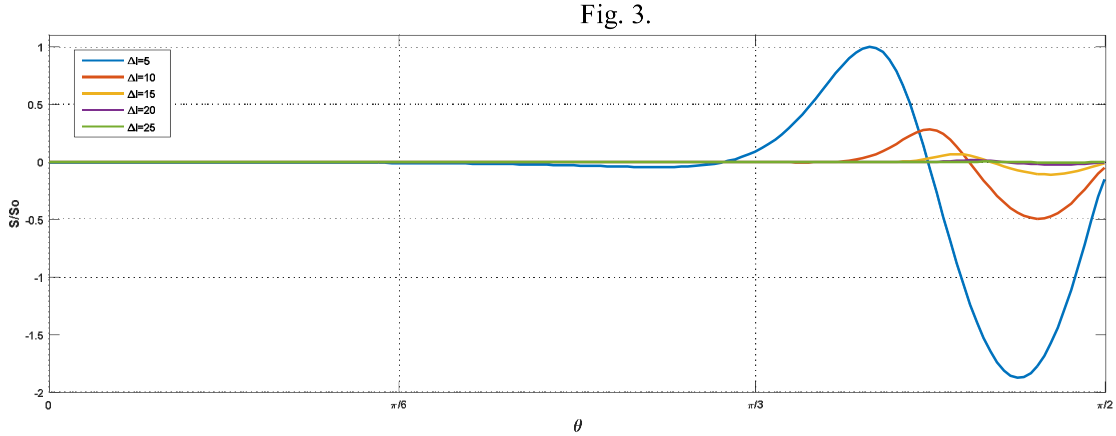

FIG. 3. Normalized Compton matrices *versus* scattering angle for LG beam at big changes in photonic OAM.

Fig. 3 confirms the low possibilities of scattering twisted photons with high changes in their POAM. Indeed, it indicates that Compton scattering at wide angles is more likely to occur with oppositely twisted orientation. Dominant negative peaks at Fig. 3 represent a tendency of large changes in photonic OAM to be in opposite orientation.

The other twisted light beam that has been investigated is the Bessel-Gaussian beam, which has a spatial wave function given by [12]:

$$R_l^{GB}(\rho,z) = A\frac{w_o}{w(z)}\exp\left[i\left(k-\frac{\kappa^2}{2k}\right)z - i\phi(z)\right]J_l\left[\frac{\kappa\rho}{(1+iz/z_R)}\right]\exp\left[\left(\frac{-1}{w(z)^2}+\frac{ik}{2R(z)}\right)\left(\rho^2+\frac{\kappa^2}{k^2}z^2\right)\right] \tag{11}$$

where A is the normalization constant, $\phi(z)=\tan^{-1}(z/z_R)$ is the associated Gouy phase, and $R(z)=z\sqrt{1+(z_R/z)^2}$ is the radius of curvature.

Eq. (11) has been implanted in Eq. (9) to compute the corresponding scattering matrix for the same parameters: a 1nm wavelength of an x-ray source, a 1μm beam waist and a non-varying Gouy phase.

The associated scattering matrix is evaluated for first-order changes in orbital angular momentum as shown in Fig. 4.

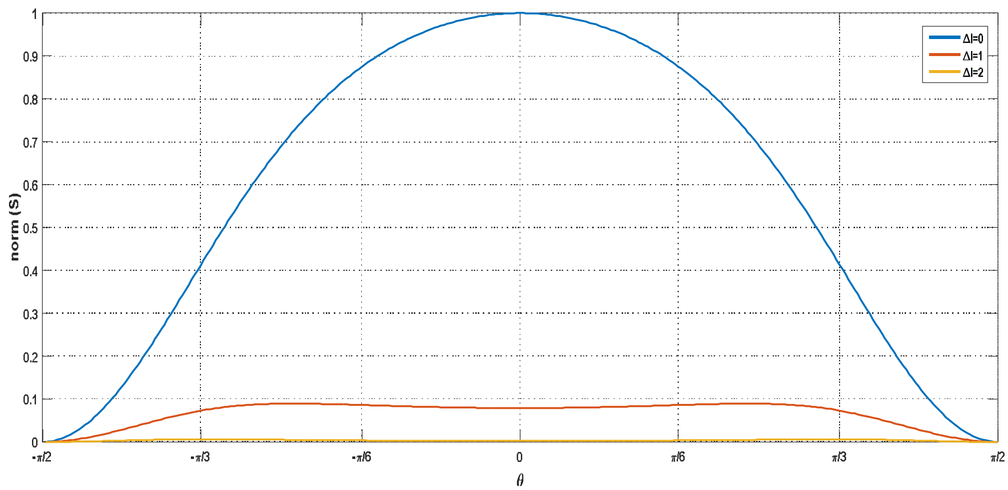

FIG. 4. Normalized Compton matrices *versus* scattering angle for BG beam at few changes in photonic OAM.

Scattering matrices of Bessel-Gauss beam which are illustrated in Fig. 4 change very slowly. Relatively high value peaks around a scattering angle of π/3 compared with small values at small scattering angles $|\theta| \leq \pi/6$. Similar to Laguerre-Gaussian mode, photonic OAM of Bessel-Gauss beam is likely to be invariant at small angle scattering. They just tend to vary in wide scattering. However, extreme minimum scattering elements are at right-angle scattering, where the classical cross-section is minimum.

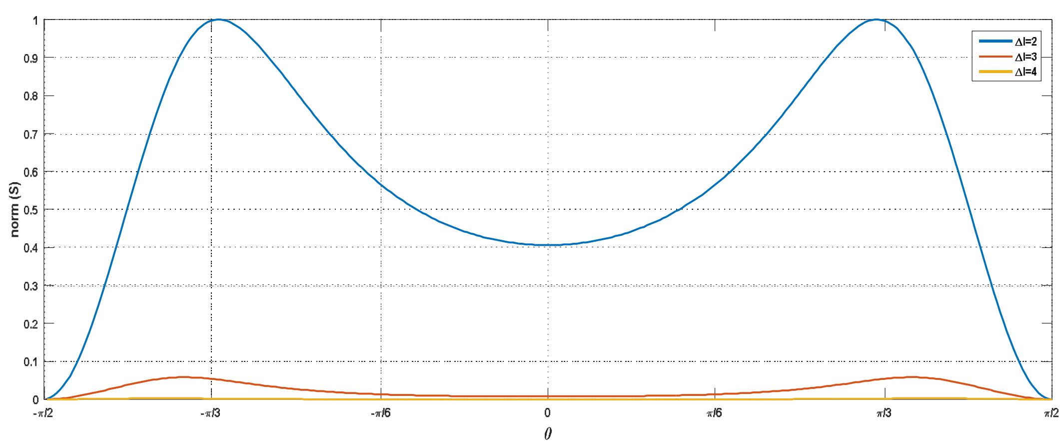

FIG. 5. Normalized Compton matrices *versus* scattering angle for BG beam at certain changes in POAM.

The scattering matrix is also computed for higher-order changes in POAM of Bessel-Gaussian mode as illustrated in Fig. 5. The order is changed by varying the associated azimuthal parameter to 2, 3 and 4, respectively.

Fig. 5 emphasizes the high possibility to exchange OAM at wide scattering compared with small angle scattering. POAM values are more consistent in Compton scattering by small angles, whereas extreme lowest values occur at right-angle scattering.

Conclusion

This study analyzes the conservation of POAM during Compton scattering. We evaluated the scattering matrix for twisted light that is scattered by free electrons. Our analysis indicates that the POAM of the twisted light can be changed through Compton scattering. Specific numerical values are determined by Compton scattering of Laguerre-Gaussian and Bessel-Gaussian light beams. It has been reported that POAM states are invariant at forward scattering as well as small-angle scattering, but they mainly vary in wide-angle scattering.

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