

Jordan Journal of Physics

ARTICLE

Applying Titius-Bode's Law on Exoplanetary Systems

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Received on: 24/7/2016;

Accepted on: 24/11/2016

Abstract: We report the application of Titius-Bode's law on 43 exoplanetary systems containing four or more planets. Due to the fact that most of these systems have their planets located within compact regions extending for less than the semi-major axis of Mercury, we found the necessity to scale down the Titius-Bode's law in each case. In this paper, we present sample calculations for three systems out of the whole set. Results show that most systems studied are verifying the applicability of the law with high accuracy. Consequently, our investigation verifies practically the scale invariance of Titius-Bode's law. The results of this study buildup the confidence in predicting positions of the exoplanets according to Titius-Bode's law besides enabling diagnosing possible reasons of deviations.

Keywords: Titius-Bode's law; Exoplanets; Scale invariance.

Introduction

The Titius-Bode's law (hereafter TB law) is a sequence of numbers that was originally set by Johann Daniel Titius in 1766. The sequence is given as 4, 7, 10, 16, 28, 52, 100, It was associated at first with the name of Johann Elert Bode, who announced it as a law in a footnote of his book. The above sequence can be taken to indicate approximately the distances of the planets from the Sun measured in AU (the semi-major axis of the Earth's orbit around the Sun). If we just divide the numbers in the sequence by 10, where we get 0.4, 0.7, 1.0, 1.6, 2.8, 5.2, 10,, this has to be compared with the actual values of the semi-major axis of the planets in the solar system measured in AU: Mercury, Venus, Earth, Mars, Jupiter and Saturn, which are known to be 0.38, 0.72, 1.0, 1.52, 5.2 and 9.54. Historically, the first support for the law came through its role in discovering Uranus, where the position of the seventh planet in our solar system was predicted at 19.6 AU by this law. In 1781, William Herschel found that there is a planet at 19.218 AU which deviates only by 1.9% from the expected distance. Uranus could have been discovered earlier if the TB relation

had been taken more seriously [1]. Although the TB law did not give an accurate prediction for the distance of Neptune from the Sun, it played an important role in discovering the Asteroid Belt, where the median of position was predicted with high accuracy. Many modified formulae were presented for the TB law because of the large deviation suffered in its prediction in the case of the outer planets. The first modification was suggested by Wurm in 1803 and the last to date was presented by Pankovic and Radacovic in 2009 [2].

During the last two decades, many exoplanetary systems have been discovered, most of which contain one or two planets, but a large number of systems contain more than three exoplanets. It would be worth to study the spacing between these planets in order to know whether the spacing satisfies the so-called Titius-Bode's law or not. A positive result will provide support for this law and may inspire more studies to find its physical foundation.

The configuration of the discovered exoplanetary systems can be compared with our solar system. In a given multiple exoplanetary

system, there could be planets that have not been discovered yet. That may occur because of the lack of sufficient data, such as planetary masses or radii that are below the detection limits [3]. Here, the TB law could be a useful tool for making predictions about the positions of exoplanets in multiple exoplanetary systems. This study aims at testing this idea. We first try to check if the multiple exoplanetary systems fit well with the TB law, then we use the law to make predictions about the positions of undetected exoplanets in systems under our study.

Several studies have considered applying the TB law to the spacing between the exoplanets discovered in multiple planetary systems. These studies followed different approaches and have different strategies. Poveda and Lara [4] tried to investigate whether the 5 planets of the 55 Cancri (also called 55Cnc) fit the TB law or not. They found that if the largest known semi-major axis to be allocated for the sixth planet, then the exponential form of the TB law fits well with the five observed semi-major axes for that system. Hence, they predict the existence of a planet at $a = 2.0$ AU in the large gap between the fourth planet ($a = 0.781$ AU) and the sixth planet (at $a = 5.22$ AU). Moreover, they predicted another new planet for the system at $a = 15.0$ AU. After publishing their paper, the semi-major axis for the most inner planet was updated from 0.038 AU to 0.015 AU [6].

Raymond *et al.* [5] mapped out the region in 55 Cancri between the known planets 55Cnc f and 55Cnc d, where an additional planet, designated g, might exist. Because there is a wide region of stability between them, they found that this region can accommodate a Saturn-mass planet which is a giant planet. So, they suggested that there could be even two or three additional planets.

Lovis *et al.* [7] applied the generalized TB relation to multiple exoplanetary systems detected by the radial velocity method only and found reasonable fits. But, they did not make any predictions for positions of new planets.

Cuntz [8] used the TB relation to predict the existence of new planets in the 55 Cancri systems. He predicted the existence of four planets at distances of 0.085, 0.41, 1.50 and 2.95 AU from their host star. He also predicted the distance of the next possible outer planet in

the system to be between 10.9 AU and 12.2 AU. None of the above predictions is observationally confirmed.

Bovaird and Lineweaver [9] studied the applicability of TB relation on a sample of multiple exoplanetary systems containing four planets and more. They found that the majority of their sample adheres to the TB relation more than the solar system does. They inferred that if any system does not follow the TB relation as close as our Solar System, there is a high probability that one or more exoplanets in this system are not detected so far. They used the TB relation to list their predictions for the existence of 141 exoplanets in 68 multiple exoplanetary systems.

Huang and Bakos [3] used the Kepler mission data to search 97 planets of those studied by Bovaird and Lineweaver [9] in 56 multi-planetary systems and found only five planetary candidates around their predictions. They considered that the remaining predicted planets could not be discovered by Kepler mission yet. They concluded that the ability of the TB law at making predictions in extrasolar planetary systems is questionable.

In this article, we present sample calculations for three exoplanetary systems and report a summary of our investigation of 43 systems, showing the results for the adherence to the TB law presented in terms of an average of the fitting percentage. Detailed calculations for the remaining 40 systems are given in Altaie *et al.* [10].

Method

The strategy of our study is based on the understanding that TB law is a configuration description rather than a law with fixed numbers. This means that the numbers given by the law 0.4, 0.7, ... etc. are only set as scale configurations. The way that the TB law was deduced confirms this understanding. Accordingly, we feel that a scaling of the configuration is always possible and therefore can be taken as a reference for the application of the TB law, as long as the relative arrangement is preserved. From this point of view, this study is a novel one that could open the door wide for other studies in the field, motivating new constraints on the way exoplanetary systems are configured.

We prepared a survey for the exoplanets reported by NASA Exoplanet Archive about the multiple exoplanetary systems under study. The confirmed exoplanets discovered up to date (21/1/2016) are 1932 planets. 259 of these exoplanets belong to 58 multiple exoplanetary systems (in which the same star hosts more than 3 planets). We have excluded 15 multiple exoplanetary systems with 66 exoplanets for lack of data about their semi-major axes. Our sample contains the remaining 193 exoplanets that are hosted by 43 stars. Thirty five systems hosting 153 of these planets were detected by the transit method, seven systems involving 36 planets were detected by the radial velocity method and only one system (HR8799) with 4 planets was detected by direct imaging. Our data on exoplanets reported in this article are obtained from the NASA Exoplanet Archive (<http://exoplanetarchive.ipac.caltech.edu>) [11].

TB Law Scaling

We noticed that most of the known exoplanetary systems under study have their planets located within regions of distances less than the semi-major axis of Mercury. This means that we need to scale the TB law for each of those systems. For this purpose, our method was to choose one of the planets of the given system to be satisfying the TB law at a given order of our choice, calculate the scaling factor which comes to be the ratio between the planet's semi-major axis and the corresponding distance in TB law and accordingly go on scaling all the system with the same factor. Such a scaling of the distances may also be understood as a scaling of the measuring units, as we can calculate this factor for any system by dividing the new astronomical unit by the corresponding distance in the TB law. Every individual system has its own scaling factor. The choice of the order of the planet to fit the TB law is done with the best fit and minimum deviation from the distance given by the law for that order.

Once we get the scaling factor, the configuration of the system becomes known to us. Accordingly, we can place the planets in their positions and the order of TB law which we have chosen up to 10 positions. This resulted, in many cases, in leaving some empty positions for planets that are not discovered yet or that even may not have formed at all, as it is the case with

the Asteroid Belt. These empty positions are considered as predictions from our point of view.

The justification for our scaling was theoretically validated by Graner and Dubrulle [12], who showed that the law is both scale - and rotation - invariant. This important work has established the theoretical basis for the invariance of TB law. In our work, we see that the scaled exoplanetary systems are indeed satisfying the TB law with high accuracy.

Fitting Results

Out of the 43 systems that we have studied, we will choose only three systems. The first is the one with the best fitting percentage of 97.78% which is Kepler-215 system. The second is arbitrarily chosen from the systems which showed values near the average fitting percent of about 90.4% and the last system is the one which showed the worst fitting percentage of 65.27%, which is Kepler-444 system. Error calculation which is taken as a measure of adherence has been presented in the tables below. This is easily done through calculating the absolute error by taking the absolute value of the difference between predicted and observed values and is presented in column (4) of each table as:

$$abs.error = |observed - predicted|;$$

where it is measured in AU. The relative error is calculated as:

$$rel.error = \frac{|abs.error|}{observed}.$$

The average relative error for all the planets in the system can be then easily calculated producing the adherence of the system to TB law. This is called in the tables the fitting percentage, which is an indication of the adherence of the system to TB law.

Fitting percentage (adherence)

$$= 100\% - rel.error.$$

These results are given in the table belonging to each system.

KEPLER-215

This is a G-type star similar to our Sun. Its mass is about 0.98 solar mass with a radius of about 1.0 solar radius and a surface temperature of about 5739 K. This exoplanetary system

harbors 4 confirmed planets which were detected by the transit method. The positions of the confirmed planets in the system are as shown in the second column of Table (1). The scaling factor for this system was calculated by positioning the second planet which has a semi-major axis of 0.113 AU in order $n=3$. Accordingly, the scaling factor should be 0.113. The planet configuration becomes as shown in Table (1).

We predict an inner planet at order $n=1$ positioned at 0.0452 AU and the rest of the predicted planets are all in the outer region at

0.5876 AU and beyond. We have shown the absolute and the relative fitting errors marking the deviation from the standard TB law in order to expose the error contribution from each planet. This might be useful in analyzing the pathology of the system and may help designating some reasons for deviations from the TB law. With the suggested configuration, the average fitting percentage for the system is calculated to be 97.78%. If we look at the configuration, we see high similarity with the solar system apart from the scale factor. The fitting result of the planets of this system is shown in Fig. (1).

TABLE 1. The predicted configuration of Kepler-215 system

n	observed (AU)	predicted (AU)	abs. error (AU)	rel. error
1	–	0.0452	–	–
2	0.084	0.0791	0.0049	0.0583
3	0.113	0.113	0.0000	0.0000
4	0.185	0.1808	0.0042	0.0227
5	0.314	0.3164	0.0024	0.0076
6	–	0.5876	–	–
7	–	1.1300	–	–
8	–	2.2148	–	–
9	–	4.3844	–	–
10	–	8.7236	–	–
Avg. rel. error				2.22%
Fitting Percent				97.78%

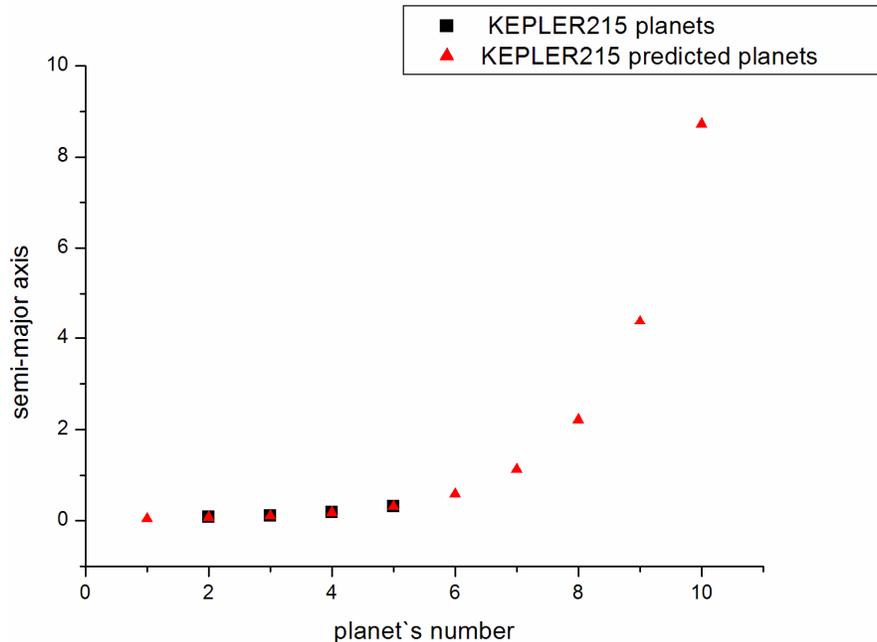


FIG. 1. Fitting of Kepler-215 system.

HD 160691

The star is a G type star, with a mass of about 1.08 solar mass. Its surface temperature is very close to that of our sun at about 5780K. It is older than our sun with an age of about 6.3 Gyr [13]. The system contains four confirmed planets that have been discovered by radial velocity method.

The semi-major axes of the confirmed planets are as shown in the second column of Table (2). The best fitting for the system is obtained when we place the 4th confirmed planet in the 9th position. This makes the scaling factor be 0.13492. Clearly, the system has a very large gap between the 2nd and the 3rd confirmed planet.

This could be because there might be planets that are not discovered yet. Consequently, the fitting allows us to predict the existence of several planets: an inner one at 0.0540 AU and three intermediate ones after the second confirmed planet at 0.1349 AU, 0.2159 AU and 0.3778AU. Two more planets are predicted in the outer region at 1.497 AU and 2.6445 AU. Outer planets at 10.416AU and more may also exist.

The average fitting percentage for this system is 90.61%, which is within the average obtained for the 43 systems under study. The fitting curve for the system is as depicted in Fig. (2).

TABLE 2. The predicted configuration of HD 160691 system

n	observed (AU)	predicted (AU)	abs. error (AU)	rel. error
1	–	0.0540	–	–
2	0.0909	0.0944	0.0035	0.0386
3	–	0.1349	–	–
4	–	0.2159	–	–
5	–	0.3778	–	–
6	0.9210	0.7016	0.2194	0.2382
7	1.497	1.3492	0.1478	0.0987
8	–	2.6445	–	–
9	5.235	5.2350	0.0000	0.0000
10	–	10.416	–	–
Avg. rel. error				9.39%
Fitting Percent				90.61%

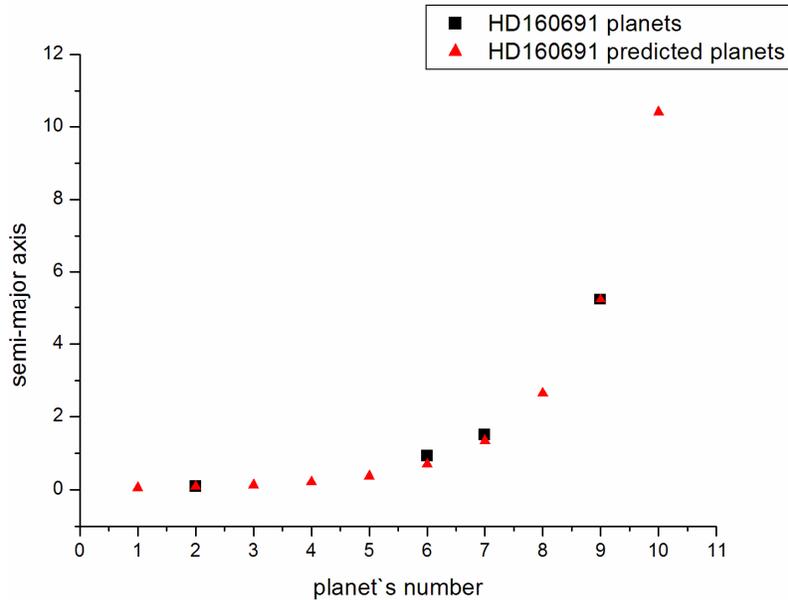


FIG. 2. Fitting of HD160169 system.

KEPLER-444

This is a system with extreme dynamical conditions. It harbors three stars (triple-star system). There are 5 confirmed planets orbiting Kepler-444A which is a K-type star in a tightly packed region. The mass of this primary star is about 0.76 solar mass with a radius of about 0.75 solar radius. This star is believed to be 11.2 Gyr old. This is the oldest star known to date harboring exoplanets. The other two stars (Kepler-444B and Kepler-444C) are small M-type stars orbiting the center of mass of the whole system at a distance of about 40 AU. So, the whole known system is composed of the three stars and the five planets with a size less than that of our solar system. As for the Kepler-444A system, the five planets orbit the host star at distances shown in Table (3). All the predicted

planets are in the outer region at 0.2262 AU and beyond.

To obtain a fitting for this system, we found that the best possible fit will be obtained if we position the 4th planet in $n = 4$ of the TB order. This gives a scaling factor of 0.04350. Consequently, we obtain the configuration given in Table (3), where we see that the confirmed planets are arranged sequentially. The fitting percentage is 65.27%, which is very poor as compared to the fitting percentages of the other planetary systems in this study. The reason is clear; the dynamical setup of the system having three stars at the play makes the system highly unstable. The orbits of this system are coinciding. Orbits of planets c and d, e and f are coinciding. This causes instability in the system. The fitting curve of the system is shown in Fig (3).

TABLE 3. The predicted configuration for Kepler-444

n	Observed (AU)	Predicted (AU)	abs. error (AU)	rel. error
1	0.0418	0.0174	0.0244	0.5835
2	0.0488	0.0304	0.0148	0.3762
3	0.0600	0.0435	0.0165	0.2750
4	0.0696	0.0696	0.0000	0.0000
5	0.0811	0.1218	0.0407	0.5018
6	–	0.2262	–	–
7	–	0.4350	–	–
8	–	0.8526	–	–
9	–	1.6878	–	–
10	–	3.3582	–	–
Avg. rel. error				34.73%
Fitting Percent				65.27

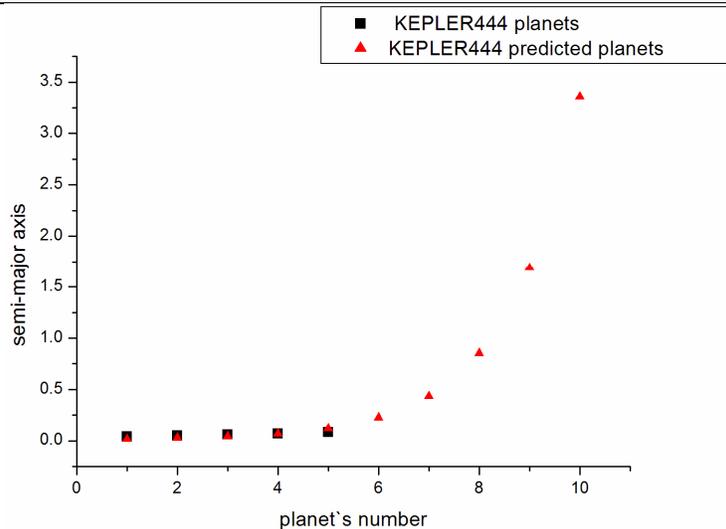


FIG. 3. Fitting of Kepler-444 system.

Discussion and Conclusions

In this article, we have chosen to present three systems out of a set of 43 exoplanetary systems which have been fitted in accordance with the TB law [10]. We find that the majority of our large set adheres to the TB law with an average adherence of about 90.40%. The examples given in this short article have been chosen to show the highest, the median and the lowest of the fitting percentages. Generally, we notice that the adherence to the TB law for exoplanetary systems under our study has different percentages. There are several reasons for these differences; one of them is the age of the system indicated by the age of the hosting star. Young systems with ages of less than 1 Gyr are normally under formation and might not have dynamically settled yet. Such systems suffer jostling during planetary system formation; this is a messy process which can include planetary scattering and migration [9]. This applies to exoplanetary system Kepler-90 (not presented here), which we found to adhere to TB law with a fitting percentage of 72.22 % and is estimated to have an age of 0.5 Gyr (Altaie *et al.* 2016). The other reason for deviating from the TB law may happen if the system under consideration may be too old, and consequently may suffer from dynamical changes that affect the positions of the planets in the system. This applies to Kepler-444 system presented above which is thought to be 11.2 Gyr old. Similarly, Kepler-11 system is found to have a fitting percentage of 73.24 % which is much below the average.

To get an idea about the adherence of our solar system to TB law using the method presented here, we note that the overall fitting percentage obtained is about 85.80%. This is

less than the average which we have obtained for the set of 43 systems that we have studied. However, it should be noted that the relative error that we get for Neptune is about 27.46%, whereas the relative errors for the other 7 planets inside the orbit of Neptune including the Asteroid Belt range from 1.0% to 5.26%. If we would include Pluto in these calculations, we find that the relative error in its position is about 95.74%. But, if we exclude Pluto and do not count it as a planet as it was decided by the IAU in 2006, then we obtain a fitting percentage of 94.89%.

The results presented in our study show that, assuming high confidence in the TB law applicability and adherence, we can have systematic speculations in respect of diagnosing the status of some pathological exoplanetary system which show low fitting percentages. Such speculative diagnosis can be tested by observations, including possible errors in observations or calculations. For example, the system Kepler-33 is found to have a fitting percentage of 78.09 % which is again much below the average percentage for the set considered, despite the fact that it is nearly an ideal system harboring 5 planets with the host star being a Sun-like star with an age of about 4.3 Gyr. This suggests that here we have a pathological case for further studies. Other systems are also analyzed in this respect and could prove that the degree of adherence to TB law might be very useful for analyzing exoplanetary systems.

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