

FIXED POINTS IN AN INTUITIONISTIC MENGER SPACE

RAJINDER SHARMA ⁽¹⁾ AND RAKESH BATRA ⁽²⁾

ABSTRACT. In this paper, we established common fixed point theorems for two pairs of self maps by using the new concept of weakly subsequential continuity (wsc) with compatibility of type (E) in an Intuitionistic Menger space (briefly IM space). We deduce important results in this line by restricting the number of mappings involved.

1. INTRODUCTION

Soon after the concept of probabilistic metric space given by Menger [5], it became one of the attractive area for the researchers involved in the field of fixed point theory and its applications, especially, where the probabilistic situation arises. Schweizer and Sklar [8, 9] stimulated the study further with their pioneering article on statistical metric spaces. Working on the same line, Sehgal and Bharucha-Reid [10] studied some fixed points of contraction mappings on probabilistic metric spaces. Stojakovic [12–14] brought forward the legacy with his pioneering work on probabilistic metric spaces and its applications. Singh and Pant [11] gave some fixed point results for commuting maps in probabilistic metric spaces. Stepping one milestone ahead, Kutucku *et al.* [6] developed probabilistic metric spaces due to Menger [5] to

1991 *Mathematics Subject Classification.* 47H10; 54H25.

Key words and phrases. fixed point, intuitionistic Menger space, compatible maps of type (E), weak subsequential continuous maps (wsc).

Copyright © Deanship of Research and Graduate Studies, Yarmouk University, Irbid, Jordan.

Received: May 16, 2017

Accepted: Jan. 8, 2018 .

intuitionistic Menger spaces and established common fixed point theorems with the help of continuous t -norm and continuous t -conorm. Rashwan and Heder [7] established some new fixed point results for compatible mappings in Menger spaces. Pant *et al.* [3] studied fixed points and its uniqueness for weakly compatible mappings in intuitionistic Menger spaces without any appeal to the continuity of mappings. Leaving aside the condition of continuity, Jain *et al.* [4] came out with some fixed point results for absorbing type of maps in intuitionistic Menger spaces.

Bouhadjera and Thobie [1] proved common fixed point theorems for pairs of sub-compatible maps. Singh *et al.* [15] introduced the notion of compatibility of type (E) and proved some common fixed point theorems for it. Recently, Beloul [2] established some fixed point theorems for two pairs of self mappings satisfying contractive conditions by using the weak subsequential mappings with compatibility of type (E). The purpose of this paper is to obtain common fixed point theorems for weak subsequential continuous mappings with compatibility of type (E) in an intuitionistic Menger space.

2. PRELIMINARIES

Definition 2.1. [3] A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is said to be t -norm if it satisfies the following conditions:

- (i) $*$ is commutative and associative,
- (ii) $*$ is continuous
- (iii) $a * 1 = a$ for all $a \in [0, 1]$ and
- (iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0, 1]$.

Definition 2.2. [3] A binary operation \diamond : $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is said to be t -conorm if it satisfies the following conditions:

- (i) \diamond is commutative and associative,

- (ii) \diamond is continuous,
- (iii) $a \diamond 0 = a$, for all $a \in [0, 1]$ and
- (iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0, 1]$.

Remark 2.1. [3] The concept of triangular norms (t -norms) and triangular conorms (t -conorms) are known as the axiomatic skeletons that we use for characterizing fuzzy intersecion and union respectively. These concepts were originally introduced by Menger [5] in his study of statistical metric spaces.

Definition 2.3. [3] A distance distribution function is function $F : R \rightarrow R^+$ which is left continuous on R , non-decreasing and $\inf_{t \in R} F(t) = 0$, $\sup_{t \in R} F(t) = 1$.

We shall denote by D the family of all distance distribution functions and by H a special distance distribution function in D given by

$$H(t) = \begin{cases} 1 & \text{if } t > 0 \\ 0 & \text{if } t \leq 0 \end{cases}$$

Definition 2.4. [3] A non-distance distribution function is function $L : R \rightarrow R^+$ which is left continuous on R , non-increasing and $\inf_{t \in R} L(t) = 1$, $\sup_{t \in R} L(t) = 0$.

We shall denote by E the family of all non distance distribution functions and by G a special non distance distribution function in E given by

$$G(t) = \begin{cases} 1 & \text{if } t \leq 0 \\ 0 & \text{if } t > 0 \end{cases}$$

Definition 2.5. [3] Given an arbitrary set X , a continuous t -norm $*$, a continuous t -conorm \diamond , a probabilistic distance F and a probabilistic non-distance L on X , the

5-tuple $(X, F, L, *, \diamond)$ is said to be an intuitionistic Menger space if the following conditions are satisfied for all $x, y, z \in X$ and $s, t \geq 0$.

$$(IM1) \quad F(x, y, t) + L(x, y, t) \leq 1,$$

$$(IM2) \quad F(x, y, 0) = 0,$$

$$(IM3) \quad F(x, y, t) = H(t) \text{ iff } x = y,$$

$$(IM4) \quad F(x, y, t) = F(y, x, t),$$

$$(IM5) \quad \text{If } F(x, y, t) = 1 \text{ and } F(y, z, s) = 1, \text{ then } F(x, z, t + s) = 1,$$

$$(IM6) \quad F(x, z, t + s) \geq F(x, y, t) * F(y, z, s),$$

$$(IM7) \quad L(x, y, 0) = 1,$$

$$(IM8) \quad L(x, y, t) = G(t) \text{ iff } x = y,$$

$$(IM9) \quad L(x, y, t) = L(y, x, t),$$

$$(IM10) \quad \text{If } L(x, y, t) = 0 \text{ and } L(y, z, s) = 0, \text{ then } L(x, z, t + s) = 0,$$

$$(IM11) \quad L(x, z, t + s) \leq L(x, y, t) \diamond L(y, z, s).$$

The functions $F(x, y, t)$ and $L(x, y, t)$ denote the degree of nearness and degree of non-nearness between x and y with respect to t , respectively.

Definition 2.6. [6] Let $(X, F, L, *, \diamond)$ be an intuitionistic Menger space with $t * t \geq t$ and $(1 - t) \diamond (1 - t) \leq (1 - t)$ for all $0 < t < 1$. A sequence $\{x_n\}$ in X is said to be convergent to $x \in X$ if, for any $\epsilon > 0$ and $k \in (0, 1)$, there exists a positive integer N such that $F(x_n, x, \epsilon) > 1 - k$ and $L(x_n, x, \epsilon) < k$ whenever $n \geq N$.

Lemma 2.1. [3] Let $(X, F, L, *, \diamond)$ be an intuitionistic Menger space. If there exists a constant $k \in (0, 1)$, and two elements $x, y \in X$ such that for all $t > 0$,

$$F(x, y, kt) \geq F(x, y, t) \text{ and } L(x, y, kt) \leq L(x, y, t),$$

Then $x = y$.

Lemma 2.2. [6] Let $(X, F, L, *, \diamond)$ be an intuitionistic Menger space with $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$ for all $0 < t < 1$. Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X converging to x and y respectively. If $t \geq 0$ is a point of continuity of $F(x, y, \cdot)$ and $L(x, y, \cdot)$, then $\lim_{n \rightarrow \infty} F(x_n, y_n, t) = F(x, y, t)$ and $\lim_{n \rightarrow \infty} L(x_n, y_n, t) = L(x, y, t)$.

Singh *et al.* [15,16] introduced the notion of compatibility of type (E), A -compatibility of type (E) and S -compatibility of type (E), in the setting of intuitionistic Menger spaces, it becomes

Definition 2.7. Self maps A and S on an intuitionistic Menger space $(X, F, L, *, \diamond)$ are said to be compatible of type (E), if $\lim_{n \rightarrow \infty} S^2 x_n = \lim_{n \rightarrow \infty} S A x_n = A z$ and $\lim_{n \rightarrow \infty} A^2 x_n = \lim_{n \rightarrow \infty} A S x_n = S z$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} A x_n = \lim_{n \rightarrow \infty} S x_n = z$ for some $z \in X$.

Definition 2.8. Self maps A and S on an intuitionistic Menger space $(X, F, L, *, \diamond)$ are said to be A -compatible of type (E), if $\lim_{n \rightarrow \infty} A^2 x_n = \lim_{n \rightarrow \infty} A S x_n = S z$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} A x_n = \lim_{n \rightarrow \infty} S x_n = z$ for some $z \in X$. Pair A and S are said to be S -compatible of type (E), if $\lim_{n \rightarrow \infty} S^2 x_n = \lim_{n \rightarrow \infty} S A x_n = A z$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} A x_n = \lim_{n \rightarrow \infty} S x_n = z$ for some $z \in X$.

Remark 2.2. It is also interesting to see that if A and S are compatible of type (E), then they are A -Compatible and S -Compatible of type (E), but the converse need not be true (see Example 1 [2]). Bouhadjera and Thobie [1] introduced the concept of subsequential continuity as follows:

Definition 2.9. Self maps A and S of a metric space (X, d) are said to be subsequentially continuous, if there exists a sequence $\{x_n\}$ such that $\lim_{n \rightarrow \infty} A x_n = \lim_{n \rightarrow \infty} S x_n = t$ for some $t \in X$ and $\lim_{n \rightarrow \infty} A S x_n = A t$ and $\lim_{n \rightarrow \infty} S A x_n = S t$.

Motivated by the Definition 2.9 and [2], we define the following in the setting of intuitionistic Menger space.

Definition 2.10. Self maps A and S defined on an intuitionistic Menger space $(X, F, L, *, \diamond)$ are said to be weakly subsequentially continuous (in short wsc), if there exists a sequence $\{x_n\}$ such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z$ for some $z \in X$ and $\lim_{n \rightarrow \infty} ASx_n = Az$ or $\lim_{n \rightarrow \infty} SAx_n = Sz$

Definition 2.11. Self maps A and S defined on an intuitionistic Menger space $(X, F, L, *, \diamond)$ are said to be S subsequentially continuous, if there exists a sequence $\{x_n\}$ such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z$ for some $z \in X$ and $\lim_{n \rightarrow \infty} SAx_n = Sz$.

Definition 2.12. Self maps A and S defined on an intuitionistic Menger space $(X, F, L, *, \diamond)$ are said to be A subsequentially continuous if there exists a sequence $\{x_n\}$ such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z$ for some $z \in X$ and $\lim_{n \rightarrow \infty} ASx_n = Az$.

Remark 2.3. If the pair of mappings $\{A, S\}$ is A -subsequentially continuous (or S -subsequentially continuous) then it is wsc (see Example 3 [2]).

3. MAIN RESULTS

Theorem 3.1. *Let A, B, S and T be four self mappings of an intuitionistic Menger space $(X, F, L, *, \diamond)$ with continuous t -norm $*$ and continuous t -conorm \diamond satisfying $t * t \geq t$ and $(1 - t)\diamond(1 - t) \leq (1 - t)$ for all $0 < t < 1$. Suppose there exists a constant $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$ the following conditions are*

satisfied:

$$F(Ax, By, kt) \geq \min\{F(Sx, Ty, t), F(Ax, Sx, t), \\ F(By, Ty, t), F(Sx, By, t), F(Ty, Ax, t)\} \quad (3.1)$$

$$L(Ax, By, kt) \leq \max\{L(Sx, Ty, t), L(Ax, Sx, t), \\ L(By, Ty, t), L(Sx, By, t), L(Ty, Ax, t)\} \quad (3.2)$$

If the pairs $\{A, S\}$ and $\{B, T\}$ are weakly subsequential continuous and compatible of type (E), then A, B, S and T have a unique common fixed point in X .

Proof. Since the pair $\{A, S\}$ is weakly subsequential continuous, we can assume that it is A -subsequentially continuous and compatible of type (E). There exists a sequence $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z$, for some $z \in X$ and $\lim_{n \rightarrow \infty} ASx_n = Az$. The compatibility of type (E) implies that $\lim_{n \rightarrow \infty} A^2x_n = \lim_{n \rightarrow \infty} ASx_n = Sz$ and $\lim_{n \rightarrow \infty} S^2x_n = \lim_{n \rightarrow \infty} SAx_n = Az$. Therefore $Az = Sz$, whereas in respect of the pair $\{B, T\}$, suppose that it is B -subsequentially continuous. Then, there exists a sequence $\{y_n\}$ in X such that $\lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = w$, for some $w \in X$ and $\lim_{n \rightarrow \infty} BTy_n = Bw$. The pair $\{B, T\}$ is compatible of type (E), so $\lim_{n \rightarrow \infty} B^2y_n = \lim_{n \rightarrow \infty} BTy_n = Tw$ and $\lim_{n \rightarrow \infty} T^2y_n = \lim_{n \rightarrow \infty} TBy_n = Bw$, for some $w \in X$. This gives $Bw = Tw$. Hence z is a coincidence point of the pair $\{A, S\}$ whereas w is a coincidence point of the pair $\{B, T\}$. Now we prove that $z = w$. Choose a $t > 0$ satisfying (3.1) and (3.2). Without loss of generality, we can assume that t and kt are points of continuity of $F(z, w, \cdot)$, $F(z, z, \cdot)$, $F(w, w, \cdot)$, $L(z, w, \cdot)$, $L(z, z, \cdot)$, $L(w, w, \cdot)$, $F(Az, w, \cdot)$, $F(Sz, w, \cdot)$, $F(z, Bz, \cdot)$, $F(z, Tz, \cdot)$, $L(Az, w, \cdot)$, $L(Sz, w, \cdot)$, $L(z, Bz, \cdot)$ and $L(z, Tz, \cdot)$. This is so because these functions are monotonic on \mathbb{R} and hence have at most countable number of discontinuities in $(0, b)$ for any $b > 0$. So we may choose t sufficiently small that $0 < kt < t < b$ and both kt and

t are points of continuity of all the functions mentioned above. By putting $x = x_n$ and $y = y_n$ in inequality (3.1), we have

$$F(Ax_n, By_n, kt) \geq \min\{F(Sx_n, Ty_n, t), F(Ax_n, Sx_n, t), F(By_n, Ty_n, t), \\ F(Sx_n, By_n, t), F(Ty_n, Ax_n, t)\}.$$

Taking the limit as $n \rightarrow \infty$ and using Lemma 2.2, we get

$$F(z, w, kt) \geq \min\{F(z, w, t), F(z, z, t), F(w, w, t), F(z, w, t), F(w, z, t)\}.$$

So, $F(z, w, kt) \geq \min\{F(z, w, t), 1, 1, F(z, w, t), F(w, z, t)\}$. This gives, for all $t > 0$,

$$F(z, w, kt) \geq F(z, w, t). \quad (3.3)$$

Again, by putting $x = x_n$ and $y = y_n$ in inequality (3.2), we have

$$L(Ax_n, By_n, kt) \leq \max\{L(Sx_n, Ty_n, t), L(Ax_n, Sx_n, t), L(By_n, Ty_n, t), \\ L(Sx_n, By_n, t), L(Ty_n, Ax_n, t)\}$$

Taking the limit as $n \rightarrow \infty$, and using Lemma 2.2 we get

$$L(z, w, kt) \leq \max\{L(z, w, t), L(z, z, t), L(w, w, t), L(z, w, t), L(w, z, t)\}.$$

So, we have for all $t > 0$,

$$L(z, w, kt) \leq \max\{L(z, w, t), 0, 0, L(z, w, t), L(w, z, t)\}.$$

$$L(z, w, kt) \leq L(z, w, t), \quad (3.4)$$

for all $t > 0$.

By Lemma 2.1, (3.3) and (3.4), we have $z = w$. Now we prove that $Az = z$. By putting $x = z$ and $y = y_n$ in the inequality (3.1), we get

$$F(Az, By_n, kt) \geq \min\{F(Sz, Ty_n, t), F(Az, Sz, t), F(By_n, Ty_n, t), \\ F(Sz, By_n, t), F(Ty_n, Az, t)\}.$$

Letting $n \rightarrow \infty$ and using Lemma 2.2, we obtain

$$F(Az, w, kt) \geq \min\{F(Sz, w, t), F(Az, Sz, t), F(w, w, t), F(Sz, w, t), F(w, Az, t)\}.$$

This gives $F(Az, w, kt) \geq \min\{F(Sz, w, t), 1, 1, F(Sz, w, t), F(w, Az, t)\}$.

But $Az = Sz$. Thus, for all $t > 0$,

$$F(Az, w, kt) \geq F(Az, w, t). \quad (3.5)$$

Again, by putting $x = z$ and $y = y_n$ in the inequality (3.2), we get

$$L(Az, By_n, kt) \leq \max\{L(Sz, Ty_n, t), L(Az, Sz, t), L(By_n, Ty_n, t), \\ L(Sz, By_n, t), L(Ty_n, Az, t)\}.$$

Taking the limit as $n \rightarrow \infty$ and using Lemma 2.2, we obtain

$$L(Az, w, kt) \leq \max\{L(Sz, w, t), L(Az, Sz, t), L(w, w, t), L(Sz, w, t), L(w, Az, t)\}.$$

This gives $L(Az, w, kt) \leq \max\{L(Sz, w, t), 0, 0, L(Sz, w, t), L(w, Az, t)\}$ and so for all $t > 0$,

$$L(Az, w, kt) \leq L(Az, w, t) \quad (3.6)$$

By Lemma 2.1, (3.5) and (3.6) we get, $Az = w$. Since $Az = Sz$, we have $Az = Sz = w = z$. Now we prove that $Bz = z$.

By putting $x = x_n$ and $y = z$ in the inequality (3.1), we get

$$F(Ax_n, Bz, kt) \geq \min\{F(Sx_n, Tz, t), F(Ax_n, Sx_n, t), F(Bz, Tz, t), \\ F(Sx_n, Bz, t), F(Tz, Ax_n, t)\}.$$

Taking the limit as $n \rightarrow \infty$ and using Lemma 2.2, we obtain

$$F(z, Bz, kt) \geq \min\{F(z, Tz, t), F(z, z, t), F(Bz, Tz, t), F(z, Bz, t), F(Tz, z, t)\}, \text{ or}$$

$$F(z, Bz, kt) \geq \min\{F(z, Tz, t), 1, 1, F(z, Bz, t), F(Tz, z, t)\}.$$

Since $z = w$ and $Bw = Tw$, then $Bz = Tz$. Thus, for all $t > 0$,

$$F(z, Bz, kt) \geq F(z, Bz, t) \tag{3.7}$$

Again, by putting $x = x_n$ and $y = z$ in the inequality (3.2), we get

$$L(Ax_n, Bz, kt) \leq \max\{L(Sx_n, Tz, t), L(Ax_n, Sx_n, t), L(Bz, Tz, t), \\ L(Sx_n, Bz, t), L(Tz, Ax_n, t)\}.$$

Taking the limit as $n \rightarrow \infty$ and using Lemma 2.2, we get

$$L(z, Bz, kt) \leq \max\{L(z, Tz, t), L(z, z, t), L(Bz, Tz, t), L(z, Bz, t), L(Tz, z, t)\},$$

which gives

$$L(z, Bz, kt) \leq \max\{L(z, Tz, t), 0, 0, L(z, Bz, t), L(Tz, z, t)\} \text{ and so for all } t > 0,$$

$$L(z, Bz, kt) \leq L(z, Bz, t) \tag{3.8}$$

By Lemma 2.1, (3.7) and (3.8), we obtain, $Bz = z$. Since $Bz = Tz$, we have $Bz = Tz = z$. So, in all $z = Az = Bz = Sz = Tz$, that is, z is a common fixed point of A, B, S and T .

To prove uniqueness, let $z^* \in X$ be such that $z^* = Az^* = Bz^* = Sz^* = Tz^*$. By putting $x = z$ and $y = z^*$ in the inequality (3.1), we get

$$F(Az, Bz^*, kt) \geq \min\{F(Sz, Tz^*, t), F(Az, Sz, t), F(Bz^*, Tz^*, t), \\ F(Sz, Bz^*, t), F(Tz^*, Az, t)\}.$$

That is, $F(z, z^*, kt) \geq \min\{F(z, z^*, t), F(z, z, t), F(z^*, z^*, t), F(z, z^*, t), F(z^*, z, t)\}$.

So

$F(z, z^*, kt) \geq \min\{F(z, z^*, t), 1, 1, F(z, z^*, t), F(z^*, z, t)\}$. Thus for all $t > 0$,

$$F(z, z^*, kt) \geq F(z, z^*, t). \tag{3.9}$$

By putting $x = z$ and $y = z^*$ in the inequality (3.2), we get

$$L(Az, Bz^*, kt) \leq \max\{L(Sz, Tz^*, t), L(Az, Sz, t), L(Bz^*, Tz^*, t), \\ L(Sz, Bz^*, t), L(Tz^*, Az, t)\}.$$

That is, $L(z, z^*, kt) \leq \max\{L(z, z^*, t), L(z, z, t), L(z^*, z^*, t), L(z, z^*, t), L(z^*, z, t)\}$.

So

$L(z, z^*, kt) \leq \max\{L(z, z^*, t), 0, 0, L(z, z^*, t), L(z^*, z, t)\}$. Thus for all $t > 0$,

$$L(z, z^*, kt) \leq L(z, z^*, t). \tag{3.10}$$

By Lemma 2.1, (3.9) and (3.10) we get, $z = z^*$. □

If we put $A = B$ in Theorem 3.1 we have the following corollary for three mappings:

Corollary 3.1. *Let A, S and T be three self mappings of an intuitionistic Menger space $(X, F, L, *, \diamond)$ with continuous t -norm $*$ and continuous t -conorm \diamond satisfying $t * t \geq t$ and $(1 - t)\diamond(1 - t) \leq (1 - t)$ for all $0 < t < 1$. Suppose there exists a*

constant $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$ the following conditions are satisfied:

$$F(Ax, Ay, kt) \geq \min\{F(Sx, Ty, t), F(Ax, Sx, t), \\ F(Ay, Ty, t), F(Sx, Ay, t), F(Ty, Ax, t)\}$$

$$L(Ax, Ay, kt) \leq \max\{L(Sx, Ty, t), L(Ax, Sx, t), \\ L(Ay, Ty, t), L(Sx, Ay, t), L(Ty, Ax, t)\}$$

If the pairs $\{A, S\}$ and $\{A, T\}$ are weakly subsequential continuous and compatible of type (E), then A , S and T have a unique common fixed point in X .

Alternatively, if we set $S = T$ in Theorem 3.1, we'll have the following corollary for three self mappings:

Corollary 3.2. *Let A , B and S be four self mappings of an intuitionistic Menger space $(X, F, L, *, \diamond)$ with continuous t -norm $*$ and continuous t -conorm \diamond satisfying $t * t \geq t$ and $(1 - t)\diamond(1 - t) \leq (1 - t)$ for all $0 < t < 1$. Suppose there exists a constant $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$ the following conditions are satisfied:*

$$F(Ax, By, kt) \geq \min\{F(Sx, Sy, t), F(Ax, Sx, t), \\ F(By, Sy, t), F(Sx, By, t), F(Sy, Ax, t)\}$$

$$L(Ax, By, kt) \leq \max\{L(Sx, Sy, t), L(Ax, Sx, t), \\ L(By, Sy, t), L(Sx, By, t), L(Sy, Ax, t)\}$$

If the pairs $\{A, S\}$ and $\{B, S\}$ are weakly subsequential continuous and compatible of type (E), then A , B and S have a unique common fixed point in X .

If we put $S = T$ in Corollary 3.1 , we have the following result for two self mappings:

Corollary 3.3. *Let A, S be two self mappings of an intuitionistic Menger space $(X, F, L, *, \diamond)$ with continuous t -norm $*$ and continuous t -conorm \diamond satisfying $t*t \geq t$ and $(1-t)\diamond(1-t) \leq (1-t)$ for all $0 < t < 1$. Suppose there exists a constant $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$ the following conditions are satisfied:*

$$F(Ax, Ay, kt) \geq \min\{F(Sx, Sy, t), F(Ax, Sx, t),$$

$$F(Ay, Sy, t), F(Sx, Ay, t), F(Sy, Ax, t)\}$$

$$L(Ax, Ay, kt) \leq \max\{L(Sx, Sy, t), L(Ax, Sx, t),$$

$$L(Ay, Sy, t), L(Sx, Ay, t), L(Sy, Ax, t)\}$$

If the pair $\{A, S\}$ is weakly subsequential continuous and compatible of type (E), then A and S have a unique common fixed point in X .

Example 3.1. Let $X = [0, 2]$ with metric $d(x, y) = |x - y|$ and for each $t \in (0, 1)$, define

$$F(x, y, t) = \begin{cases} \frac{t}{t + |x-y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0, \end{cases}$$

$$L(x, y, t) = \begin{cases} \frac{|x-y|}{t + |x-y|}, & \text{if } t > 0; \\ 1, & \text{if } t = 0, \end{cases}$$

for all $x, y \in X$. Clearly $(X, F, L, *, \diamond)$ is an intuitionistic Menger space where $*$ is defined by $t*t \geq t$ and \diamond defined by $(1-t)\diamond(1-t) \leq (1-t)$. Let us define mappings

A, B, S and T as follows:

$$A(x) = B(x) = \begin{cases} 1, & 0 \leq x \leq 1; \\ \frac{3}{4}, & 1 < x \leq 2, \end{cases}$$

$$S(x) = T(x) = \begin{cases} \frac{x+1}{2}, & 0 \leq x \leq 1; \\ 2, & 1 < x \leq 2, \end{cases}$$

Let us consider a sequence $\{x_n\}$ in X defined by $x_n = 1 - \frac{1}{n}$ for $n \in N$ such that $\lim_{n \rightarrow \infty} Ax_n = 1 = \lim_{n \rightarrow \infty} Sx_n$ and $\lim_{n \rightarrow \infty} ASx_n = 1 = A(1)$; $\lim_{n \rightarrow \infty} A^2x_n = 1 = S(1)$, $\lim_{n \rightarrow \infty} S^2x_n = 1 = A(1)$. Hence $\{A, S\}$ is weakly subsequentially continuous and compatible of type (E). Proceeding in the same way, we can easily show that $\{B, T\}$ is weakly subsequentially continuous and compatible of type (E). Next, for $k = \frac{3}{4}$, we'll consider the different cases for which the following inequalities holds:

$$F(Ax, By, kt) \geq \min\{F(Sx, Ty, t), F(Ax, Sx, t), F(By, Ty, t), \\ F(Sx, By, t), F(Ty, Ax, t)\}$$

and

$$L(Ax, By, kt) \leq \max\{L(Sx, Ty, t), L(Ax, Sx, t), L(By, Ty, t), \\ L(Sx, By, t), L(Ty, Ax, t)\}.$$

1. If $x, y \in [0, 1]$, we have

$$F(Ax, By, kt) = 1 \geq \frac{t}{t + \frac{|x-y|}{2}} = F(Sx, Ty, t)$$

and

$$L(Ax, By, kt) = 0 \leq \frac{\frac{|x-y|}{2}}{t + \frac{|x-y|}{2}} = F(Sx, Ty, t).$$

2. If $x \in [0, 1]$ and $y \in (1, 2]$ we have

$$F(Ax, By, kt) = \frac{kt}{kt+0.25} \geq \frac{t}{t+1.25} = F(By, Ty, t)$$

and

$$L(Ax, By, kt) = \frac{0.25}{kt+0.25} \leq \frac{1.25}{t+1.25} = F(By, Ty, t).$$

3. If $x \in (1, 2]$ and $y \in [0, 1]$ we have

$$F(Ax, By, kt) = \frac{kt}{kt+0.25} \geq \frac{t}{t+1.25} = F(Ax, Sx, t)$$

and

$$L(Ax, By, kt) = \frac{0.25}{kt+0.25} \leq \frac{1.25}{t+1.25} = F(Ax, Sx, t).$$

4. If $x, y \in (1, 2]$, we have

$$F(Ax, By, kt) = 1 \geq \frac{t}{t+1.25} = F(Sx, By, t)$$

and

$$L(Ax, By, kt) = 0 \leq \frac{1.25}{t+1.25} = F(Sx, By, t).$$

Thus, all the conditions of Theorem (3.1) are satisfied and A,B,S,T have a unique common fixed point $x = 1$.

Theorem 3.2. *Let A, B, S and T be four self mappings of an intuitionistic Menger space $(X, F, L, *, \diamond)$ with continuous t -norm $*$ and continuous t -conorm \diamond satisfying $t * t \geq t$ and $(1 - t)\diamond(1 - t) \leq (1 - t)$ for all $0 < t < 1$. Suppose there exists a constant $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$ satisfying (3.1) and (3.2). Assume that*

(i) *the pair $\{A, S\}$ is A -compatible of type (E) and A -subsequentially continuous.*

(ii) *the pair $\{B, T\}$ is B -compatible of type (E) and B -subsequentially continuous.*

Then A, B, S and T have a unique common fixed point in X .

Proof. The proof is obvious as on the lines of Theorem 3.1. □

Acknowledgement

The authors would like to offer their sincere gratitude to the editor and the anonymous referees for their valuable comments and suggestions which helps alot to put the article in its present form.

REFERENCES

- [1] Bouhadjera H. and Thobie C.G. : Common fixed point for pair of sub-compatible maps , Hal-00356516. 1 (2009), 1-16.
- [2] Beloul S. : Common fixed point theorems for weakly sub-sequentially continuous generalized contractions with applications, App. Math. E-notes, 15 (2015), 173-186.
- [3] Pant, B.D.: Chauhan, S., Pant, V.: Common fixed point theorems in intuitionistic Menger spaces, Journal of Advanced Studies in Topology, 1 (2010), 54-62.
- [4] Jain,A., Gupta V.K. and Bhide R. : Fixed points in intuitionistic Menger space, Italian J. Pure Appl. Math., 35 (2015), 557-568.
- [5] Menger, K. : Statistical Metrics, Pro. Nat. Acad. Sci. USA, 28 (3), (1942), 535- 537.
- [6] Kutukcu, S., Tuna, A. and Yakut, A.T.: Generalized contraction mapping principle in intuitionistic Menger space and application to differential equations, Appl. Math. Mech., 28 (2007), 799-809.
- [7] Rashwan, R.A., Hedar, A.: On common fixed point theorems of compatible mappings in Menger spaces, Demonstratio Math., 31 (1998), 537-546.
- [8] Schweizer, B. and Sklar, A. : Statistical metric spaces, Pacific. J. Math., 10(1960), 313-334.
- [9] Schweizer, B. and Sklar, A. : Probabilistic metric spaces, Amsterdam; North Holland, (1983).
- [10] Sehgal V.M. and Bharucha-Reid A.T. : Fixed points of contraction mappings on probabilistic metric spaces, Math. Systems Theory 6 (1972), 97-102.
- [11] Singh, S.L. and Pant, B.D. : Fixed point theorems for commuting mappings in probabilistic metric space, Honam Math. J., 5 (1983),139-150.
- [12] Stojakovic, M. : Fixed point theorem in probabilistic metric spaces, Kobe J. Math., 2 (1985), 1-9.
- [13] Stojakovic, M. : Common fixed point theorem in complete metric and probabilistic metric spaces, Bull. Austral. Math. Soc., 36 (1987), 73-88.
- [14] Stojakovic, M. : A common fixed point theorem in probabilistic metric spaces and its applications, Glasnic Math., 23 (1988), 203-211.
- [15] Singh M.R. and Mahendra Singh Y. : Compatible mappings of type (E) and common fixed point theorems of Meir-Keller type, Int. J. Math. Sci. Engg. Appl. 1(2) (2007), 299-315.

- [16] Singh M.R. and Mahendra Singh Y. : On various types of compatible maps and fixed point theorems of Meir-Keller type, Hacet. J. Math. Stat., 40(4)(2011), 503-513.

(1) SOHAR COLLEGE OF APPLIED SCINCES , MATHEMATICS SECTION, PO BOX-135, P.C-311,SOHAR, OMAN.

E-mail address: rajind.math@gmail.com

(2) HANS RAJ COLLEGE, UNIVERSITY OF DELHI, DELHI-110007.

E-mail address: rakeshbatra.30@gmail.com