Jordan Journal of Mathematics and Statistics (JJMS) 12(2), 2019, pp 135 - 149

GENERALIZED FUZZY SLACKS-BASED MEASURES OF EFFICIENCY AND ITS APPLICATIONS

ALI ASHRAFI [(1)] AND MOZHGAN MANSOURI KALEIBAR [(2)]

Abstract. Data envelopment analysis (DEA) is a mathematical approach for evaluating the efficiency of decision making units (DMUs). Additionally, slacks-based measures (SBM) of efficiency are used for direct assessment of efficiency in the presence of imprecise data with slack values. Traditional DEA models assume that all input and output data are known exactly. In many situations, however, some inputs and/or outputs take fuzzy data. Fuzzy DEA (FDEA) models emerge as another class of DEA models to account for imprecise inputs and outputs for decision making units. Although several approaches for solving fuzzy DEA models have been developed, numerous deficiencies including the α-cut approaches and types of fuzzy numbers must still be improved. Moreover, a fuzzy sample DMU (SDMU) still cannot be evaluated for the FDEA model. Therefore, the present paper proposes a generalized Fuzzy SBM (GFSBM) model which can evaluate SDMU and the traditional FSBM model. A numerical experiment is used to demonstrate and show the application of the proposed GFSBM approach.

1. Introduction

Data envelopment analysis is a non-parametric technique for evaluating the relative efficiency of decision making units (DMUs) that use multiple inputs to produce multiple outputs [4, 5]. DEA has been used in various environments and numerous

2000 *Mathematics Subject Classification.* 90C05, 90C70.

Key words and phrases. Efficiency, Fuzzy Mathematical programming, Fuzzy Data, Generalized fuzzy DEA methods, Sample decision making unit (SDMU), Slacks-Based Measure model.

Received: Feb. 18, 2018 Accepted: Jan. 6, 2019 .

applications [15, 23, 31]. One of the main objectives of DEA is to measure the efficiency of decision making unit. Slacks based measure deals directly with the input excesses and the output shortfalls of decision making unit concerned [17]. Their proposed DEA models were initially developed for crisp data and were then extended to fuzzy data. Theoretically, their proposed DEA models were able to work with fuzzy data, but their models had some fundamental drawbacks.

In most real world situations, the possible values of parameters of mathematical models are often only imprecisely or ambiguously known to the experts. It would be certainly more appropriate to interpret the experts understanding of the parameters as fuzzy numerical data which can be represented by means of fuzzy sets of the real line known as fuzzy numbers. Some papers proposes a method using fuzzy set theory in real life application such as fuzzy mathematics [1, 7].

Some papers propose a method using alpha cutting measure which changes fuzzy DEA model into primary firm model. Using such method, it is needed to solve several linear planning problems to estimate membership function and then assess performance of a decision making unit. In more general cases, the data for evaluation is imprecise. Thus, several studies proposed the fuzzy DEA model for input and output data [18]. And large number of studies introduced the application of the FDEA model [10, 12, 18].

The studies on FDEA models still focus on the special DEA model and fuzzy number and often apply only a single fuzzy number and the α-cut approach to one FDEA model. The ranking methods still have several limitations, and the selected DMUs, after applying α-cut, still remain as special DMUs. More important to the above conclusions is that these evaluation methods still cannot analyze a fuzzy sample DMU (SDMU). To address the limitations of the FDEA model, the present paper proposes a generalized FDEA (GFDEA) SBM model. At last, application of method are illustrated.

Their proposed DEA approach considers provided to both generalize the types of fuzzy numbers, the selected special point and the α-cut approach and to improve the traditional evaluating method of FDEA model.

This paper is organized as follows. In Section 2, we present SBM models for measuring the efficiencies of the DMUs. In Section 3, we develop SBM models for dealing with fuzzy data. In section 4 generalized SBM models are introduced. In Section 5 numerical experiment of mentioned models is exhibited followed by conclusions in the last.

2. SBM models for measuring efficiencies with crisp data

Assume that there are n DMUs to be evaluated, each consisting of m inputs and s outputs. $X_{ij}, \quad i=1,\cdots,m$ and $Y_{rj}, \quad r=1,\cdots,s$ denote the input and output values of DMUj $(j=1,\cdots,n)$, all of which are known and non-negative. Then, the production possibility set is defined as follows:

$$T=\left\{(X,Y)|\ \sum_{j=1}^{n}\lambda_j X_j\le X,\ \sum_{j=1}^{n}\lambda_j Y_j\ge Y,\ \lambda_j\ge 0,\quad j=1,\cdots,n\right\} \tag{2.1}$$

T is a closed and convex set. Boundary points of T are defined as the efficient production frontier.

For direct assessment of the efficiency with slack values, an efficiency SBM was proposed by [17]. According to the concept of the efficient production frontier, the SBM model is defined as follows:

$$\text{Minimize}\quad \rho=\frac{1-\dfrac{1}{m}\dfrac{\sum_{i=1}^{m}s_i^-}{x_{io}}}{1+\dfrac{1}{s}\dfrac{\sum_{r=1}^{s}s_r^+}{y_{ro}}} \tag{2.2}$$

$$\begin{aligned}\text{subject to}\quad & \textstyle\sum_{j=1}^{n}\lambda_j x_{ij}+s_i^-=x_{io},\quad i=1,\cdots,m\,,\\ & \textstyle\sum_{j=1}^{n}\lambda_j y_{rj}-s_r^+=y_{ro},\quad r=1,\cdots,s,\\ & \lambda_j\ge 0,\quad j=1,\cdots,n,\quad s_i^-\ge 0,\quad i=1,\cdots,m,\quad s_r^+\ge 0,\quad r=1,\cdots,s.\end{aligned}$$

Where DMUo denotes the DMU under evaluation. s_i^- $(i = 1, \cdots, m)$ and s_r^+ $(r = 1, \cdots, s)$ are called slacks. If $x_{io} = 0$, then the term $\dfrac{s_i^-}{x_{io}}$ is eliminated. If $y_{ro} = 0$, then it is replaced by a very small number, so that the term $\dfrac{s_r^+}{y_{ro}}$ has compensatory effect. Using scale transformation, model (2) can be converted into the following linear programming (LP) model:

(2.3)

$$\begin{array}{ll}
\text{Minimize} & \tau = t - \dfrac{1}{m}\dfrac{\sum_{i=1}^{m} s_i^-}{x_{io}} \\
\text{subject to} & \sum_{j=1}^{n} \Lambda_j x_{ij} + s_i^- = t x_{io}, \quad i = 1, \cdots, m, \\
& \sum_{j=1}^{n} \Lambda_j y_{rj} - s_r^+ = t y_{ro}, \quad r = 1, \cdots, s, \\
& 1 = t + \dfrac{1}{s}\dfrac{\sum_{r=1}^{s} s_r^+}{y_{ro}}, \\
& \Lambda_j \geq 0, \quad j = 1, \cdots, n, \ s_i^- \geq 0, \ i = 1, \cdots, m, \ s_r^+ \geq 0, \ r = 1, \cdots, s, \ t > 0.
\end{array}$$

An important property of efficiency SBM is that τ is independent of the measurement unit used for inputs and outputs, and is monotonically decreasing in each input and output slack. If the optimal value of ρ occurs, i.e. $\rho^* = 1$, the respective DMU is called efficient; otherwise it is called non-efficient.

3. SBM models for measuring the efficiencies with fuzzy data

In this section, we introduce SBM models for measuring the efficiencies in fuzzy environment.

3.1. **SBM models for measuring the efficiencies with fuzzy data.**

In fuzzy DEA, it is assumed that some input values $\tilde{X}_{ij}$ and output values $\tilde{Y}_{ik}$ are approximately known and can be represented by fuzzy sets with membership functions $\mu_{\tilde{X}_{ij}}$ and $\mu_{\tilde{Y}_{ik}}$, respectively Without loss of generality, we will assume that all observations are fuzzy, since crisp values can be represented by degenerated membership functions which only have one value in their domain. To deal with such an uncertain situation, we present the following LP models for obtaining the upper and lower bounds of efficiency with slack values. These models measure the efficiencies of the DMUs. Hence, a fuzzy SBM model can be formulated as:

(3.1)

$$\begin{array}{ll} \text{Minimize} & \tilde{\tau} = t - \dfrac{1}{m}\dfrac{\sum_{i=1}^{m} s_i^-}{\tilde{x}_{io}} \\ \text{subject to} & \sum_{j=1}^{n} \Lambda_j \tilde{x}_{ij} + s_i^- = t\tilde{x}_{io}, \quad i = 1, \cdots, m, \\ & \sum_{j=1}^{n} \Lambda_j y_{rj} - s_r^+ = t y_{ro}, \quad r = 1, \cdots, s, \\ & 1 = t + \dfrac{1}{s}\dfrac{\sum_{r=1}^{s} s_r^+}{\tilde{y}_{ro}} \\ & \Lambda_j \geq 0,\ j = 1, \cdots, n,\ s_i^- \geq 0,\ i = 1, \cdots, m,\ s_r^+ \geq 0,\ r = 1, \cdots, s,\ t > 0. \end{array}$$

Let $S(X_{ij})$ and $S(Y_{ik})$ denote the support of $\tilde{X}_{ij}$ and $\tilde{Y}_{ik}$. The α-cuts of $\tilde{X}_{ij}$ and $\tilde{Y}_{ik}$ are defined as

$$(X_{ij})_\alpha = \left\{ X_{ij} \in S(\tilde{X}_{ij}) \left| \mu_{\tilde{X}_{ij}}(X_{ij}) \geq \alpha \right. \right\} \quad \forall\, i, j$$
$$(Y_{ik})_\alpha = \left\{ Y_{ik} \in S(\tilde{Y}_{ik}) \left| \mu_{\tilde{Y}_{ik}}(Y_{ik}) \geq \alpha \right. \right\} \quad \forall\, i, k$$

Note that $(X_{ij})_\alpha$ and $(Y_{ik})_\alpha$ are crisp sets. Using α-cuts, also called α-level sets, the inputs and outputs can be represented by different levels of confidence intervals. The fuzzy DEA model is thus transformed to a family of crisp DEA models with different α-level sets $\{(X_{ij})_\alpha\,|0 < \alpha \leq 1\}$ and $\{(Y_{ik})_\alpha\,|0 < \alpha \leq 1\}$. These sets represent sets of movable boundaries, and they form nested structures for expressing the relationship between ordinary sets and fuzzy sets [18].

The α-level sets defined are crisp intervals which can be expressed in the form:

$$(X_{ij})_\alpha = \left[\min_{X_{ij}} \left\{ X_{ij} \in S(\tilde{X}_{ij}) \left| \mu_{\tilde{X}_{ij}}(X_{ij}) \geq \alpha \right. \right\},\ \max_{X_{ij}} \left\{ X_{ij} \in S(\tilde{X}_{ij}) \left| \mu_{\tilde{X}_{ij}}(X_{ij}) \geq \alpha \right. \right\} \right]$$
$$(Y_{ij})_\alpha = \left[\min_{Y_{ik}} \left\{ Y_{ik} \in S(\tilde{Y}_{ik}) \left| \mu_{\tilde{Y}_{ik}}(Y_{ik}) \geq \alpha \right. \right\},\ \max_{Y_{ik}} \left\{ Y_{ik} \in S(\tilde{Y}_{ik}) \left| \mu_{\tilde{Y}_{ik}}(Y_{ik}) \geq \alpha \right. \right\} \right]$$

Based on Zadeh's extension principle [29], the efficiency of DMUr can be defined as:

(3.2)

$$\begin{aligned}
\text{Minimize} \quad & \tau_o^U = t - \frac{1}{m}\frac{\sum_{i=1}^m s_i^-}{x_{io}^L} \\
\text{subject to} \quad & \textstyle\sum_{j=1}^n \Lambda_j x_{ij}^L + s_i^- = t x_{io}^L, \quad i = 1, \cdots, m, \\
& \textstyle\sum_{j=1}^n \Lambda_j y_{rj}^U - s_r^+ = t y_{ro}^U, \quad r = 1, \cdots, s, \\
& 1 = t + \frac{1}{s}\frac{\sum_{r=1}^s s_r^+}{y_{ro}^U}, \\
& \Lambda_j \geq 0, \quad j = 1, \cdots, n, \ s_i^- \geq 0, \ i = 1, \cdots, m, \ s_r^+ \geq 0, \ r = 1, \cdots, s, \ t > 0.
\end{aligned}$$

(3.3)

$$\begin{aligned}
\text{Minimize} \quad & \tau_o^L = t - \frac{1}{m}\frac{\sum_{i=1}^m s_i^-}{x_{io}^U} \\
\text{subject to} \quad & \textstyle\sum_{j=1}^n \Lambda_j x_{ij}^L + s_i^- = t x_{io}^U, \quad i = 1, \cdots, m, \\
& \textstyle\sum_{j=1}^n \Lambda_j y_{rj}^U - s_r^+ = t y_{ro}^L, \quad r = 1, \cdots, s, \\
& 1 = t + \frac{1}{s}\frac{\sum_{r=1}^s s_r^+}{y_{ro}^L}, \\
& \Lambda_j \geq 0, \quad j = 1, \cdots, n, \ s_i^- \geq 0, \ i = 1, \cdots, m, \ s_r^+ \geq 0, \ r = 1, \cdots, s, \ t > 0.
\end{aligned}$$

τ_o^U is the efficiency under the most favourable conditions and τ_o^L is the efficiency under the most unfavourable conditions for DMUo. They form the efficiency interval $[\tau_o^L, \tau_o^U]$. If $\tau_o^{U*} = 1$, then DMUo is called efficient.

4. Generalized fuzzy SBM models for measuring the efficiencies with fuzzy data

In this section, we extend SBM models for measuring the efficiencies in fuzzy environment to generalized models.

4.1. Generalized fuzzy SBM models for measuring the efficiencies.

In this section, we extend SBM models in fuzzy environment to generalized models.

Definition 4.1. Suppose DMU is one decision making unit in a decision making problem, all the data that in which have the same input and output data with DMU is called sample decision making unit (SDMU) based on this decision making problem [16].

The distinctions between an SDMU and DMU are presented as follows:

- A DMU must be in the production possibility set, whereas SDMU may be outside of the production possibility set.
- The efficiency value of the DMU must be equal to or below 1, whereas that of the SDMU can be equal to, smaller than, or greater than 1.
- A DMU must appear in the constraints, whereas an SDMU can either appear within or not be among the constraints.
- The reference sets in FDEA model are the efficient FDMUs, while in the generalized FDEA model, they can be the efficient FDMUs, normal FDMUs, inefficient FDMUs, special FDMUs, non-FDMUs. These five types of DMUs are called fuzzy sample DMUs (FSDMUs).

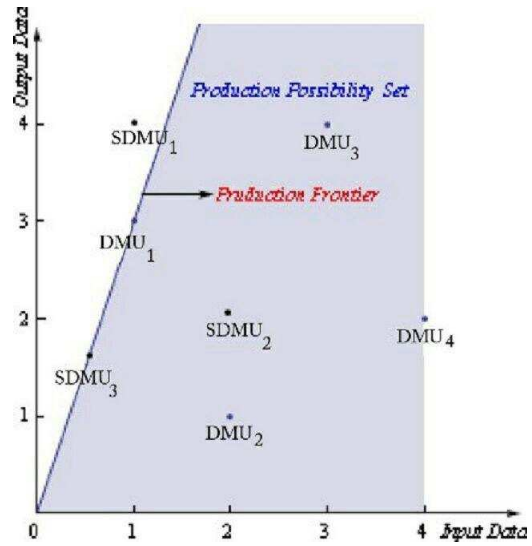

FIGURE 1. SDMUs of the CCR model

Fig. 1 shows the SDMU of a CCR model. For the FDEA model, a number of studies found the efficiency value to be greater than 1. This condition results from the fact that all the constraints, including the target DMU, will select the worst DMUs, whereas the target DMU selects the best DMU to be evaluated. Thus, the evaluated DMU is an SDMU, not a DMU.

Fig. 2 shows an FCCR model with four FDMUs. The target DMU is assumed to be an FDMU1. According to the ranking approach, all the FDMUs of the constraints will select the worst points, a_1, a_2, a_3, a_4, and the target FDMU1 will use the best point A to be evaluated. Therefore, point A is an SDMU, not a DMU [16].

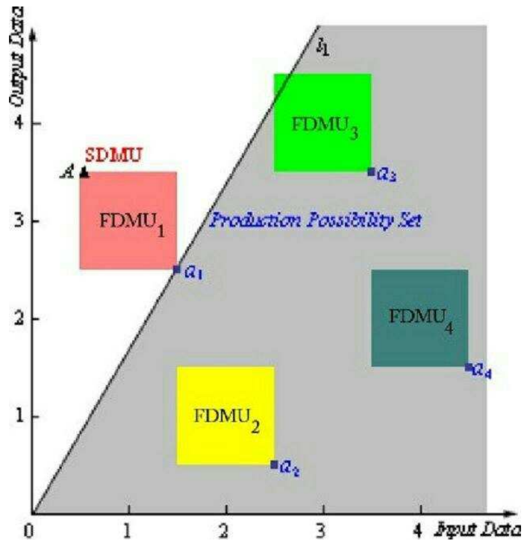

FIGURE 2. DMUs of the FCCR model

After improving FDMUo to FSDMUo, the generalized fuzzy SBM model can be easily obtained. The generalized FSBM model is shown in Eq. (4.1).

(4.1)

$$\begin{aligned} \text{Minimize} \quad & \tau = t - \frac{1}{m}\frac{\sum_{i=1}^{m} s_i^-}{x_{So}} \\ \text{subject to} \quad & \textstyle\sum_{j=1}^{n} \Lambda_j x_{ij} + s_i^- = t x_{io},\ i = 1, \cdots, m, \\ & \textstyle\sum_{j=1}^{n} \Lambda_j y_{rj} - s_r^+ = t y_{ro}, \quad r = 1, \cdots, s, \\ & 1 = t + \frac{1}{s}\frac{\sum_{r=1}^{s} s_r^+}{y_{So}}, \\ & \Lambda_j \geq 0, \quad j = 1, \cdots, n,\ s_i^- \geq 0,\ i = 1, \cdots, m,\ s_r^+ \geq 0,\ r = 1, \cdots, s,\ t > 0. \end{aligned}$$

The generalized FSBM model is shown in Eq. (4.2).

(4.2)

$$\begin{array}{ll} \text{Maximize} & \varphi = t + \dfrac{1}{m}\dfrac{\sum_{i=1}^{m} s_i^+}{x_{So}} \\ \text{subject to} & \sum_{j=1}^{n} \Lambda_j x_{ij} - s_i^+ = t x_{io}, \quad i = 1, \cdots, m, \\ & \sum_{j=1}^{n} \Lambda_j y_{rj} + s_r^- = t y_{ro}, \quad r = 1, \cdots, s, \\ & 1 = t - \dfrac{1}{s}\dfrac{\sum_{r=1}^{s} s_r^-}{y_{So}}, \\ & \Lambda_j \geq 0, \quad j = 1, \cdots, n, \ s_i^+ \geq 0, \ i = 1, \cdots, m, \ s_r^- \geq 0, \ r = 1, \cdots, s, \ t > 0. \end{array}$$

4.2. **Efficiency analysis of the GFSBM model.**

In this section, efficiency measuring method for the GFSBM model are given.

Method: The FSDMU is replaced by one of the SDMUs of the FSDMU and FDMUi by one of the DMU is of the FDMUi. Once the model becomes a crisp DEA model, it can be solved using the appropriate software. The selected SDMU or DMUi can be any point of the domain. Among these points are the following seven special points [16]:

Best DMU, Worst DMU, Max DMU, Min DMU, Center DMU, 1-Cut DMU and Vertex DMU.

When evaluating the target FDMU of the FSBM model, either the best or the worst DMU is selected. The remaining five special DMUs are never selected. In the proposed method, the seven special DMUs or any DMU of the domain that the decision maker prefers can be selected.

Definition 4.2. If the efficiency value of the SDMU is equal to or greater than 1 when the FSDMU is replaced by the worst SDMU and the FDMUs are replaced by the best DMUs, the FSDMU is said to be strongly efficient.

Definition 4.3. If the efficiency value of the DMU is equal to or greater than 1 when the FSDMU and FDMUs are replaced by one of the DMUs, the FSDMU is said to be efficient.

Definition 4.4. If the efficiency value of the SDMU is equal to or greater than 1 when the FSDMU is replaced by the best SDMU and the FDMUs are replaced by the worst DMUs, the FSDMU is said to be weakly efficient.

Definition 4.5. If FSDMU is not weakly efficient, it is said to be inefficient.

We can easily prove that FSDMU is efficient if it is strongly efficient and that FSDMU weakly efficient if it is efficient. However, the converse is false. All possible efficiency values of FSDMU change from strong to weak.

5. Numerical experiment

This research firstly estimates the VaR values of banks, and use the Generalized Fuzzy SBM (GFSBM) model to estimate the efficiency values of the sample banks. Table 1 shows the data for 30 banks. In this research, the VaR is a triangular membership function. After selecting the different FSDMU and FDMU points, the FSDUM and FDMU of the GFSBM model are evaluated using generalized method. In this experiment we assume that all DMUs are SDMU. After using the MATLAB software, different efficiency values of FSDMU are obtained, as shown in second Table .

Table 1. Data for 30 commercial banks

DMU	(I)	(I)	(I)	(I)	(O)	(O)	(O)
	Staff	Total fixed assets	Total deposits	VaR	Total loans	Total investments	Handling fees and commissions
1	6357	1,326,533,000	1,050,190,000	(70,529,416.955; 78,807,369.000; 84,939,256.998)	974,943,000	243,653,636,000	24,964,000
2	7087	1,710,707,000	1,287,330,000	(106,858,247.144; 119,478,198.000; 128,818,105.258)	1,152,060,000	347,215,787,000	35,068,000
3	7054	1,719,297,000	1,318,371,000	(125,572,158.135; 140,237,214.000; 151,133,746.121)	1,114,366,000	339,244,517,000	37,643,000
4	619	302,961,000	29,834,000	(36,615,960.067; 40,599,569.000; 43,536,997.803)	78,758,000	117,615,019,000	6,013,000
5	5103	1,951,405,000	1,289,290,000	(119,208,049.328; 132,442,637.000; 142,214,397.413)	1,303,503,000	262,799,160,000	39,554,000
6	4554	486,452,000	390,918,000	(60,148,426.724; 67,459,310.000; 72,441,632.297)	309,643,000	162,408,129,000	29,925,000
7	2057	263,525,000	240,894,000	(2,690,746.149; 3,006,908.000; 3,249,640.602)	201,832,000	15,489,524,000	11,217,000
8	8660	1,495,246,000	1,100,243,000	(140,833,824.365; 128,880,642.000; 166,447,808.328)	838,473,000	464,253,083,000	132,530,000
9	5910	1,287,367,000	1,020,416,000	(63,920,629.897; 71,386,010.000; 76,937,362.099)	809,587,000	383,290,422,000	37,837,000
10	6259	1,144,145,000	835,647,000	(64,333,945.741; 71,857,095.000; 77,470,993.458)	752,384,000	135,871,482,000	44,377,000

DMU	(I)	(I)	(I)	(I)	(O)	(O)	(O)
	Staff	Total fixed assets	Total deposits	VaR	Total loans	Total investments	Handling fees and commissions
11	5109	1,127,815,000	945,385,000	(23,138,413.488; 25,843,708.000; 27,887,187.847)	878,770,000	143,826,345,000	13,244,000
12	887	156,853,000	130,526,000	(1,218,610.301; 1,362,854.000; 1,475,856.728)	136,244,000	22,504,931,000	4,562,000
13	3388	363,637,000	286,768,000	(4,840,110.110; 5,416,305.000; 5,868,685.009)	178,254,000	99,894,696,000	13,826,000
14	4986	999,939,000	811,336,000	(19,730,865.362; 22,043,240.000; 23,773,976.241)	628,204,000	190,743,070,000	21,153,000
15	3956	793,935,000	621,534,000	(72,253,195.559; 80,909,553.000; 87,255,243.926)	532,833,000	137,988,571,000	27,546,000
16	2865	345,832,000	273,644,000	(17,059,173.302; 19,022,491.000; 20,468,053.950)	235,411,000	62,967,373,000	8,955,000
17	7029	922,248,000	730,199,000	(29,130,040.429; 32,501,226.000; 34,952,896.035)	517,193,000	82,864,223,000	51,064,000
18	2327	360,972,000	281,299,000	(15,070,948.811; 16,806,072.000; 18,088,254.187)	210,523,000	33,423,633,000	11,590,000
19	2662	314,171,000	240,961,000	(7,918,030.617; 8,842,479.000; 9,513,124.336)	218,440,000	15,382,449,000	11,830,000
20	2057	282,356,000	214,779,000	(10,455,868.813; 11,676,926.000; 12,605,004.466)	68,862,000	47,285,483,000	7,405,000
21	3264	385,703,000	329,084,000	(18,750,447.446; 21,063,506.000; 22,551,675.739)	278,853,000	45,843,179,000	11,400,000
22	2459	244,797,000	210,391,000	(2,049,796.528; 2,294,035.000; 2,487,135.188)	164,816,000	14,130,703,000	4,059,000
23	2267	601,748,000	422,033,000	(33,692,235.857; 37,616,815.000; 40,523,263.768)	326,869,000	205,799,438,000	15,672,000
24	1974	250,195,000	214,344,000	(7,406,408.591; 8,280,655.000; 8,928,622.163)	136,151,000	8,176,279,000	5,893,000
25	977	103,845,000	90,772,000	(2,176,761.992; 2,413,018.000; 2,582,166.537)	80,928,000	4,459,217,000	4,840,000
26	1167	103,575,000	93,359,000	(2,873,551.616; 3,158,796.000; 3,367,802.933)	77,650,000	5,089,629,000	4,622,000
27	8792	2,440,706,000	1,998,654,000	(74,570,485.906; 83,266,015.000; 89,697,570.698)	1,823,898,000	175,139,720,000	26,579,000
28	8219	3,087,269,000	2,509,014,000	(97,328,763.805; 108,717,690.000; 117,299,074.842)	1,981,786,000	354,095,568,000	23,412,000
29	1459	171,934,000	146,151,000	(1,968,698.956; 2,199,182.000; 2,369,657.752)	119,632,000	4,529,515,000	3,215,000
30	398	41,241,000	35,808,000	(3,244,061.428; 3,622,866.000; 3,903,951.098)	26,916,000	5,345,923,000	146,000

TABLE 2. Efficiency measures with GFSBM models

DMU	GFSBM (optimistic) worst	GFSBM (optimistic) best	GFSBM (optimistic) center	GFSBM (optimistic) 1-cut
1	0.4694007	0.4675121	0.4469491	0.4675121
2	0.4362531	0.4338996	0.4228676	0.4338996
3	0.3943271	0.2010138	0.3854403	0.2010138
4	1.000000	1.000000	1.000000	1.000000
5	0.4309795	0.4303778	0.4205701	0.4303778
6	0.5240129	0.5228750	0.5079335	0.5228750
7	1.000000	1.000000	1.000000	1.000000
8	1.000000	1.000000	1.000000	1.000000
9	0.8789818	0.8983021	0.8641374	0.8983021
10	0.5323229	0.5287498	0.4815450	0.5287498
11	0.5785811	0.5777702	0.5622465	0.5777702

DMU	GFSBM (optimistic)	GFSBM (optimistic)	GFSBM (optimistic)	GFSBM (optimistic)
	worst	best	center	1-cut
12	1.000000	1.000000	1.000000	1.000000
13	1.000000	1.000000	1.000000	1.000000
14	0.8491007	0.8590758	0.8490316	0.8590758
15	0.3988561	0.3973987	0.3835619	0.3973987
16	0.5093565	0.5075717	0.4785682	0.5075717
17	1.000000	1.000000	1.000000	1.000000
18	0.4793465	0.4789690	0.451397	0.4789690
19	0.3824198	0.3820368	0.3782024	0.3820368
20	0.3412241	0.3409182	0.3402758	0.3409182
21	0.4716393	0.4690391	0.4335034	0.4690391
22	0.4536641	0.4536382	0.4535841	0.4536382
23	1.000000	1.000000	1.000000	1.000000
24	0.2402042	0.2400306	0.2396660	0.2400306
25	0.6957883	1.000000	0.6093805	1.000000
26	0.4497900	0.4551135	0.4047742	0.4551135
27	0.4198307	0.4190282	0.4130571	0.4190282
28	0.4180016	0.4166677	0.3959204	0.4166677
29	0.2657442	0.2657757	0.2658417	0.2657757
30	0.1009084	0.1004816	0.0926031	0.1004816

From the empirical research results shown in Table 2, it can be seen that difference between the upper and lower bounds of the efficiency value for all DMUs. We suppose all the FDMUs are replaced by the center, worst, best and 1-cut DMUs when FDMUs requires replace. The efficiency value of FDMUs after using evaluating model and evaluating method are shown in Table 2.

6. Conclusion

This paper attempts to extend the traditional DEA model to a fuzzy framework, thus proposing a fuzzy SBM model based on α-cut approach and Zadeh's extension principle to deal with the efficiency measuring problem with given fuzzy input and output data. We introduced new model, the generalized FSBM model is the generalization of FSBM model. It can not only evaluate the inner DMU, but also arbitrarily evaluate the given sample DMU. The extension and use of the different fuzzy numbers in one FSBM model make the FSBM model more general. Furthermore, the different evaluation methods of the FSBM model provide more understanding on the FSDMU. The generalization of our evaluating method are applied for calculating the efficiency measure of 30 banks and we obtained results.

References

[1] M. F. Abbod, D. G. Von Keyserlingk, D. A. Linkens, M. Mahfouf, Survey of Utilization of Fuzzy Technology in Medicine and Healthcare, Fuzzy sets and system. 120 (2001) 331-349.

[2] A. Ashrafi, M. Mansouri Kaleibar, Cost, revenue and profit efficiency models in generalized fuzzy data envelopment analysis, Fuzzy Information and Engineering. 9 (2017) 237-246.

[3] H. Azizi, S. Kordrostami, A. Amirteimoori, Slacks-based measures of efficiency in imprecise data envelopment analysis: An approach based on data envelopment analysis with double frontiers, Computers & Industrial Engineering. 79 (2015) 42-51.

[4] R. D. Banker, A. Charnes, W. W. Cooper, Some models for estimating technical and scale inefficiencies in data envelopment analysis, Management Science. 30 (9) (1984) 1078-1092.

[5] A. Charnes, W. W. Cooper, E. Rhodes, Measuring the efficiency of decision making units, European Journal of Operational Research. 2(6) (1978) 429-444.

[6] T. Entani, Y. Maeda, H. Tanaka, Dual models of interval DEA and its extension to interval data, European Journal of Operational Research. 136(1) (2002) 32–45.

[7] A.F. Jameel, N.R. Anakira, A.K. Alomari, D.M. Alsharo, A. Saaban, New semi-analytical method for solving two point nth order fuzzy boundary value problem, Int. J. Mathematical Modelling and Numerical Optimisation. 9(1) (2019) 21-31.

[8] C. Kao, Interval efficiency measures in data envelopment analysis with imprecise data, European Journal of Operational Research. 174(2) (2006) 1087-1099.

[9] C. Kao, S.N. Hwang, Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan, European Journal of Operational Research. 185(1) (2008) 418-429.

[10] C. Kao, S.T. Liu, Fuzzy efficiency measures in data envelopment analysis, Fuzzy Sets and Systems. 113(3) (2000) 427-437.

[11] C. Kao, S.T. Liu, predicting bank performance with financial forecasts: A case of Taiwan commercial banks, Journal of Banking & Finance. 28(10) (2004) 2353-2368.

[12] S.H. Kim, C.G. Park, K.S. Park, An application of data envelopment analysis in telephone offices evaluation with partial data, Computers & Operations Research. 26(1) (1999) 59-72.

[13] Y. K. Lee, K. S. Park, S. H. Kim, Identification of inefficiencies in an additive model based IDEA (imprecise data envelopment analysis), Computers & Operations Research. 29(12) (2002) 1661-1676.

[14] F. H. Liu, C. L Chen, The worst-practice DEA model with slack-based measurement, Computers & Industrial Engineering. 57(2) (2009) 496-505.

[15] S. Lozano, E. Gutiérrez, P. Moreno, Network DEA approach to airports performance assessment considering undesirable outputs, Applied Mathematical Modelling. 37(4) (2013) 1665–1676.

[16] Z. Muren, Wei Cui, Generalized fuzzy data envelopment analysis methods, Applied Soft Computing. 19 (2014) 215-225

[17] K. Tone, A slacks-based measure of efficiency in data envelopment analysis, European Journal of Operational Research. 130(3) (2001) 498-509.

[18] Y.M. Wang, K.S. Chin, A new approach for the selection of advanced manufacturing technologies: DEA with double frontiers, International Journal of Production Research. 47(23) (2009) 6663-6679.

[19] Y.M. Wang, K.S. Chin, Fuzzy data envelopment analysis: A fuzzy expected value approach, Expert Systems with Applications. 38(9) (2011) 11678-11685.

[20] Y.M. Wang, K.S. Chin, P. Jiang, Weight determination in the cross efficiency evaluation, Computers & Industrial Engineering. 61(3) (2011) 497-502.

[21] Y.M. Wang, K.S. Chin, J.B. Yang, Measuring the performances of decision-making units using geometric average efficiency, Journal of the Operational Research Society. 58(7) (2007) 929-937.

[22] Y.M. Wang, R. Great banks, J.B. Yang, Interval efficiency assessment using data envelopment analysis, Fuzzy Sets and Systems. 153(3) (2005) 347-370.

[23] Y.M. Wang, Y.X. Lan, Measuring Malmquist productivity index: A new approach based on double frontiers data envelopment analysis. Mathematical and Computer Modelling. 54(11-12) (2011) 2760-2771.

[24] Y.M. Wang, Y.X. Lan, Estimating most productive scale size with double frontiers data envelopment analysis, Economic Modelling. 33 (2013) 182-186.

[25] Y.M., Wang, Y. Luo, DEA efficiency assessment using ideal and anti-ideal decision making units, Applied Mathematics and Computation. 173(2) (2006) 902-915.

[26] Y.M. Wang, Y. Luo, L. Liang, Fuzzy data envelopment analysis based upon fuzzy arithmetic with an application to performance assessment of manufacturing enterprises, Expert Systems with Applications. 36(3) (2009) 5205-5211.

[27] Y.M., Wang, J.B. Yang, Measuring the performances of decision-making units using interval efficiencies, Journal of Computational and Applied Mathematics. 198(1) (2007) 253-267.

[28] Y.M. Wang, J.B. Yang, D.L. Xu, Interval weight generation approaches based on consistency test and interval comparison matrices, Applied Mathematics and Computation. 167(1) (2005a) 252-273.

[29] Y.M. Wang, J.B. Yang, D.L. Xu, A preference aggregation method through the estimation of utility intervals, Computers & Operations Research. 32(8) (2005b) 2027-2049.

[30] N.S. Wang, R.H. Yi, W. Wang, Evaluating the performances of decision making units based on interval efficiencies. Journal of Computational and Applied Mathematics. 216(2) (2008) 328-343.

[31] J. Wu, Q. An, S. Ali, L. Liang, DEA based resource allocation considering environmental factors, Mathematical and Computer Modelling. 58(5-6), (2013) 1128-1137.

[32] L.A. Zadeh, Fuzzy Sets as a basis for a theory of possibility, Fuzzy Sets and Systems. 1 (1978) 3-28.

[33] J. Zhu, Imprecise data envelopment analysis (IDEA): A review and improvement with an application, European Journal of Operational Research. 144(3) (2003) 513-529.

(1) Faculty of Mathematics, Statistics and Computer Science, Semnan University, Semnan, Iran

E-mail address: a_ashrafi@semnan.ac.ir

(2) Faculty of Mathematics, Statistics and Computer Science, Semnan University, Semnan, Iran

E-mail address: Mozhganmansouri953@gmail.com