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COUPLED FIXED POINT THEOREM FOR HYBRID PAIRS OF MAPPINGS UNDER $\varphi - \psi$ CONTRACTION IN FUZZY METRIC SPACES

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ABSTRACT. In this paper, we establish some common coupled fixed point theorems for two hybrid pairs of mappings under $\varphi - \psi$ contraction on noncomplete fuzzy metric spaces using the property (EA) introduced by Deshpande and Handa [3]. We improve, extend and generalize the results of Deshpande and Handa [3] and several other known results in metric spaces to fuzzy metric spaces.

1. INTRODUCTION

The theory of fuzzy sets was introduced by Zadeh [5] in 1965. Later many authors have introduced the concept of a fuzzy metric space in different ways. George and Veeramani [1] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [11] and defined a Hausdorff topology on fuzzy metric spaces. They also showed that every metric induces a fuzzy metric.

Many authors studied the existence of fixed points for various multivalued contractive mappings under different conditions. The theory of multivalued mappings find some of its applications in control theory, convex optimization, differential inclusions, and economics. Nadler [13] extended the famous Banach contraction principle

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[12] from single-valued mapping to multivalued mapping and proved the fixed point theorem for the multivalued contraction.

The coupled coincidence and coupled fixed point theorems for two mappings were discussed by Bhaskar and Lakshmikantham [15], Lakshmikantham and Ćirić [16]. These results were used to study the existence of a unique solution to a periodic boundary value problem. Luong and Thuan [10] generalized the results of Bhaskar and Lakshmikantham [15]. Berinde [17] extended the results of Bhaskar and Lakshmikantham [15] and Luong and Thuan [10]. Lakshmikantham and Ćirić [16] proved coupled coincidence and common coupled fixed point theorems for nonlinear contractive mappings in partially ordered complete metric spaces and extended the results of Bhaskar and Lakshmikantham [15]. Jain et al [9] extended and generalized the results of Berinde [17], Bhaskar and Lakshmikantham [15], Lakshmikantham and Ćirić [16], and Luong and Thuan [10].

These concepts were extended by Abbas et al [6] to multivalued mappings and who obtained coupled coincidence point and common coupled fixed point theorems involving hybrid pair of mappings satisfying generalized contractive conditions in complete metric spaces.

Aamri and El Moutawakil [7] defined (EA) property for self-mappings which contained the class of noncompatible mappings. Kamran [14] extended the (EA) property for hybrid pair of mappings. Liu et al [18] introduced common (EA) property for hybrid pairs of single and multivalued mappings and gave some new common fixed point theorems under hybrid contractive conditions.

In this paper, we establish some common coupled fixed point theorems for two hybrid pairs of mappings under $\varphi - \psi$ contraction on noncomplete fuzzy metric spaces. The $\varphi - \psi$ contraction is weaker contraction than the contraction defined in Bhaskar and Lakshmikantham [15] and Luong and Thuan [10]. We improve, extend, and generalize the results of Berinde [17], Bhaskar and Lakshmikantham [15], Jain et al [9], Lakshmikantham and Ćirić [16], Liu et al [18], and Luong and Thuan [10], Deshpande and Handa [3] to fuzzy metric spaces. The results of this paper generalize the common fixed point theorems for hybrid pairs of mappings and essentially contain fixed point theorems for hybrid pair of mappings in fuzzy metric spaces.

2. Preliminaries

Let (X, d) be a metric space and suppose that $CB(X)$ denotes the set of non-empty, closed and bounded subsets of X.

For $A, B \in CB(X)$, we denote

$$\begin{aligned} D(A,B) &= \inf\{d(a,b) : a \in A, b \in B\} \\ D(x,A) &= \inf\{d(x,a) : a \in A\} \\ H(A,B) &= \max\{\sup\{D(a,B) : a \in A\}, \sup\{D(A,b) : b \in B\}\} \end{aligned}$$

It is well known that $(CB(X), H)$ is a metric space with the distance function H. Moreover, $(CB(X), H)$ is complete in the event that (X, d) is complete.

Definition 2.1. [5, Definitions, p. 339] Let X be any set. A fuzzy set A in X is a function with domain X and values in $[0, 1]$.

Definition 2.2. [2, Definition 1.1, p. 315] A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if $([0,1], *)$ is an Abelian topological monoid with the unit 1 such that $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$. Examples of t-norms are $a * b = ab$ and $a * b = \min\{a, b\}$.

Definition 2.3. [1, Definition 2.4, p. 395] The 3-tuple $(X, M, *)$ is called a fuzzy metric space (shortly FM-space) if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $t, s > 0$, (fm-1) $M(x,y,t) > 0$,

(fm-2) $M(x,y,t) = 1$ for all $t > 0$ iff $x = y$,

(fm-3) $M(x,y,t) = M(y,x,t)$,

(fm-4) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$,

(fm-5) $M(x,y,.) : X^2 \times [0,\infty) \to [0,1]$ is continuous.

Note that $M(x,y,t)$ can be thought as the degree of nearness between x and y with respect to t. We identify $x = y$ with $M(x,y,t) = 1$ for all $t > 0$ and $M(x,y,t) = 0$ with ∞ and we can find some topological properties and examples of fuzzy metric spaces in [1].

Lemma 2.1. [8, Lemma 4, p. 386] *For all $x, y \in X$, $M(x,y,.)$ is nondecreasing.*

Example 2.1. [1, Example 2.7, p. 396] *Let (X,d) be a metric space wherein $a*b = ab$ for all $a,b \in [0,1]$. Define a fuzzy metric $M_d(x,y,t)$ by $M_d(x,y,t) = \dfrac{t}{t + d(x,y)}$ for each $x,y \in X$ and $t > 0$. Then $(X, M_d, *)$ is a fuzzy metric space.*

Definition 2.4. [4, Definition 2, p. 2] Let $CB(X)$ be the set of all nonempty closed bounded subsets of a fuzzy metric space $(X, M, *)$ Then for every $A, B, C \in CB(X)$ and $t > 0$,

$$M(A,B,t) = \min\left\{\min_{a\in A} M(a,B,t), \min_{b\in B} M(A,b,t)\right\}$$

where $M(C,y,t) = \max\{M(z,y,t) : z \in C\}$.

Obviously, $M(A,B,t) \leq M(a,B,t)$ whenever $a \in A$ and $M(A,B,t) = 1$ iff $A = B$ and $1 = M(A,B,t) \leq M(a,B,t)$ for all $a \in A$.

Definition 2.5. [6, Definition 4, p. 2] Let X be a nonempty set, $F : X \times X \to 2^X$ (a collection of all nonempty subsets of X) and let g be a self-mapping on X. An element $(x,y) \in X \times X$ is called

(i) a coupled coincidence point of hybrid pair $\{F, g\}$ if $gx \in F(x, y)$ and $gy \in F(y, x)$,

(ii) a common coupled fixed point of hybrid pair $\{F, g\}$ if $x = gx \in F(x, y)$ and $y = gy \in F(y, x)$.

We denote the set of coupled coincidence points of mappings F and g by $C\{F, g\}$. Note that if $(x, y) \in C\{F, g\}$ then (y, x) is also in $C\{F, g\}$.

Definition 2.6. [6, Definition 5, p. 2] Let $F : X \times X \to 2^X$ be a multivalued mapping and let g be a self-mapping on X. The mapping g is called $F-$weakly commuting at some point $(x, y) \in X \times X$ if $g^2x \in F(gx, gy)$ and $g^2y \in F(gy, gx)$.

Definition 2.7. [3, Definition 4, p. 2] The mappings $f : X \to X$ and $F : X \times X \to CB(X)$ are said to satisfy the property (EA) if there exist two sequences $\{x_n\}, \{y_n\}$ in $X, u, v \in X$ and $A, B \in CB(X)$ such that-

$$\begin{aligned} \lim_{n\to\infty} fx_n &= u \in A = \lim_{n\to\infty} F(x_n, y_n), \\ \lim_{n\to\infty} fy_n &= v \in B = \lim_{n\to\infty} F(y_n, x_n) \end{aligned}$$

Definition 2.8. [3, Definition 5, p. 2] Let $f, g : X \to X$ and $F, G : X \times X \to CB(X)$. The pairs $\{F, f\}$ and $\{G, g\}$ are said to satisfy the property (EA) if there exist sequences $\{x_n\}, \{y_n\}, \{u_n\}, \{v_n\}$ in $X, u, v \in X$ and $A, B, C, D \in CB(X)$ such that

$$\begin{aligned} \lim_{n\to\infty} F(x_n, y_n) &= A, \lim_{n\to\infty} G(u_n, v_n) = B \\ \lim_{n\to\infty} fx_n &= \lim_{n\to\infty} gu_n = u \in A \cap B, \\ \lim_{n\to\infty} F(y_n, x_n) &= C, \lim_{n\to\infty} G(v_n, u_n) = D, \\ \lim_{n\to\infty} fy_n &= \lim_{n\to\infty} gv_n = v \in C \cap D. \end{aligned}$$

Example 2.2. *Let* $X = [1, +\infty)$ *with usual metric* $d(x, y)$. *Define* $f, g : X \to X$ *and* $F, G : X \times X \to CB(X)$ *by*

$$\begin{aligned} fx &= 1 + x, gx = x + 2 \\ F(x, y) &= [1, 2 + x + 3y], G(x, y) = [2, 3 + x + y] \end{aligned}$$

for all $x, y \in X$. *Consider the sequences*

$$\begin{aligned} \{x_n\} &= \left\{2 + \frac{1}{n}\right\}, y_n = \left\{3 + \frac{1}{n}\right\} \\ \{u_n\} &= \left\{1 + \frac{2}{n}\right\}, v_n = \left\{2 + \frac{3}{n}\right\}. \end{aligned}$$

Then

$$\begin{aligned} \lim_{n\to\infty} F(x_n, y_n) &= [1, 13] = A, \\ \lim_{n\to\infty} G(u_n, v_n) &= [2, 6] = B, \\ \lim_{n\to\infty} fx_n &= \lim_{n\to\infty} gu_n = 3 \in A \cap B, \\ \lim_{n\to\infty} F(y_n, x_n) &= [1, 11] = C, \\ \lim_{n\to\infty} G(v_n, u_n) &= [2, 6] = D, \\ \lim_{n\to\infty} fy_n &= \lim_{n\to\infty} gv_n = 4 \in C \cap D. \end{aligned}$$

Therefore the pairs $\{F, f\}$ *and* $\{G, g\}$ *satisfy the property* (EA).

Let Φ denote the set of all functions $\varphi : [0, +\infty) \to [0, +\infty)$ satisfying the following conditions:

(i_φ) φ is continuous and strictly increasing,

(ii_φ) $\varphi(t) < t$ for all $t > 0$,

(iii_φ) $\varphi(t + s) \leq \varphi(t) + \varphi(s)$ for all $t, s > 0$.

And let Ψ denote the set of all functions $\psi : [0, +\infty) \to [0, +\infty)$ satisfying the following conditions:

(i_ψ) $\lim_{t \to r} \psi(t) > 0$ for all $r > 0$ and $\lim_{t \to 0+} \psi(t) = 0$,

(ii_ψ) $\psi(t) > 0$ for all $t > 0$ and $\psi(0) = 0$.

3. Main Results

Theorem 3.1. *Let $(X, M, *)$ be a fuzzy metric space. Let $f, g : X \to X$ and $F, G : X \times X \to CB(X)$ be mappings satisfying the following conditions-*

(1.1) *$\{F, f\}$ and $\{G, g\}$ satisfy the property (EA),*

(1.2) *for all $x, y, u, v \in X$, there exist some $\varphi \in \Phi$ and some $\psi \in \Psi$ such that*

$$\begin{aligned} & \varphi\left(\frac{M(F(x,y), G(u,v), t) + M(F(y,x), G(v,u), t)}{2}\right) \\ \leq \; & \varphi\left(\frac{M(fx, gu, t) + M(fy, gv, t)}{2}\right) - \psi\left(\frac{M(fx, gu, t) + M(fy, gv, t)}{2}\right) \end{aligned}$$

(1.3) *$f(X)$ and $g(X)$ are closed subsets of X. Then*

(a) F and f have a coupled coincidence point,

(b) G and g have a coupled coincidence point,

(c) if f is $F-$weakly commuting at (x, y), and $f^2x = fx$ and $f^2y = gy$ for $(x, y) \in C\{F, f\}$, then F and f have a common coupled fixed point,

(d) if g is $G-$weakly commuting at (x, y), and $g^2x' = gx'$ and $g^2y' = gy'$ for $(x', y') \in C\{G, g\}$, then G and g have a common coupled fixed point,

(e) F, G, f and g have common coupled fixed point provided that both (c) and (d) are true.

Proof. Since the pairs $\{F, f\}$ and $\{G, g\}$ satisfy the property (EA), there exist sequences $\{x_n\}, \{y_n\}, \{u_n\}, \{v_n\}$ in $X, u, v \in X$ and $A, B, C, D \in CB(X)$ such that

$$\begin{aligned} \lim_{n\to\infty} F(x_n, y_n) &= A, \lim_{n\to\infty} G(u_n, v_n) = B \\ \lim_{n\to\infty} fx_n &= \lim_{n\to\infty} gu_n = u \in A \cap B, \end{aligned}$$

$$\lim_{n\to\infty} F(y_n, x_n) = C, \lim_{n\to\infty} G(v_n, u_n) = D,$$
$$\lim_{n\to\infty} fy_n = \lim_{n\to\infty} gv_n = v \in C \cap D.$$

Since $f(X)$ and $g(X)$ are closed subsets of X, there exist $x, y, x^{/}, y^{/} \in X$ such that

$$u = fx = gx^{/}, v = fy = gy^{/}. \tag{2}$$

Using (1.2), we get

$$\varphi\left(\frac{M(F(x,y), G(u_n, v_n), t) + M(F(y,x), G(v_n, u_n), t)}{2}\right)$$
$$\leq \varphi\left(\frac{M(fx, gu_n, t) + M(fy, gv_n, t)}{2}\right) - \psi\left(\frac{M(fx, gu_n, t) + M(fy, gv_n, t)}{2}\right)$$

As $n \to \infty$, we get by using $(1), (2), (i_\varphi), (ii_\varphi)$ and (i_ψ),

$$\varphi\left(\frac{M(F(x,y), B, t) + M(F(y,x), D, t)}{2}\right) \leq \varphi(0) - \psi(0) = 0,$$

which by (i_φ) and (ii_φ) implies

$$M(F(x,y), B, t) = 0, M(F(y,x), D, t) = 0.$$

Since $fx \in B, fy \in D$, it follows that

$$fx \in F(x,y), fy \in F(y,x).$$

That is (x, y) is a coupled coincidence point of F and f. This proves (a).

Again using (1.2),

$$\varphi\left(\frac{M\left(F(x_n, y_n), G\left(x^{/}, y^{/}\right), t\right) + M\left(F(y_n, x_n), G\left(x^{/}, y^{/}\right), t\right)}{2}\right)$$
$$\leq \varphi\left(\frac{M\left(fx_n, gx^{/}, t\right) + M\left(fy_n, gy^{/}, t\right)}{2}\right) - \psi\left(\frac{M\left(fx_n, gx^{/}, t\right) + M\left(fy_n, gy^{/}, t\right)}{2}\right).$$

As $n \to \infty$, we get by using $(1), (2), (i_\varphi), (ii_\varphi)$ and (i_ψ)

$$\varphi\left(\frac{M\left(A, G\left(x', y'\right), t\right) + M\left(C, G\left(x', y'\right), t\right)}{2}\right) \leq \varphi(0) - \psi(0) = 0,$$

which by (i_φ) and (ii_φ) implies

$$M\left(A, G\left(x', y'\right), t\right) = 0, M\left(C, G\left(x', y'\right), t\right) = 0.$$

Since $gx' \in A, gy' \in C$, it follows that

$$gx' \in G\left(x', y'\right), gy' \in G\left(y', x'\right).$$

That is $\left(x', y'\right)$ is a coupled fixed point of G and g. This proves (b).

Further, since f is $F-$weakly commuting at (x, y) and $f^2x = fx, f^2y = fy$, we have

$$f^2x \in F(fx, fy), f^2(y) \in F(fy, fx).$$

Thus

$$fx = f^2x \in F(fx, fy) \text{ and } fy = f^2y \in F(fy, fx),$$

i.e.

$$u = fu \in F(u, v), v = fv \in F(v, u).$$

This proves (c).

Similar argument proves (d). Then (e) holds immediately. □

Put $f = g$ in Theorem 3.1 we get the following result:

Corollary 3.1. *Let $(X, M, *)$ be a fuzzy metric space. Let $g : X \to X$ and $F, G : X \times X \to CB(X)$ be mappings satisfying the following conditions-*

(2.1) *$\{F, g\}$ and $\{G, g\}$ satisfy the property (EA),* (2.2) *for all* $x, y, u, v \in X$, *there exist some* $\varphi \in \Phi$ *and some* $\psi \in \Psi$ *such that*

$$\begin{aligned}&\varphi\left(\frac{M\left(F\left(x,y\right),G\left(u,v\right),t\right)+M\left(F\left(y,x\right),G\left(v,u\right),t\right)}{2}\right)\\ \leq\ &\varphi\left(\frac{M\left(gx,gu,t\right)+M\left(gy,gv,t\right)}{2}\right)-\psi\left(\frac{M\left(gx,gu,t\right)+M\left(gy,gv,t\right)}{2}\right)\end{aligned}$$

(2.3) $g(X)$ *are closed subsets of* X. *Then*

(a) F *and* g *have a coupled coincidence point,*

(b) G *and* g *have a coupled coincidence point,*

(c) *if* g *is* $F-$*weakly commuting at* (x, y), *and* $g^2x = gx$ *and* $g^2y = gy$ *for* $(x, y) \in C\{F, g\}$, *then* F *and* g *have a common coupled fixed point,*

(d) *if* g *is* $G-$*weakly commuting at* (x, y), *and* $g^2x^{/} = gx^{/}$ *and* $g^2y^{/} = gy^{/}$ *for* $\left(x^{/}, y^{/}\right) \in C\{G, g\}$, *then* G *and* g *have a common coupled fixed point,*

(e) F, G,*and* g *have common coupled fixed point provided that both* (c) *and* (d) *are true.*

Put $F = G$ and $f = g$ in Corollary 3.1 we get the following result:

Corollary 3.2. *Let* $(X, M, *)$ *be a fuzzy metric space. Let* $g : X \to X$ *and* $F : X \times X \to CB(X)$ *be mappings satisfying the following conditions-*

(3.1) $\{F, g\}$ *satisfies the property* (EA),

(3.2) *for all* $x, y, u, v \in X$, *there exist some* $\varphi \in \Phi$ *and some* $\psi \in \Psi$ *such that*

$$\begin{aligned}&\varphi\left(\frac{M\left(F\left(x,y\right),F\left(u,v\right),t\right)+M\left(F\left(y,x\right),F\left(v,u\right),t\right)}{2}\right)\\ \leq\ &\varphi\left(\frac{M\left(gx,gu,t\right)+M\left(gy,gv,t\right)}{2}\right)-\psi\left(\frac{M\left(gx,gu,t\right)+M\left(gy,gv,t\right)}{2}\right)\end{aligned}$$

(3.3) $g(X)$ *are closed subsets of* X. *Then*

(a) F *and* g *have a coupled coincidence point,*

(b) *if* g *is* $F-$*weakly commuting at* (x, y) *and* $g^2x = gx$ *and* $g^2y = gy$ *for* $(x, y) \in C\{F, g\}$, *then* F *and* g *have a common coupled fixed point.*

Corollary 3.3. *Let $(X, M, *)$ be a fuzzy metric space. Let $f, g : X \to X$ and $F, G : X \times X \to CB(X)$ be mappings satisfying the following conditions-*

(4.1) *$\{F, f\}$ and $\{G, g\}$ satisfy the property (EA),*

(4.2) *for all $x, y, u, v \in X$, there exist some $\psi \in \Psi$ such that*

$$\begin{aligned} & M(F(x,y), G(u,v), t) + M(F(y,x), G(v,u), t) \\ \leq \ & M(fx, gu, t) + M(fy, gv, t) - 2\psi\left(\frac{M(fx, gu, t) + M(fy, gv, t)}{2}\right) \end{aligned}$$

(4.3) *$f(X)$ and $g(X)$ are closed subsets of X. Then*

(a) F and f have a coupled coincidence point,

(b) G and g have a coupled coincidence point,

(c) if f is $F-$weakly commuting at (x, y) and $f^2x = fx$ and $f^2y = fy$ for $(x, y) \in C\{F, f\}$, then F and f have a common coupled fixed point,

(d) if g is $G-$weakly commuting at (x, y) and $g^2x^/ = gx^/$ and $g^2y^/ = gy^/$ for $(x^/, y^/) \in C\{G, g\}$, then G and g have a common coupled fixed point,

(e) F, G, f and g have common coupled fixed point provided that both (c) and (d) are true.

Proof. If $\psi \in \Psi$, the for all $k > 0, k\psi \in \Psi$. Divide equation (4.2) by 4 and take $\varphi(t) = (1/2)t, t \in [0, \infty)$. Then the above condition reduces to condition (1.2) of the Theorem 3.1 with $\psi_1 = (1/2)\psi$. Hence by Theorem 3.1, we get the result. □

Put $f = g$ in Corollary 3.3, we get the following result:

Corollary 3.4. *Let $(X, M, *)$ be a fuzzy metric space. Let $g : X \to X$ and $F, G : X \times X \to CB(X)$ be mappings satisfying the following conditions-*

(5.1) *$\{F, g\}$ and $\{G, g\}$ satisfy the property (EA),* (5.2) *for all* $x, y, u, v \in X$, *there exist some* $\psi \in \Psi$ *such that*

$$\begin{aligned} & M(F(x,y), G(u,v), t) + M(F(y,x), G(v,u), t) \\ \leq\ & M(gx, gu, t) + M(gy, gv, t) - 2\psi\left(\frac{M(gx, gu, t) + M(gy, gv, t)}{2}\right) \end{aligned}$$

(5.3) $g(X)$ *is closed subset of* X. *Then*

(a) F *and* g *have a coupled coincidence point,*

(b) G *and* g *have a coupled coincidence point,*

(c) *if* g *is* $F-$*weakly commuting at* (x, y) *and* $g^2x = gx$ *and* $g^2y = gy$ *for* $(x, y) \in C\{F, g\}$, *then* F *and* g *have a common coupled fixed point,*

(d) *if* g *is* $G-$*weakly commuting at* (x, y) *and* $g^2x^/ = gx^/$ *and* $g^2y^/ = gy^/$ *for* $(x^/, y^/) \in C\{G, g\}$, *then* G *and* g *have a common coupled fixed point,*

(e) F, G *and* g *have common coupled fixed point provided that both* (c) *and* (d) *are true.*

Put $F = G$ and $f = g$ in Corollary 3.4, we get the following result:

Corollary 3.5. *Let* $(X, M, *)$ *be a fuzzy metric space. Let* $g : X \to X$ *and* $F : X \times X \to CB(X)$ *be mappings satisfying the following conditions-*

(6.1) $\{F, g\}$ *satisfies the property* (EA),

(6.2) *for all* $x, y, u, v \in X$, *there exist some* $\psi \in \Psi$ *such that*

$$\begin{aligned} & M(F(x,y), F(u,v), t) + M(F(y,x), F(v,u), t) \\ \leq\ & M(gx, gu, t) + M(gy, gv, t) - 2\psi\left(\frac{M(gx, gu, t) + M(gy, gv, t)}{2}\right) \end{aligned}$$

(6.3) $g(X)$ *is closed subset of* X. *Then*

(a) F *and* g *have a coupled coincidence point,*

(b) *if* g *is* $F-$*weakly commuting at* (x, y) *and* $g^2x = gx$ *and* $g^2y = gy$ *for* $(x, y) \in C\{F, g\}$. *then* F *and* g *have a common coupled fixed point.*

Example 3.1. *Let* $(X, M, *)$ *be a fuzzy metric space where* $X = [0, 1]$, $M(x, y, t) = \frac{t}{t + d(x, y)}$, $d : X \times X \to [0, +\infty)$ *defined as* $d(x, y) = \max\{x, y\}$ *and* $d(x, x) = 0$ *for all* $x, y \in X$. *Define* $f, g : X \to X$ *and* $F, G : X \times X \to CB(X)$ *by*

$$\begin{aligned} fx &= \frac{x}{4}, gx = \frac{x}{8} \\ F(x, y) &= [0, \frac{x+y}{4}], G(x, y) = [0, \frac{x+y}{8}] \end{aligned}$$

for all $x, y \in X$. *Define* $\varphi : [0, +\infty) \to [0, +\infty)$ *and* $\psi : [0, +\infty) \to [0, +\infty)$ *by*

$$\varphi(t) = \frac{t}{2}, \psi(t) = \frac{t}{4}$$

Consider the sequences

$$\begin{aligned} \{x_n\} &= \left\{2 + \frac{1}{n}\right\}, y_n = \left\{1 + \frac{1}{n}\right\} \\ \{u_n\} &= \left\{4 + \frac{1}{n}\right\}, v_n = \left\{2 + \frac{2}{n}\right\}. \end{aligned}$$

Then

$$\begin{aligned} \lim_{n\to\infty} F(x_n, y_n) &= [0, \frac{3}{4}] = A, \\ \lim_{n\to\infty} G(u_n, v_n) &= [0, \frac{6}{8}] = B, \\ \lim_{n\to\infty} fx_n &= \lim_{n\to\infty} gu_n = 1/2 \in A \cap B, \\ \lim_{n\to\infty} F(y_n, x_n) &= [0, \frac{3}{4}] = C, \\ \lim_{n\to\infty} G(v_n, u_n) &= [0, \frac{6}{8}] = D, \\ \lim_{n\to\infty} fy_n &= \lim_{n\to\infty} gv_n = 1/4 \in C \cap D, \end{aligned}$$

i.e. the pairs $\{F, f\}$ *and* $\{G, g\}$ *satisfy the property* (EA). *Therefore* (1.1) *of Theorem 3.1 is satisfied.*

Now for all $x, y, u, v \in X$,

$$\varphi\left(\frac{M(F(x, y), G(u, v), t) + M(F(y, x), G(v, u), t)}{2}\right) = \frac{1}{2}$$

and

$$\begin{aligned}
&\varphi\left(\frac{M(fx,gu,t)+M(fy,gv,t)}{2}\right)-\psi\left(\frac{M(fx,gu,t)+M(fy,gv,t)}{2}\right)\\
=\ &\frac{1}{4}\left[\frac{t}{t+x/4}+\frac{t}{t+y/4}\right]-\frac{1}{8}\left[\frac{t}{t+x/4}+\frac{t}{t+y/4}\right]\\
=\ &2t\left[\frac{8t+x+y}{(4t+x)(4t+y)}\right]\\
\leq\ &\left[\frac{8t+x+y}{8t}\right]\\
=\ &1+\frac{x+y}{8t}.
\end{aligned}$$

Therefore $x,y,u,v\in X$,

$$\begin{aligned}
&\varphi\left(\frac{M(F(x,y),G(u,v),t)+M(F(y,x),G(v,u),t)}{2}\right)\\
\leq\ &\varphi\left(\frac{M(fx,gu,t)+M(fy,gv,t)}{2}\right)-\psi\left(\frac{M(fx,gu,t)+M(fy,gv,t)}{2}\right)
\end{aligned}$$

Therefore (1.2) *of Theorem 3.1 is satisfied.*

Also $f([0,1])=[0,1/4]$ *and* $f([0,1])=[0,1/8]$ *are closed subsets of* X.

All other conditions of the Theorem 3.1 are satisfied. The common coupled fixed point of F,G,f *and* g *is* $z=(0,0)$.

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