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SOME PROPERTIES OF FUZZY STAR GRAPH AND FUZZY LINE GRAPH

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Abstract. In 1973, Kauffman [8] introduced the concept of fuzzy graphs. The Wiener index of a graph was introduced in 1947 by Wiener in [24]. The Wiener index of a fuzzy graph was introduced by Mordeson and Mathew in [14]. In this paper, we've achieved the fuzzy Wiener index, the fuzzy hyper-Wiener index, the fuzzy reverse-Wiener index of fuzzy star graph and fuzzy line graph. We also study the concept of the first and second fuzzy Zagreb indices and coindices and mention the relation between fuzzy Wiener index, fuzzy hyper-Wiener index and fuzzy Zagreb indices and coindices of the fuzzy star graph.

1. Introduction

The first definition of fuzzy graph was introduced by Kauffman [8] in the year 1973. But, Rosenfeld [19] described fuzzy relations on fuzzy sets and developed some theories of fuzzy graphs. The importance of fuzziness in different environments, and the concept of fuzzy logic as introduced by Lofti.A.Zadeh [25].

A number of places fuzzy graphs have been studied such as [4], [11], [15] and [20]. Fuzzy graphs structures are important because they have some possible applications in various branches of knowledge like Environmental Science, Social Science, Geography, Linguistics etc.

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In 1947, Harold Wiener [24] defined $W(G)$ as the sum of the distances between all the pairs of vertices of graph G (see also [12]). The hyper-Wiener index of a cyclic graph was introduced by Randic in 1993. Then Klein et al. [9] generalized Randic's definition for all connected graphs, as a generalization of the Wiener index. The reverse-Wiener index was proposed by Balaban et al. in 2000 [3].

The first and second Zagreb indices were introduced by Gutman et al. in [7]. These coindices were introduced by Doslic in [5]. The hyper-Zagreb index $HM_1(G)$ was introduced by Shirdel et al. in [21]. The hyper Zagreb coindex $\overline{HM}_1(G)$ was introduced by Veylaki et al. in [23]. The second hyper-Zagreb index $HM_2(G)$ was introduced by Farhani et al. in [6]. Kulli in [10], introduced the second hyper-Zagreb coindex $\overline{HM}_2(G)$.

Now we state some definitions and notations used throughout this paper.

Definition 1.1. A fuzzy graph $G = (V, \sigma, \mu)$ is a nonempty set V together with a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ such that for all $x, y \in V$, $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$, where $x \wedge y = min\{x, y\}$.

A weakest arc of $G = (V, \sigma, \mu)$ is an arc with least membership value.

A path P of length n is a sequence of distinct nodes $u_0, u_1, ..., u_n$ such that $\mu(u_{i-1}, u_i) > 0$, $i = 1, 2, 3, ..., n$ and the degree of membership of a weakest arc in the path is defined as its strength.

A leaf is a node in a tree with degree 1.

Two nodes that are joined by a path are said to be connected. A fuzzy graph is connected if for any two vertices x and y, there is a path whose endpoints are x and y.

Definition 1.2. Let $G = (V, \sigma, \mu)$ be a connected fuzzy graph. For any path P : $u_0 - u_1 - u_2 - u_3 - ... - u_n$, length of P is denoted by $L(P)$, if $n = 0$, define $L(P) = 0$

and for $n \geq 1$, $L(P)$ is defined as the sum of the weights of the arcs in P, i.e.

$$L(P) = \sum_{i=1}^{n} \mu(u_{i-1}, u_i).$$

For any two nodes u, v in G, let $P = \{P_i : P_i \text{ is a } u - v \text{ path}, i = 1, 2, 3, ...\}$.

The sum distance between u and v is defined as

$$d_S(u, v) = \min\{L(P_i)| \ P_i \in P, i = 1, 2, 3, ...\}.$$

(See also [22]).

Definition 1.3. [1] Let $G = (V, \sigma, \mu)$ be a connected fuzzy graph. The fuzzy diameter $d(G)$ of G is defined by

$$d(G) = max_{\{u,v\} \subseteq V(G)} d_S(u, v).$$

Definition 1.4. Let $G = (V, \sigma, \mu)$ be a fuzzy graph. The degree of a vertex u is defined by

$$deg(u) = \sum_{u \neq v} \mu(u, v).$$

Since $\mu(u, v) > 0$ for $uv \in E$ and $\mu(u, v) = 0$ for $uv \notin E$, this is equivalent to

$$deg(u) = \sum_{uv \in E} \mu(u, v).$$

(See also [16], [17], [18]).

Definition 1.5. A fuzzy star graph consists of two vertex sets $V = \{v\}$ and $U = \{u_1, ..., u_n\}$ such that $\mu_i = \mu(v, u_i) > 0$ for $1 \leq i \leq n$ and $\mu(u_j, u_k) = 0$ for $1 \leq j, k \leq n$ and $j \neq k$. It is denoted by FS_n. (See also [2]).

Mordeson in [13], defined the fuzzy line graph as follows.

Definition 1.6. A fuzzy line graph is a path with n vertices, of which exactly two vertices are leaves and denoted by FP_n.

Throughout this paper, for vertices $u_1, ..., u_n$ of FP_n, μ_i is $\mu(u_i, u_{i+1})$ for $1 \leq i \leq n-1$.

In section 2, we obtain the fuzzy Wiener index and fuzzy hyper-Wiener index of fuzzy star graph and fuzzy line graph.

In section 3, we obtain the fuzzy degree distance and fuzzy reverse-Wiener index of fuzzy star graph and fuzzy line graph.

In section 4, 5 we define the first and second fuzzy Zagreb indices and coindices of fuzzy graphs and the first and second fuzzy hyper-Zagreb indices and coindices of fuzzy graphs also we state some theorems for fuzzy star graph and fuzzy line graph.

2. The fuzzy Wiener index, fuzzy hyper-Wiener index of fuzzy graphs

We begin this section by the following lemma.

Lemma 2.1. *Let FS_n be a fuzzy star graph and FP_n be a fuzzy line graph. Then*

(i) $d(FS_n) = \max\{\mu_i + \mu_j | \ 1 \leq i, j \leq n, i \neq j\}$.

(ii) $d(FP_n) = \sum_{i=1}^{n-1} \mu_i$.

Proof. The proof is obvious. □

Lemma 2.2. *Let FS_n be a fuzzy star graph. Then*

$$deg(v) = \sum_{i=1}^{n} \mu_i, \ deg(u_i) = \mu_i$$

where $1 \leq i \leq n$.

Proof. The proof is clear. □

Now we mention the definitions of fuzzy Wiener index and fuzzy average distance of a fuzzy graph.

Definition 2.1. Let $G = (V, \sigma, \mu)$ be a connected fuzzy graph. The fuzzy Wiener index $W(G)$ of G is defined by

$$W(G) = \sum_{\{u,v\}\subseteq V(G)} d_S(u, v).$$

Definition 2.2. Let $G = (V, \sigma, \mu)$ be a connected fuzzy graph. The fuzzy average distance $\mu(G)$ between the vertices of G is defined by

$$\mu(G) = \frac{W(G)}{\binom{|V(G)|}{2}}.$$

Theorem 2.1. *Let FS_n be a fuzzy star graph. Then*

$$W(FS_n) = n \sum_{i=1}^{n} \mu_i.$$

Proof. The proof is obvious. □

Theorem 2.2. *Let FP_n be a fuzzy line graph. Then*

$$W(FP_n) = \sum_{i=1}^{n-1} i(n-i)\mu_i.$$

Proof. It is clear. □

The next two corollaries follow from the definition of fuzzy average distance, and theorems 2.1 and 2.2, respectively.

Corollary 2.1. *Let FS_n be a fuzzy star graph. Then*

$$\mu(FS_n) = \frac{2\sum_{i=1}^{n} \mu_i}{n+1}.$$

Corollary 2.2. *Let FP_n be a fuzzy line graph. Then*

$$\mu(FP_n) = \frac{2\sum_{i=1}^{n-1} i(n-i)\mu_i}{n(n-1)}.$$

Now we mention the definition of fuzzy hyper-Wiener index of a fuzzy graph.

Definition 2.3. Let $G=(V,\sigma,\mu)$ be a connected fuzzy graph. The fuzzy hyper-Wiener index $WW(G)$ of G is defined by

$$WW(G)=\frac{1}{2}\sum_{\{v,u\}\subseteq V(G)}[d_S(u,v)+d_S(u,v)^2]$$

Theorem 2.3. *Let FS_n be a fuzzy star graph. Then*

$$WW(FS_n)=\frac{1}{2}(n\sum_{i=1}^{n}(\mu_i+\mu_i^2)+2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\mu_i\mu_j).$$

Proof. By definition,

$$WW(FS_n)=\frac{1}{2}(\sum_{i=1}^{n}d_S(v,u_i)+d_S(v,u_i)^2+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}d_S(u_i,u_j)+d_S(u_i,u_j)^2)$$

$$=\frac{1}{2}(\sum_{i=1}^{n}(\mu_i+\mu_i^2)+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(\mu_i+\mu_j)+(\mu_i+\mu_j)^2)$$

$$=\frac{1}{2}(n\sum_{i=1}^{n}(\mu_i+\mu_i^2)+2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\mu_i\mu_j).$$

□

3. The fuzzy degree distance and fuzzy reverse-Wiener index of fuzzy graphs

Definition 3.1. Let $G=(V,\sigma,\mu)$ be a connected fuzzy graph. The fuzzy degree distance $DD(G)$ of G is defined by

$$DD(G)=\sum_{\{v,u\}\subseteq V(G)}(deg(u)+deg(v))d_S(u,v)$$

Theorem 3.1. *Let FS_n be a fuzzy star graph. Then*

$$DD(FS_n)=(n+1)\sum_{i=1}^{n}\mu_i^2+4\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\mu_i\mu_j.$$

Proof. It is similar to the idea in the proof of Theorem 2.3. □

Theorem 3.2. *Let FP_n be a fuzzy line graph. Then*

$$DD(FP_n) = n\sum_{i=1}^{n-1}\mu_i^2 + \sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1} 2(n-(j-i))\mu_i\mu_j.$$

Proof.

$$DD(FP_n) = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(deg(u_i)+deg(u_j))d_S(u_i,u_j)$$

$$= \sum_{j=2}^{n}(deg(u_1)+deg(u_j))d_S(u_1,u_j)$$

$$+\sum_{i=2}^{n-2}\sum_{j=i+1}^{n}(deg(u_i)+deg(u_j))d_S(u_i,u_j) + (deg(u_{n-1})+deg(u_n))d_S(u_{n-1},u_n)$$

$$= \sum_{j=2}^{n}(\mu_1+\mu_{j-1}+\mu_j)(\mu_1+...+\mu_{j-1})$$

$$+\sum_{i=2}^{n-2}\sum_{j=i+1}^{n}(\mu_{i-1}+\mu_i+\mu_{j-1}+\mu_j)(\mu_i+...+\mu_{j-1}) + (\mu_{n-2}+\mu_{n-1}+\mu_{n-1})\mu_{n-1}$$

$$= n\sum_{i=1}^{n-1}\mu_i^2 + \sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1} 2(n-(j-i))\mu_i\mu_j.$$

□

Definition 3.2. Let $G = (V, \sigma, \mu)$ be a connected fuzzy graph. The fuzzy reverse-Wiener index $\Lambda(G)$ is defined by

$$\Lambda(G) = \frac{1}{2}n(n-1)d(G) - W(G).$$

where n is the number of vertices and $d(G)$ is the fuzzy diameter of G.

Lemma 3.1. *Let FS_n be a fuzzy star graph. Then*

$$\Lambda(FS_n) = \frac{1}{2}(n+1)n.\ max\{\mu_i+\mu_j | 1 \le i,j \le n, i \ne j\} - n\sum_{i=1}^{n}\mu_i.$$

Proof. It is obtained from Lemma 2.1(i) and Theorem 2.1. □

Lemma 3.2. *Let FP_n be a fuzzy line graph. Then*

$$\Lambda(FP_n) = \frac{1}{2}n(n-1)\sum_{i=1}^{n-1}\mu_i - \sum_{i=1}^{n-1} i(n-i)\mu_i.$$

Proof. It is obtained from Lemma 2.1(ii) and Theorem 2.2. □

4. The first and second fuzzy Zagreb indices and coindices of fuzzy graphs

Definition 4.1. Let $G = (V, \sigma, \mu)$ be a connected fuzzy graph. The first fuzzy Zagreb index $M_1(G)$ and the second fuzzy Zagreb index $M_2(G)$ of G are defined as

$$M_1(G) = \sum_{uv\in E(G)} (deg(u) + deg(v)),\ M_2(G) = \sum_{uv\in E(G)} deg(u)deg(v).$$

Definition 4.2. Let $G = (V, \sigma, \mu)$ be a connected fuzzy graph. The first fuzzy Zagreb coindex $\overline{M}_1(G)$ and the second fuzzy Zagreb coindex $\overline{M}_2(G)$ of G are defined as

$$\overline{M}_1(G) = \sum_{uv\notin E(G)} (deg(u) + deg(v)),\ \overline{M}_2(G) = \sum_{uv\notin E(G)} deg(u)deg(v).$$

Theorem 4.1. *Let FS_n be a fuzzy star graph. Then*

(i) $M_1(FS_n) = (n+1)\sum_{i=1}^{n}\mu_i$,

(ii) $M_2(FS_n) = (\sum_{i=1}^{n}\mu_i)^2$,

(iii) $\overline{M}_1(FS_n) = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(\mu_i + \mu_j) = (n-1)\sum_{i=1}^{n}\mu_i$,

(iv) $\overline{M}_2(FS_n) = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\mu_i\mu_i$.

Proof. (i) From the definition, we have

$$M_1(FS_n) = \sum_{i=1}^{n} deg(v) + deg(u_i) = \sum_{i=1}^{n}(\sum_{j=1}^{n}\mu_j) + \mu_i = (n+1)\sum_{i=1}^{n}\mu_i.$$

(ii)

$$M_2(FS_n) = \sum_{i=1}^{n} deg(v).deg(u_i)$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_j.\mu_i = \sum_{j=1}^{n}\mu_j.\sum_{i=1}^{n}\mu_i = (\sum_{i=1}^{n}\mu_i)^2.$$

(iii)

$$\overline{M}_1(FS_n) = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} deg(u_i) + deg(u_j)$$
$$= \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \mu_i + \mu_j = (n-1)\sum_{i=1}^{n}\mu_i.$$

(iv)

$$\overline{M}_2(FS_n) = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} deg(u_i).deg(u_j) = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\mu_i\mu_j.$$

□

Corollary 4.1. *Let FS_n be a fuzzy star graph. Then*

(i) $M_1(FS_n) = W(FS_n) + deg(v)$,

(ii) $M_2(FS_n) = (deg(v))^2$,

(iii) $\overline{M}_1(FS_n) = W(FS_n) - deg(v)$,

(iv) $\overline{M}_2(FS_n) = WW(FS_n) - \frac{1}{2}W(FS_n) - \frac{1}{2}n\Sigma_{i=1}^{n}\mu_i^2$.

Proof. It is obtained from Lemma 2.2, Theorem 2.1, Theorem 2.3 and Theorem 4.1.

□

Theorem 4.2. *Let FP_n be a fuzzy line graph. Then*

(i) $M_1(FP_n) = 3\mu_1 + 4\sum_{i=2}^{n-2}\mu_i + 3\mu_{n-1}$,

(ii) $M_2(FP_n) = \mu_1(\mu_1+\mu_2) + \sum_{i=2}^{n-2}(\mu_{i-1}+\mu_i)(\mu_i+\mu_{i+1}) + (\mu_{n-2}+\mu_{n-1})\mu_{n-1}$,

(iii) $\overline{M}_1(FP_n) = \sum_{i=2}^{n-2}(\mu_1+\mu_i+\mu_{i+1}) + \mu_1 + \mu_{n-1} + \sum_{j=2}^{n-3}\sum_{i=j+1}^{n-2}\mu_{j-1}+\mu_j+\mu_i+\mu_{i+1} + \sum_{j=2}^{n-3}\mu_{j-1}+\mu_j+\mu_{n-1}+\mu_{n-3}+\mu_{n-2}+\mu_{n-1}$,

(iv) $\overline{M}_2(FP_n) = \sum_{i=2}^{n-2}\mu_1(\mu_i+\mu_{i+1}) + (\mu_1.\mu_{n-1}) + \sum_{j=2}^{n-3}\sum_{i=j+1}^{n-2}(\mu_{j-1}+\mu_j)(\mu_i+\mu_{i+1}) + \sum_{j=2}^{n-3}(\mu_{j-1}+\mu_j)\mu_{n-1} + (\mu_{n-3}+\mu_{n-2})\mu_{n-1}$.

Proof. (i) From the definition, we have

$$M_1(FP_n) = \sum_{i=1}^{n-1} deg(u_i) + deg(u_{i+1})$$

$$= \mu_1 + \mu_1 + \mu_2 + \sum_{i=2}^{n-2}(\mu_{i-1} + \mu_i + \mu_i + \mu_{i+1}) + \mu_{n-2} + \mu_{n-1} + \mu_{n-1}$$

$$= 3\mu_1 + 4\sum_{i=2}^{n-2}\mu_i + 3\mu_{n-1}.$$

(ii)

$$M_2(FP_n) = \sum_{i=1}^{n-1} deg(u_i).deg(u_{i+1})$$

$$= \mu_1(\mu_1 + \mu_2) + \sum_{i=2}^{n-2}(\mu_{i-1} + \mu_i)(\mu_i + \mu_{i+1}) + (\mu_{n-2} + \mu_{n-1})\mu_{n-1}.$$

(iii)

$$\overline{M}_1(FP_n) = \sum_{i=2}^{n-2} deg(u_1) + deg(u_{i+1}) + deg(u_1) + deg(u_n)$$

$$+\sum_{j=2}^{n-3}\sum_{i=j+1}^{n-2} deg(u_j) + deg(u_{i+1}) + \sum_{j=2}^{n-3} deg(u_j) + deg(u_n) + deg(u_{n-2}) + deg(u_n)$$

$$= \sum_{i=2}^{n-2}(\mu_1 + \mu_i + \mu_{i+1}) + \mu_1 + \mu_{n-1} + \sum_{j=2}^{n-3}\sum_{i=j+1}^{n-2} \mu_{j-1} + \mu_j + \mu_i + \mu_{i+1}$$

$$+\sum_{j=2}^{n-3}\mu_{j-1} + \mu_j + \mu_{n-1} + \mu_{n-3} + \mu_{n-2} + \mu_{n-1}.$$

(iv)

$$\overline{M}_2(FP_n) = \sum_{i=2}^{n-2} deg(u_1).deg(u_{i+1}) + deg(u_1).deg(u_n)$$

$$+\sum_{j=2}^{n-3}\sum_{i=j+1}^{n-2} deg(u_j).deg(u_{i+1}) + \sum_{j=2}^{n-3} deg(u_j).deg(u_n) + deg(u_{n-2}).deg(u_n)$$

$$= \sum_{i=2}^{n-2}\mu_1(\mu_i + \mu_{i+1}) + (\mu_1\mu_{n-1}) + \sum_{j=2}^{n-3}\sum_{i=j+1}^{n-2}(\mu_{j-1} + \mu_j)(\mu_i + \mu_{i+1})$$

$$+\sum_{j=2}^{n-3}(\mu_{j-1} + \mu_j)\mu_{n-1} + (\mu_{n-3} + \mu_{n-2})\mu_{n-1}.$$

□

5. The first and second fuzzy hyper-Zagreb indices and coindices of fuzzy graphs

Definition 5.1. Let $G = (V, \sigma, \mu)$ be a connected fuzzy graph. The first fuzzy hyper-Zagreb index $HM_1(G)$ and the second fuzzy hyper-Zagreb index $HM_2(G)$ of G are defined as

$$HM_1(G) = \sum_{uv \in E(G)} (deg(u) + deg(v))^2, \ HM_2(G) = \sum_{uv \in E(G)} (deg(u)deg(v))^2.$$

Definition 5.2. Let $G = (V, \sigma, \mu)$ be a connected fuzzy graph. The first fuzzy hyper-Zagreb coindex $\overline{HM}_1(G)$ and the second fuzzy hyper-Zagreb coindex $\overline{HM}_2(G)$ of G are defined as

$$\overline{HM}_1(G) = \sum_{uv \notin E(G)} (deg(u) + deg(v))^2, \ \overline{HM}_2(G) = \sum_{uv \notin E(G)} (deg(u)deg(v))^2.$$

We have following theorem.

Theorem 5.1. *Let FS_n be a fuzzy star graph. Then*

(i) $HM_1(FS_n) = \sum_{i=1}^{n}(\sum_{j=1}^{n} \mu_j + \mu_i)^2$,

(ii) $HM_2(FS_n) = \sum_{i=1}^{n}(\sum_{j=1}^{n} \mu_j.\mu_i)^2$,

(iii) $\overline{HM}_1(FS_n) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n}(\mu_i + \mu_j)^2$,

(iv) $\overline{HM}_2(FS_n) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n}(\mu_i.\mu_j)^2$.

Proof. The proof of this theorem is similar to the proof of Theorem 4.1. □

Theorem 5.2. *Let FP_n be a fuzzy line graph. Then*

(i) $HM_1(FP_n) = (2\mu_1 + \mu_2)^2 + \sum_{i=2}^{n-2}(\mu_{i-1} + 2\mu_i + \mu_{i+1})^2 + (2\mu_{n-1} + \mu_{n-2})^2$,

(ii) $HM_2(FP_n) = (\mu_1.(\mu_1 + \mu_2))^2 + \sum_{i=2}^{n-2}((\mu_{i-1} + \mu_i).(\mu_i + \mu_{i+1}))^2 + ((\mu_{n-2} + \mu_{n-1}).\mu_{n-1})^2$

(iii) $\overline{HM}_1(FP_n) = \sum_{i=2}^{n-2}(\mu_1 + \mu_i + \mu_{i+1})^2 + (\mu_1 + \mu_{n-1})^2 + \sum_{j=2}^{n-3} \sum_{i=j+1}^{n-2}(\mu_{j-1} + \mu_j + \mu_i + \mu_{i+1})^2 + \sum_{j=2}^{n-3}(\mu_{j-1} + \mu_j + \mu_{n-1})^2 + (\mu_{n-3} + \mu_{n-2} + \mu_{n-1})^2$,

(iv) $\overline{HM}_2(FP_n) = \sum_{i=2}^{n-2}(\mu_1(\mu_i + \mu_{i+1}))^2 + (\mu_1.\mu_{n-1})^2 + \sum_{j=2}^{n-3}\sum_{i=j+1}^{n-2}((\mu_{j-1} + \mu_j)(\mu_i + \mu_{i+1}))^2 + \sum_{j=2}^{n-3}((\mu_{j-1} + \mu_j)\mu_{n-1})^2 + ((\mu_{n-3} + \mu_{n-2})\mu_{n-1})^2$.

Proof. The proof of this theorem is similar to the proof of Theorem 4.2. □

References

[1] M. Akram, and M. G. karunambigai, Metric in bipolar fuzzy graphs, *World Applied Sciences Journal* **14 12** (2011), 1920-1927.

[2] K. Ameenal, M. Devi, Fuzzy B-magic labeling on cycle graph and star graph, *global Journal of Pure and Applied Mathematics.* **6** (2017), 1975-1985.

[3] A. T. Balaban, D. Mills, O. Ivanciuc, S. C. Basak, Reverse Wiener indices, *Croat Chem. Acta* **73** (2000), 923-941.

[4] P. Bhattacharya, Some remarks on fuzzy graphs, *Pattern Recognition Letters,* **6** (1987), 297-302.

[5] T. Doslic, Vertex weighted Wiener polynomials for composite graphs, *Ars Mathematica Contemporanea*, 1 (1), (2008), 66-80.

[6] M. R. Farhani, M. R. R. Kanna and R. Pradeep Kumar , On the hyper-Zagreb indices of nano-structures, *Asian Academic Research J. Multidisciplinary,* **3(1)** (2016).

[7] I. Gutman and N. Trinajstic, Graph theory and molecular orbitals. Total π-electron energy of alternant hydrocarbons, *Chem. Phys. Lett.* **17** (1972), 535-538.

[8] A. Kauffman, "Introduction a la Theorie des Sous-emsembles Flous". *Masson et Cie Editeurs*, Paris .(1973).

[9] D. J. Klein, I. Lukovits, I. Gutman, On the definition of the hyper-Wiener index for cycle-containing structures, *J. Chem. Inf. Comput. Sci* **35** (1995), 50-52.

[10] V. R. Kulli, On the second hyper-Zagreb coindex of a graph, submitted.

[11] S. Mathew, M. S. Sunitha, Some remarks on fuzzy bipartite graphs, *Adv. Fuzzy Sets Syst* **10** (2011), no. 1, 45-56.

[12] B. Mohar, T. Pisansky, How to compute the Weiner index of a graph, *Journal of Mathematical Chemistry* **2** (1988), 267 277.

[13] J. N. Mordeson, Fuzzy line graphs, *Pattern Recognition Letters* **14** (1993), 381-384.

[14] J. N. Mordeson, S. Mathew , Advanced topics in fuzzy graph theory, (2018).

[15] J. N. Mordeson, P. S. Nair, Fuzzy Graphs and Fuzzy Hypergraphs, Physica Verlag, (2000).

[16] G. A. Nagoor, A. M. Basheer, Order and size in fuzzy graph, *Bull. Pure Appl. Sci. Sect. E Math. Stat* **22** (2003), no. 1, 145-148.

[17] G. A. Nagoor, K. Radha, On Regular Fuzzy Graphs, *Journal of physical Sciences* **12** (2008), 33-40.

[18] G. A. Nagoor, K. Radha, The Degree of a Vertex in some Fuzzy Graphs, *International Journal of Algorithms, Computing and Mathematics* **2**(2009), no. 3, 107-116.

[19] A. Rosenfeld, Fuzzy graphs, in: L.A. Zadeh, K.S. Fu,, K. Tanaka, M. Shimura (Eds), "Fuzzy Sets and Their Applications to Cognitive and Decision Processes, ", *Academic Press*, New York. (1975) 77-95.

[20] S. Samanta, B. Sarkar, D. Shin, and M. Pal, Completeness and regularity of generalized fuzzy graphs, *Springer Plus* **5** (2016), 1-15.

[21] G. H. Shirdel, H. Rezapour and A. M. Sayadi, The hyper-Zagreb index of graph operations, *Iran J. Math. Chem.* **4(2)** (2013), 213-220.

[22] M. Tom, M. S. Sunitha, Sum Distance in Fuzzy Graphs, *Annals of Pure and Applied Mathematics* **7** (2014), no. 2, 73-89.

[23] M. Veylaki, M. J. Nikmehr and H. R. Tavallaee, The third and hyper-Zagreb coindices of some graph operations, *J. Appl. Math. Comput* (2015).

[24] H. Wiener, J. Amer. Chem. Soc. **69** (1947), 17.

[25] L. A. Zadeh, Fuzzy sets, *Inform. And Control.* **8** (1965), 338-353.

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