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CHINMAYI INDICES OF SOME GRAPH OPERATIONS

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Abstract. In this paper, we investigate the mathematical properties of a set of new novel topological indices named as *Chinmayi indices.* We obtain closed formulae for *Chinmayi indices* of some class of graphs. Further, we obtain *first Chinmayi index* of some graph operations viz., Cartesian product, composition, tensor product, and corona product of two graphs. Lastly, as an application of graph operations, we derive explicit formulae for some chemically important structures and nano materials.

1. Introduction

Let $G=(V,E)$ be a finite, undirected graph without loops and multiple edges with V as vertex set and E as edge set. Let $|V|=n$ and $|E|=m$. The neighbourhood of a vertex $u \in V(G)$ is defined as the set $N_G(u)$ consisting of all points v which are adjacent with u. The closed neighbourhood is $N_G[u]=N_G(u)\cup\{u\}$. The degree of a vertex $u \in V(G)$, denoted by $d_G(u)$ and is defined as $|N_G(u)|$. Let $S_G(u)=\sum\limits_{v\in N_G(u)} d_G(v)$ be the degree sum of neighbourhood vertices. Here we define *closed neighbourhood degree sum* of vertices denoted by $S_G[u]$ and defined as $S_G[u]=\sum\limits_{v\in N_G[u]} d_G(v)$. A graph G is said to be r-regular if the degree of each vertex in G is

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equal to $r(\in Z^+)$. For undefined graph theoretic terminologies and notations we refer [7, 14].

Topological indices are numerical values associated to the molecular graphs. These are graph invariants. In mathematical chemistry, these are known as molecular descriptors. Topological indices play a vital role in mathematical chemistry specially, in chemical documentation, isomer discrimination, quantitative structure-property relationships (QSPR) and quantitative structure-activity relationships (QSAR) analysis. *Wiener index* is the first topological index used by H. Wiener [24] in the year 1947, to calculate the boiling point of paraffins. Then after Gutman and Trinajstić defined *Zagreb indices* in 1972, which now are most popular and have many applications in chemistry [13]. The first and second Zagreb indices of a graph G are defined as follows [12]:

$$M_1(G) = \sum_{v \in V(G)} d_G(v)^2 \quad and \quad M_2(G) = \sum_{uv \in E(G)} d_G(u) \cdot d_G(v),$$

respectively. The first Zagreb index [20] can also be expressed as

$$M_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)].$$

Later, it was Randić who gave most chemically efficient topological index called *Randić index* [23], which is defined as

$$R(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) d_G(v)}}.$$

In [8], Graovac et al. defined *fifth M -Zagreb indices* as

$$M_1G_5(G) = \sum_{uv \in E(G)} (S_G(u) + S_G(v)) \text{ and } M_2G_5(G) = \sum_{uv \in E(G)} (S_G(u) \cdot S_G(v)).$$

A new version of third Zagreb index called fifth M_3-Zagreb index is defined by Kulli in [17] as

$$M_3G_5(G) = \sum_{uv \in E(G)} (|S_G(u) - S_G(v)|).$$

In literature, mathematical chemists used combinations of vertex degree, edge degree, open neighbourhood degree sum, edge neighbourhood degree sum to define several topological indices and to study their applications in chemistry. Recently, Basavanagoud et al. defined new topological indices based on neighbourhood degree sum of a vertex called the *first neighbourhood Zagreb index* [5] as

$$NM_1(G) = \sum_{v \in V(G)} S_G(v)^2$$

and topological indices based on edge neighbourhood degree sum called Kulli-Basava indices [6] defined as

$$\text{first Kulli-Basava index:} \qquad KB_1(G) = \sum_{uv \in E(G)} \big(S_e(u) + S_e(v)\big),$$

$$\text{modified first Kulli-Basava index:} \qquad KB_1^*(G) = \sum_{v \in V(G)} S_e(v)^2,$$

$$\text{second Kulli-Basava index:} \qquad KB_2(G) = \sum_{uv \in E(G)} S_e(u) \cdot S_e(v),$$

$$\text{third Kulli-Basava index:} \qquad KB_3(G) = \sum_{uv \in E(G)} \big(|S_e(u) - S_e(v)|\big).$$

The authors found chemical applications of these new indices. Now, to study the effect of neighbourhood vertices along with the vertex itself, we proceed to introduce new topological indices based on closed neighbourhood degree sum of a vertex and would name them as first, second and third *Chinmayi index* which are defined respectively as

$$\text{first Chinmayi index:} \qquad C_1(G) = \sum_{uv \in E(G)} \big(S_G[u] + S_G[v]\big), \tag{1.1}$$

$$\text{second Chinmayi index:} \qquad C_2(G) = \sum_{uv \in E(G)} S_G[u] \cdot S_G[v], \tag{1.2}$$

$$\text{third Chinmayi index:} \qquad C_3(G) = \sum_{uv \in E(G)} \big(|S_G[u] - S_G[v]|\big). \tag{1.3}$$

In this paper, we investigate mathematical properties of *Chinmayi indices* with some standard class of graphs and first *Chinmayi index* some graph operations. The rest of the paper is organized as follows. In section 2, we obtain explicit formulae for Chinamayi indices of some standard class of graphs. In section 3, we derive first *Chinmayi index* of some graph operation and we end with the conclusion, followed by references.

2. Chinmayi indices of some standard class of graphs

In this section, we first obtain relation between the new *Chinmayi indices* and existing topological indices. Next, we obtain explicit formulae of *Chinmayi indices* for various standard classes of graphs. We use familiar notations like P_n, C_n, K_n, $K_{r,s}$ and $K_{1,s}$ to denote a path, a cycle, a complete graph, a complete bipartite graph and a star graph respectively.

In the following proposition, *Chinmayi indices* are expressed in terms of other topological indices which are already in the literature.

Proposition 2.1. *If G is a graph with n vertices and m edges, then*

$$\begin{aligned} C_1(G) &= M_1G_5(G) + M_1(G), C_2(G) = \\ M_2G_5(G) + M_2(G) + NM_1(G), \\ C_3(G) &= M_3G_5(G) + M_3(G). \end{aligned}$$

By definition of first Chinmayi index, we have

$$\begin{aligned} C_1(G) &= \sum_{uv \in E(G)} (S_G[u] + S_G[v]) \\ &= \sum_{uv \in E(G)} (S_G(u) + d_G(u) + S_G(v) + d_G(v)) \\ &= \sum_{uv \in E(G)} (S_G(u) + S_G(v)) + \sum_{uv \in E(G)} (d_G(u) + d_G(v)) \\ &= M_1G_5(G) + M_1(G). \end{aligned}$$

By definition of second Chinmayi index, we have

$$\begin{aligned}
C_2(G) = \sum_{uv\in E(G)} (S_G[u]\cdot S_G[v]) &= \sum_{uv\in E(G)} ((S_G(u)+d_G(u))\cdot(S_G(v)+d_G(v))) \\
&= \sum_{uv\in E(G)} S_G(u)\cdot S_G(v) + \sum_{uv\in E(G)} d_G(u)\cdot d_G(v) \\
&\quad + \sum_{uv\in E(G)} (S_G(u)d_G(v)+d_G(u)S_G(v)) \\
&= M_2G_5(G) + M_2(G) + NM_1(G).
\end{aligned}$$

By definition of third Chinmayi index, we have

$$\begin{aligned}
C_3(G) = \sum_{uv\in E(G)} (|S_G[u]-S_G[v]|) &= \sum_{uv\in E(G)} (|S_G(u)+d_G(u)-(S_G(v)+d_G(v))|) \\
&= \sum_{uv\in E(G)} (|S_G(u)-S_G(v)|) + \sum_{uv\in E(G)} (|d_G(u)-d_G(v)|) \\
&= M_3G_5(G) + M_3(G).
\end{aligned}$$

Proposition 2.2. *If P_n is a path with n vertices, then*

$$C_1(P_n) = \begin{cases} 4 & \text{if } n=2, \\ 12n-22 & \text{otherwise.} \end{cases}$$

$$C_2(P_n) = \begin{cases} 4 & \text{if } n=2, \\ 24 & \text{if } n=3, \\ 55 & \text{if } n=4, \\ 18(2n-5) & \text{otherwise.} \end{cases}$$

$$C_3(P_n) = \begin{cases} 0 & \text{if } n=2, \\ 2 & \text{if } n=3, \\ 4 & \text{if } n=4, \\ 6 & \text{otherwise.} \end{cases}$$

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If $n = 2$, then P_2 is an edge xy. Thus, $S_{P_2}[x] = 2$ and $S_{P_2}[y] = 2$. Therefore, by Equation (1.1) we have, $C_1(P_2) = 4$.
For any path P_n with $n \geq 3$,
$S_G[v] = 3$, if v is a pendant vertex.
$S_G[v] = 5$, if v is adjacent to pendant vertex.
$S_G[v] = 6$, otherwise.
Therefore, by substituting these values in Equations. (1.1), (1.2) and (1.3) we get the desired results.

Proposition 2.3. *If G be a r-regular graph with n vertices and m edges then,*

$$\begin{aligned} C_1(G) &= nr^2(r+1), \\ C_2(G) &= \frac{nr^3(r+1)^2}{2}, \\ C_3(G) &= 0. \end{aligned}$$

Let G be a r-regular graph with n vertices and m edges. Every vertex of G is adjacent to a vertex of degree r and $S_G[v] = r(r+1)$, for $v \in G$ and $m = \frac{nr}{2}$. Therefore,

$$\begin{aligned} C_1(G) &= \sum_{uv \in E(G)} \big(S_G[u] + S_G[v]\big) \\ &= \sum_{uv \in E(G)} \big(2r(r+1)\big) \\ &= 2mr(r+1) \\ &= nr^2(r+1). \end{aligned}$$

$$\begin{aligned}
C_2(G) &= \sum_{uv\in E(G)} \big(S_G[u]\cdot S_G[v]\big)\\
&= \sum_{uv\in E(G)} \big(r(r+1)^2\big)\\
&= mr(r+1)^2\\
&= \frac{nr^3(r+1)^2}{2}.
\end{aligned}$$

$$\begin{aligned}
C_3(G) &= \sum_{uv\in E(G)} \big(|S_G[u]-S_G[v]|\big)\\
&= \sum_{uv\in E(G)} \big(r(r+1)-r(r+1)\big)\\
&= 0.
\end{aligned}$$

Corollary 2.1. *If C_n is a cycle with n vertices, then*

$$\begin{aligned}
C_1(C_n) &= 12n,\\
C_2(C_n) &= 36n,\\
C_3(C_n) &= 0.
\end{aligned}$$

Corollary 2.2. *If K_n is a complete graph with n vertices, then*

$$\begin{aligned}
C_1(K_n) &= (n(n-1))^2,\\
C_2(K_n) &= \frac{(n(n-1))^3}{2},\\
C_3(K_n) &= 0.
\end{aligned}$$

Proposition 2.4. *If $G = K_{r,s}$ be a complete bipartite graph with $r+s$ vertices and rs edges for $1 \leq r \leq s$ and $s \geq 2$ then,*

$$\begin{aligned} C_1(K_{r,s}) &= rs(2rs+s+r), \\ C_2(K_{r,s}) &= r^2s^2(r+1)(s+1), \\ C_3(K_{r,s}) &= |rs(s-r)|. \end{aligned}$$

Let $G = K_{r,s}$ be a complete bipartite graph with $r+s$ vertices and rs edges. Let V_1 and V_2 be the partition of vertex set, $V(G)$ with $|V_1| = r$ and $|V_2| = s$. Then, $S_G[v] = rs+s$, for $v \in V_1$ and $S_G[v] = rs+r$, for $v \in V_2$. Therefore,

$$\begin{aligned} C_1(G) &= \sum_{uv \in E(G)} \big(S_G[u] + S_G[v]\big) \\ &= \sum_{uv \in E(G)} \big(rs+s+rs+r\big) \\ &= \sum_{uv \in E(G)} \big(2rs+s+r\big) \\ &= rs(2rs+s+r). \\ C_2(G) &= \sum_{uv \in E(G)} \big(S_G[u] \cdot S_G[v]\big). \\ &= \sum_{uv \in E(G)} \big((rs+s)(rs+r)\big) \\ &= r^2s^2(r+1)(s+1). \\ C_3(G) &= \sum_{uv \in E(G)} \big(|S_G[u] - S_G[v]|\big). \\ &= \sum_{uv \in E(G)} \big(|rs+s-rs-r|)\big) \\ &= |rs(s-r)|. \end{aligned}$$

Corollary 2.3. *Let $K_{1,s}$ be a star graph with $s \geq 2$. Then,*

$$\begin{aligned} C_1(K_{1,s}) &= s(3s+1), \\ C_2(K_{1,s}) &= 2s^2(s+1), \\ C_3(K_{r,r}) &= s(s-1). \end{aligned}$$

Corollary 2.4. *Let $K_{r,r}$ be a complete bipartite graph. Then,*

$$\begin{aligned} C_1(K_{r,r}) &= 2r^3(r+1), \\ C_2(K_{1,s}) &= r^4(r+1)^2, \\ C_3(K_{r,r}) &= s(s-1). \end{aligned}$$

3. First Chinmayi index of some graph operations

Graph operations play an important role as some important graphs can be obtained by simple graph operations. For example, C_4-nanotorus, C_4-nanotube, planar grids, n-prism and Rook's graph are obtained by Cartesian product of $C_m \times C_n$, $P_m \times C_n$, $P_n \times P_m$, $K_2 \times C_m$ and $K_n \times K_m$ respectively. Fence graph, closed fence graph and Catlin graph are obtained by composition of graphs. Definitions of all these graphs can be found in [11]. For more on product graph operations we refer the book by Imrich and Klavažar [18].

There are several papers devoted to topological index of graph operations. To mention few, Khalifeh et al., obtained first and second Zagreb index of graph operations in [16], Akhter al., obtained F-index in [1], Basavanagoud et al., obtained hyper-Zagreb index in [3, 4], N. De et al., obtained F-coindex in [9], Fath-Tabar derived GA_2 index in [10], Nadjafi-Arani et al., obtained degree distance based topological indices of tensor product of graphs in [19], Paulraj et al., computed degree distance of product graphs in [22], Yarahmadi et al., computed Szeged, vertex PI, first and second Zagreb indices of corona product of graphs in [25]. For some other topological indices of graph operations on can refer [2, 15, 21, 25, 26] and references cited there in.

At this stage, we evaluate first *Chinmayi index* for some product operations such as Cartesian product, composition, tensor product and corona product of graphs.

We now, proceed with the following lemma which is directly follows from the definition of the closed neighbourhood degree sum.

Lemma 3.1. *For a graph G, we have*
(i) $\sum_{v\in V(G)} S_G[v] = 2m + M_1(G)$,
(ii) $\sum_{v\in V(G)} d_G(v)S_G[v] = M_1(G) + 2M_2(G)$.

Definition 3.1. The *Cartesian product* [14] $G \times H$ of graphs G and H has the vertex set $V(G \times H) = V(G) \times V(H)$ and $(a, x)(b, y)$ is an edge of $G \times H$ if and only if $[a = b$ and $xy \in E(H)]$ or $[x = y$ and $ab \in E(G)]$.

The following lemma is direct consequence of above definition.

Lemma 3.2. *For graphs G_1 and G_2 of order n_1 and n_2 and size m_1 and m_2 respectively, we have,*
(i) $S_{G_1\times G_2}[(u, v)] = S_{G_1}[u] + S_{G_2}[v] + 2d_{G_1}(u)d_{G_2}(v)$,
(ii) $|E(G_1 \times G_2)| = n_2m_1 + n_1m_2$.

Theorem 3.1. *The first Chinmayi index of $G_1 \times G_2$ is given by*

$$C_1(G_1 \times G_2) = n_2C_1(G_1) + n_1C_1(G_2) + 6m_2M_1(G_1) + 6m_1M_1(G_2) + 8m_1m_2.$$

By definition, $C_1(G_1 \times G_2) = \sum\limits_{(a,x)(b,y) \in E(G_1 \times G_2)} \big(S_{G_1 \times G_2}[(a,x)] + S_{G_1 \times G_2}[(b,y)]\big).$

Using Lemma 3.2 we get,

$$= \sum_{a \in V(G_1)} \sum_{xy \in E(G_2)} \big(S_{G_1}[a] + S_{G_2}[x] + 2d_{G_1}(a)d_{G_2}(x) + S_{G_1}[a] + S_{G_2}[y] + 2d_{G_1}(a)d_{G_2}(y)\big)$$

$$+ \sum_{x \in V(G_2)} \sum_{ab \in E(G_1)} \big(S_{G_2}[x] + S_{G_1}[a] + 2d_{G_2}(x)d_{G_1}(a) + S_{G_2}[x] + S_{G_1}[b] + 2d_{G_2}(x)d_{G_1}(b)\big)$$

$$= \sum_{a \in V(G_1)} \sum_{xy \in E(G_2)} \big(2S_{G_1}[a] + (S_{G_2}[x] + S_{G_2}[y]) + 2d_{G_1}(a)(d_{G_2}(x) + d_{G_2}(y))\big)$$

$$+ \sum_{x \in V(G_2)} \sum_{ab \in E(G_1)} \big(2S_{G_2}[x] + (S_{G_1}[a] + S_{G_1}[b]) + 2d_{G_2}(x)(d_{G_1}(a) + d_{G_1}(b)\big)$$

$$= 2m_2 \sum_{a \in V(G_1)} S_{G_1}[a] + n_1 \sum_{xy \in E(G_2)} (S_{G_2}[x] + S_{G_2}[y]) + 4m_1 M_1(G_2)$$

$$+2m_1 \sum_{a \in V(G_2)} S_{G_2}[x] + n_2 \sum_{ab \in E(G_1)} (S_{G_1}[a] + S_{G_1}[b]) + 4m_2 M_1(G_1).$$

Using Lemma 3.1 and by definition of first Chinmayi index we get the desired result.

As an application of Theorem 3.1, we list explicit formulae for *Chinmayi index* of ladder graph L_n, C_4-nanotorus, C_4-nanotube, planar grids, n-prism and Rook's graph. The formulae follow from Theorem 3.1 and using the following expressions:
$M_1(P_n) = 4n - 6, n \geq 1$; $M_1(C_n) = 4n, n \geq 3$; $M_1(K_n) = n(n-1)^2$.
$C_1(P_n) = 12n - 22$; $C_1(C_n) = 12n$; and $C_1(K_n) = (n(n-1))^2$.

Corollary 3.1. *The ladder graph L_n is defined as the Cartesian product of P_2 and P_n. From Theorem 3.1, we derive the following result.*

$$C_1(L_n) = 64n - 28.$$

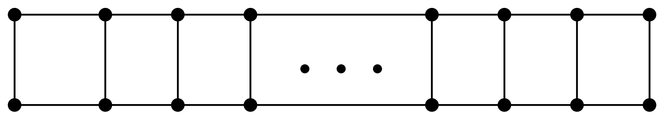


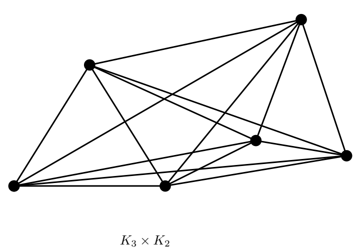
FIGURE 1. The ladder graph L_n.

Corollary 3.2. *For a C_4-nanotorus, $TC_4(m,n) = C_m \times C_n$, the Chinmayi index is given by*

$$C_1(TC_4(m,n)) = 80mn.$$

Corollary 3.3. *The Cartesian product of P_n and C_m yields a C_4-nanotube, $TUC_4(m,n) = P_n \times C_m$. Its Chinmayi index is given by*

$$C_1(TUC_4(m,n)) = 80mn - 82m.$$



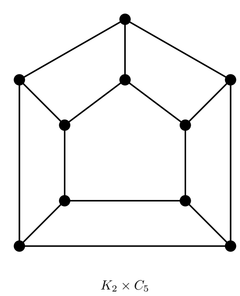


FIGURE 2. The example of Rook's graph ($K_3 \times K_2$) and n-Prism graph ($n = 5$)

Corollary 3.4. *The Chinmayi index of planar grids $P_n \times P_m$ is given by*

$$C_1(P_n \times P_m) = 80(mn+1) - 90(m+n).$$

Corollary 3.5. *For a n-prism, $K_2 \times C_n$, the Chinmayi index is given by*

$$C_1(K_2 \times C_n) = 72n.$$

Corollary 3.6. *The Cartesian product of K_n and K_m yields the Rook's graph. Its Chinmayi index is given by*

$$C_1(K_n \times K_m) = mn\{n(n-1)^2 + m(m-1)^2 + (m-1)(n-1)\{3(n+m) - 4\}\}.$$

Definition 3.2. The *composition* [14] $G[H]$ of graphs G and H with disjoint vertex sets $V(G)$ and $V(H)$ and edge sets $E(G)$ and $E(H)$ is the graph with vertex set $V(G[H]) = V(G) \times V(H)$ and $(a,x)(b,y)$ is an edge of $G[H]$ if and only if [a is adjacent to b in G] or [$a = b$ and x is adjacent to y in H].

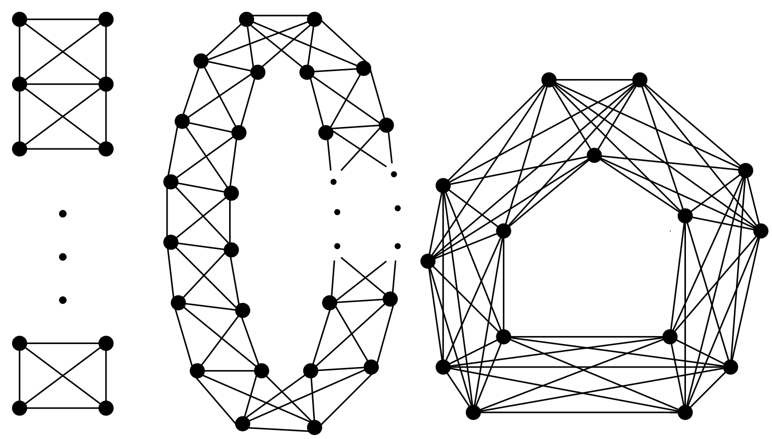


FIGURE 3. Fence graph ($P_n[P_2]$), closed Fence graph ($C_n[P_2]$) and Catlin graph ($C_n[P_2]$)

Lemma 3.3. *For graphs G_1 and G_2 of order n_1 and n_2 and size m_1 and m_2 respectively, we have,*

$S_{G_1[G_2]}[(u,v)] = n_2 S_{G_1}[u] + S_{G_2}[v] + n_2(n_2-1)S_{G_1}(u) + 2m_2 d_{G_1}(u) + n_2 d_{G_1}(u) d_{G_2}(v).$

Theorem 3.2. *The first Chinmayi index of $G_1[G_2]$ is given by*

$$\begin{aligned} C_1(G_1[G_2]) &= n_2^3 C_1(G_1) + n_1 C_1(G_2) + 4m_1 n_2 M_1(G_2) + n_2^3(n_2-1) M_1 G_5(G_1) + 8m_1 m_2 n_2 \\ &\quad + \Big(4m_2 n_2^2 + 2n_2 m_2 + 2n_2 m_2(n_2-1)\Big) M_1(G_1) + 8m_1 m_2^2. \end{aligned}$$

By definition, $C_1(G_1[G_2]) = \sum\limits_{(a,x)(b,y)\in E(G_1[G_2])} \big(S_{G_1[G_2]}[(a,x)] + S_{G_1[G_2]}[(b,y)]\big)$
Using Lemma 3.3 we get,

$$\begin{aligned}
C_1(G_1[G_2]) &= \sum_{x\in V(G_2)}\sum_{y\in V(G_2)}\sum_{ab\in E(G_1)} \big(n_2S_{G_1}[a] + S_{G_2}[x] + n_2(n_2-1)S_{G_1}(a) + 2m_2d_{G_1}(a) \\
&\quad + n_2d_{G_1}(a)d_{G_2}(x) + n_2S_{G_1}[b] + S_{G_2}[y] + n_2(n_2-1)S_{G_1}(b) + 2m_2d_{G_1}(b) \\
&\quad + n_2d_{G_1}(b)d_{G_2}(y)\big) \\
&\quad + \sum_{a\in V(G_2)}\sum_{xy\in E(G_2)} \big(n_2S_{G_1}[a] + S_{G_2}[x] + n_2(n_2-1)S_{G_1}(a) + 2m_2d_{G_1}(a) \\
&\quad + n_2d_{G_1}(a)d_{G_2}(x) + n_2S_{G_1}[a] + S_{G_2}[y] + n_2(n_2-1)S_{G_1}(a) + 2m_2d_{G_1}(a) \\
&\quad + n_2d_{G_1}(a)d_{G_2}(y)\big) \\
&= \sum_{x\in V(G_2)}\sum_{y\in V(G_2)}\sum_{ab\in E(G_1)} \big(n_2S_{G_1}[a] + n_2S_{G_1}[b] + S_{G_2}[x] + S_{G_2}[y] \\
&\quad + n_2(n_2-1)(S_{G_1}(a) + S_{G_1}(b)) + 2m_2(d_{G_1}(a) + d_{G_1}(b)) \\
&\quad + n_2d_{G_1}(a)d_{G_2}(x) + n_2d_{G_1}(b)d_{G_2}(y)\big) \\
&\quad + \sum_{a\in V(G_2)}\sum_{xy\in E(G_2)} \big(2n_2S_{G_1}[a] + S_{G_2}[x] + S_{G_2}[y] + 2n_2(n_2-1)S_{G_1}(a) \\
&\quad + 4m_2d_{G_1}(a)).
\end{aligned}$$

Using Lemma 3.1 and by definition of first Chinmayi index we get the desired result.

As an application of Theorem 3.2, we derive explicit formulae for *Chinmayi index* of Fence graph [11], closed Fence graph [11] and Catlin graph [7].

Corollary 3.7. *The first Chinmayi index of Fence graph* $(P_n[P_2])$ *and closed Fence graph* $(C_n[P_2])$ *are given by*
(i) $C_1(P_n[P_2]) = 8n^2 + 28n - 426, n \geq 4,$
(ii) $C_1(C_n[P_2]) = 300n, n \geq 3.$

Corollary 3.8. *The first Chinmayi index of Catlin graph* $(C_5[K_3])$ *is given by*

$$C_1(C_5[K_3]) = 8640.$$

Definition 3.3. The *corona* [14] $G_1 \circ G_2$ of two graphs G_1 and G_2 of order n_1 and n_2 respectively, is defined as the graph obtained by taking one copy of G_1 and n_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

Lemma 3.4. *For graphs* G_1 *and* G_2 *of order* n_1 *and* n_2 *and size* m_1 *and* m_2 *respectively, we have,*
(i) $S_{G_1 \circ G_2}[v] = S_{G_1}[v] + n_2 d_{G_1}(v) + 2(n_2 + m_2)$ *for* $v \in V(G_1)$*,*
(ii) $S_{G_1 \circ G_2}[v] = S_{G_2}[v] + d_{G_2}(v) + d_{G_1}(v_i) + n_2 + 1$ *for* $v \in V(G_2)$ *and* $v_i \in V(G_1)$*.*

Theorem 3.3. *The first Chinmayi index of* $G_1 \circ G_2$ *is given by*

$$\begin{aligned} C_1(G_1 \circ G_2) &= C_1(G_1) + n_1 C_1(G_2) + 2m_2 M_1(G_1) + 2n_1 M_1(G_2) + 8n_2 m_1 + 8m_1 m_2 \\ &\quad + 4n_1 n_2 m_2 + 6n_1 m_2 + 2m_1 n_2^2 + 3n_1 n_2^2 + n_1 n_2. \end{aligned}$$

By definition, $C_1(G_1 \circ G_2) = \sum_{(u,v) \in E(G_1 \circ G_2)} \big(S_{G_1 \circ G_2}[u] + S_{G_1 \circ G_2}[v]\big)$
Using Lemma 3.4 we get,

$$\begin{aligned} C_1(G_1 \circ G_2) &= \sum_{uv \in E(G_1)} \big(S_{G_1}[u] + n_2 d_{G_1}(u) + 2(n_2 + m_2) + S_{G_1}[v] + n_2 d_{G_1}(v) + 2(n_2 + m_2)\big) \\ &\quad + \sum_{uv \in E(G_2)} \sum_{v_i \in V(G_1)} \big(S_{G_2}[u] + d_{G_2}(u) + S_{G_2}[v] + d_{G_2}(v) + 2d_{G_1}(v_i) + 2n_2 + 2\big) \\ &\quad + \sum_{u \in V(G_1)} \sum_{v \in V(G_2)} \Bigg(S_{G_1}[v] + n_2 d_{G_1}(v) + 2(n_2 + m_2) \\ &\quad + S_{G_2}[v] + d_{G_2}(v) + d_{G_1}(v_i) + n_2 + 1\Bigg). \end{aligned}$$

Using Lemma 3.1 and by definition of first Chinmayi index we get the desired result.

As an application of Theorem 3.3, we obtain explicit formulae for first *Chinmayi index* of k-thorny cycle $C_n \circ \overline{K_k}$.

Corollary 3.9. $C_1(C_n \circ \overline{K_k}) = 5nk^2 + 8nk + nk + 12n.$

Definition 3.4. The *tensor product* [18] $G_1 \circledast G_2$ of two graphs G_1 and G_2 of order n_1 and n_2 respectively, is defined as the graph with vertex set $V(G_1) \times V(G_2)$ and (u_1, v_1) is adjacent with (u_2, v_2) if and only if $u_1u_2 \in E(G_1)$ and $v_1v_2 \in E(G_2)$.

Lemma 3.5. *For graphs G_1 and G_2 of order n_1 and n_2 and size m_1 and m_2 respectively, we have,*

$$S_{G_1 \circledast G_2}[(u,v)] = S_{G_1}(u)S_{G_2}(v) + d_{G_1}(u)d_{G_2}(v).$$

Theorem 3.4. *The first Chinmayi index of $G_1 \circledast G_2$ is given by*

$$C_1(G_1 \circledast G_2) \quad = \quad M_1G_5(G_1)M_1G_5(G_2) + M_1(G_1)M_1(G_2).$$

By definition, $C_1(G_1 \circledast G_2) = \sum\limits_{(a,x)(b,y)\in E(G_1 \circledast G_2)} \big(S_{G_1 \circledast G_2}[(a,x)] + S_{G_1 \circledast G_2}[(b,y)]\big).$
Using Lemma 3.5 we get,

$$\begin{aligned}
C_1(G_1 \circledast G_2) &= \sum_{ab\in E(G_1)} \sum_{xy\in E(G_2)} \big(S_{G_1}(a)S_{G_2}(x) + d_{G_1}(a)d_{G_2}(x) + S_{G_1}(b)S_{G_2}(y) + d_{G_1}(b)d_{G_2}(y)\big) \\
&\quad + \sum_{ba\in E(G_1)} \sum_{yx\in E(G_2)} \big(S_{G_1}(b)S_{G_2}(x) + d_{G_1}(b)d_{G_2}(x) + S_{G_1}(a)S_{G_2}(y) + d_{G_1}(a)d_{G_2}(y)\big) \\
&= \sum_{ab\in E(G_1)} \sum_{xy\in E(G_2)} \Big[S_{G_1}(a)(S_{G_2}(x) + S_{G_2}(y)) + S_{G_1}(b)(S_{G_2}(x) + S_{G_2}(y)) \\
&\quad + d_{G_1}(a)(d_{G_2}(x) + d_{G_2}(y)) + d_{G_1}(b)(d_{G_2}(x) + d_{G_2}(y))\Big] \\
&= M_1G_5(G_1)M_1G_5(G_2) + M_1(G_1)M_1(G_2).
\end{aligned}$$

As an application of Theorem 3.4, we obtain following computations.

Corollary 3.10. *Let P_n, C_n, K_n be a path, a cycle and a complete graph of order n respectively, then*

(i) $C_1(P_n \circledast P_m) = 64(n-2)(m-2) + 4(2n-3)(2m-3), m, n \geq 5$

(ii) $C_1(P_n \circledast C_m) = 64m(n-2) + 4m(4n-6), m, n \geq 5.$

(iii) $C_1(P_n \circledast K_m) = 8m(n-2)(m-1)^3 + m(4n-6)(m-1)^2, m, n \geq 5.$

(iv) $C_1(C_n \circledast C_m) = 80mn, m, n \geq 3.$

(v) $C_1(C_n \circledast K_m) = 4mn(m-1)^2(2m-1), m, n \geq 3.$

(vi) $C_1(K_n \circledast K_m) = mn(n-1)^2(m-1)^2((nm-n-m+2).$

Similarly, we can obtain second and third *Chinmayi index* of graph operations.

4. Conclusion

In this paper, we proposed a set of new topological indices called *Chinmayi indices*. We have explicitly studied first *Chinmayi index* of some standard class of graphs. Further, we have obtained *Chinmayi index* of some graph operations. In future, we derive the results for some other graph operations and create some linear models with other indices having good correlation with different physicochemical properties of molecules.

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