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# GENERALIZED COMPLEX SPACE FORMS

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ABSTRACT. In this paper, we find an eigen value of Ricci operator corresponding to scalar curvature r of a generalized complex space form and we give conditions for the existence of a generalized complex space form.

## 1. Introduction

A complex space form is an $n$-dimensional Kahler manifold of constant holomorphic sectional curvature $c$. In 1981, Tricerri and Vanhecke [17] introduced the notion of generalized complex space form and proved the condition for an almost Hermitian manifold to be a generalized complex space form. Further in ([10])the author proved that a $2n(n \geq 3)$ dimensional generalized complex space form is a real or a complex space form. Later Bagewadi and co-authors ([2], [11], [13]) made an extensive study of these space forms. The local symmetry is an useful notion in Riemannian geometry. This class of Riemannian symmetric manifolds is a very natural generalization of the class of manifolds of constant curvature. As a generalization of locally symmetric spaces, many geometers have considered semi-symmetric spaces and inturn their generalizations. We would like to have these notions, local symmetry, semi-symmetry in complex manifolds. In particular in generalized complex space forms.

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The W-curvature tensor introduced by Pokhriyal ([14]) has been studied by a number of authors in different contexts. In particular Ahsan et al. ([1]) have made a detailed study of this tensor on the spacetime of general relativity. In most of the cases, Schouten tensor is an Einstein tensor. Motivated by the all important roles of local symmetry, semi symmetry and W-curvature tensor in the study of different differential geometric structures, we have made a study of these notions in generalized complex space forms.

In this paper we study conformally flat, $W_2$-flat, locally-symmetric, generalized recurrent, semi-symmetric and pseudo-symmetric generalized complex space forms. The paper is organized as follows. In Section 2, we give definitions, notations of generalized complex space forms. Section 3 deals with conformally flat and $W_2$-flat generalized complex space forms. In Section 4, we deal with locally symmetric and generalized recurrent generalized complex space forms. Lastly, in Section 5, we study semi-symmetric and Ricci generalized pseudo symmetric generalized complex space forms.

## 2. Preliminaries

In this section we recall some general definitions and basic formulas which will be used later. Let $M$ be an $n$-dimensional Kahler manifold with the complex structure $J$ and the metric $g$ satisfying the following conditions:

(2.1)

$$J^2X = -X, \ \ g(JX, JY) = -g(X, Y), \ \ (\nabla_X J)(Y) = 0, \ \ g(JX, Y) = -g(X, JY),$$

for all $X$, $Y$, $Z \in TM$, where $TM$ denotes the Lie algebra of smooth vector fields on $M$. In a Kahler manifold the following relations hold [3]:

$$R(X, Y) = R(JX, JY) \tag{2.2}$$

$$S(X,Y) = S(JX,JY) \tag{2.3}$$

$$S(X,JY) + S(JX,Y) = 0 \tag{2.4}$$

$$(\nabla_X R)(Y,Z,U,V) = (\nabla_X R)(JY,JZ,U,V) \tag{2.5}$$

$$(\nabla_X R)(Y,Z,,U,V) = (\nabla_X R)(Y,Z,JU,JV), \tag{2.6}$$

where $R$ and $S$ are respectively Riemannian curvature tensor and Ricci tensor. The holomorphic sectional curvature of $M$ is given by $R(X,JY,X,JY) = R(JX,Y,JX,Y)$. An $n$-dimensional Kahler manifold of constant holomorphic sectional curvature $c$ is called a complex space form. The curvature tensor of a complex space form is given by

(2.7)

$$R(X,Y)Z = \frac{c}{4}\big[g(Y,Z)X - g(X,Z)Y + g(X,JZ)JY - g(Y,JZ)JX + 2g(X,JY)JZ\big].$$

The models of complex space form are complex Euclidean space $C^n$, complex projective space $CP^n$ and complex hyperbolic space $CH^n$, depending on $c = 0$, $c > 0$ or $c < 0$.

In the following we give generalization of the notion of complex space form.

**Definition 2.1.** An $2n(n \geq 2)$-dimensional almost Hermitian manifold $(M,J,g)$ is called a generalized complex space form $M(f_1,f_2)$ if its Riemannian curvature tensor $R$ takes the form

$$\begin{aligned} R(X,Y)Z =& f_1\big[g(Y,Z)X - g(X,Z)Y\big] \\ &+ f_2\big[g(X,JZ)JY - g(Y,JZ)JX + 2g(X,JY)JZ\big], \end{aligned} \tag{2.8}$$

for any $X$, $Y$, $Z \in TM$, where $f_1$ and $f_2$ are smooth functions on $M$.

In an $n$-dimensional Riemannian manifold $(M, g)(n \geq 3)$, the Schouten tensor $L$ is given by

$$LX = \frac{-1}{2n-1}QX + \frac{r}{4n(2n-1)}X. \tag{2.9}$$

In an almost Hermitian manifold $M$, the Weyl conformal curvature tensor is given by

$$W(X,Y)Z = R(X,Y)Z - \big[g(LX,Z)Y - g(Y,Z)LX - g(LY,Z)X + g(X,Z)LY\big], \tag{2.10}$$

for any $X$, $Y$, $Z \in TM$.

In 1970, Pokhariyal and Mishra [14] introduced the notion of $W_2$-curvature tensor field. It is defined by

$$W_2(X,Y)Z = R(X,Y)Z + \frac{1}{n-1}\big[g(X,Z)QY - g(Y,Z)QX\big]. \tag{2.11}$$

A generalized complex space form $M(f_1, f_2)$ is said to be locally symmetric if

$$(\nabla_W R)(X,Y)Z = 0, \quad X, Y, Z, W \in TM. \tag{2.12}$$

The notion of generalized recurrent manifolds was introduced and studied by De and Guha [7].

**Definition 2.2.** A generalized complex space form $M(f_1, f_2)$ is called generalized recurrent [8] if its curvature tensor $R$ satisfies

$$(\nabla_W R)(X,Y)Z = A(W)R(X,Y)Z + B(W)G(X,Y)Z, \tag{2.13}$$

for all $X$, $Y$, $Z$ , $W \in TM$, where $A$ and $B$ are two non-vanishing 1-forms such that $A(X) = g(X, \rho_1)$, $B(X) = g(X, \rho_2)$. Here $\rho_1$and $\rho_2$ are vector fields associated with 1-forms $A$ and $B$ respectively and the tensor $G$ is defined by

$$G(X,Y)Z = g(Y,Z)X - g(X,Z)Y, \tag{2.14}$$

for all $X$, $Y$, $Z \in TM$.

For a $(0,k)$-tensor $(k \geq 1)$ field $T$ on $(M,g)$, we define a $(0,k+2)$-tensor field $R\cdot T$ by the condition

$$(R\cdot T)(Y,Z,X_1,X_2.....X_k) = -\sum_{i=1}^{k} T(X_1,....,R(Y,Z)X_i,.....,X_k)$$

and a $(0,k+2)$-tensor field $(k \geq 1)$ $Q(g,T)$ is defined by

$$Q(g,T)(Y,Z,X_1,.....,X_k) = -\sum_{i=1}^{r} T(X_1,....,(Y\wedge_g Z)X_i,....,X_k).$$

This tensor gives a formulation of the notions of various pseudo-symmetry type curvature conditions.

A Riemannian manifold $M$ is said to be semi-symmetric if its curvature tensor $R$ satisfies $R(X,Y)\cdot R = 0$, where $R(X,Y)$ acts on $R$ as derivation ( [15], [16]).

A complete intrinsic classification of these was given by Szabo [15]. A Riemannian manifold is said to be pseudo symmetric ([6], [9]) if

$$R\cdot R = L_R Q(g,R),$$

holds on the set $U_R = \left\{x \in M/R - \frac{r}{n(n-1)}G \neq 0 \ \text{at} \ x\right\}$, where $G$ is the (0,4)-tensor defined by $G(X_1,X_2,X_3,X_4) = g((X_1\wedge X_2)X_3,X_4)$ and $L_R$ is some function on $U_R$, where $(X\wedge_g Y)$ is the endomorphism given by

$$(X\wedge_g Y)Z = g(Y,Z)X - g(X,Z)Y.$$

A Riemannian manifold is said to be Ricci generalized pseudo symmetric if

$$R\cdot S = L_S Q(g,S), \tag{2.15}$$

holds on the set $U_S = \{x \in M : Q(g,S) \neq 0 \ \text{at} \ x\}$ and $L_S$ is some function on $U_S$.

A symmetric tensor field $T$ of type (1,1) in $M(f_1,f_2)$ is said to be of Lie codazzi type if

$$(\nabla_X L)(Y) - (\nabla_Y L)(X) = 0. \tag{2.16}$$

In [4] S. Bochner defined the Bochner curvature tensor $B$ on a Kahler manifold $M$ by

(2.17)

$$\begin{aligned} B(X,Y,Z,U) =& R(X,Y,Z,U) - \frac{1}{2n+4}\Big[S(X,U)g(Y,Z) - S(X,Z)g(Y,U) \\ &+ S(JX,U)g(JY,Z) - S(JX,Z)g(JY,U) + S(Y,Z)g(X,U) \\ &- S(Y,U)g(X,Z) + S(JY,Z)g(JX,U) - S(JY,U)g(JX,Z) \\ &- 2S(JX,Y)g(JZ,U) - 2S(JZ,U)g(JX,Y)\Big] \\ &+ \frac{r}{(2n+2)(2n+4)}\Big[g(Y,Z)g(X,U) - g(X,Z)g(Y,U) \\ &+ g(JY,Z)g(JX,U) - g(JX,Z)g(JY,U) - 2g(JX,Y)g(JZ,U)\Big], \end{aligned}$$

for all $X, Y, Z, U \in TM$.

## 3. Conformally flat and $W_2$-flat Generalized Complex Space Forms

Let $M(f_1, f_2)$ be an $n$-dimensional generalized complex space form and let $\{e_i\}_{i=1}^n$ be an orthonormal basis of the tangent space at each point of the manifold. We derive from (2.8), the following:

$$S(Y,Z) = \big((n-1)f_1 + 3f_2\big)g(Y,Z) \tag{3.1}$$

$$QY = \big((n-1)f_1 + 3f_2\big)Y \tag{3.2}$$

$$r = n\big((n-1)f_1 + 3f_2\big), \tag{3.3}$$

where $Q$ is the Ricci operator and $r$ is scalar curvature of the space form $M(f_1, f_2)$. Using (3.2) and (3.3) in (2.9), we get

$$LX = \frac{-3}{4}\Big[\frac{(n-1)f_1 + 3f_2}{2n-1}\Big]X. \tag{3.4}$$

**Theorem 3.1.** *In an $n$-dimensional generalized complex space form $M(f_1, f_2)$ the following hold:*
*(i) the Schouten tensor $L$ is of Lie-Coddazi type if and only if $(n-1)f_1 + 3f_2$ is a constant*
*(ii) if $(n-1)f_1 + 3f_2$ is a constant then the Schouten tensor $L$ is Solenoidal if and only if $traceL$ is constant.*

*Proof.* Differentiating (3.4) covariantly with respect to $X$, we obtain

$$(3.5)\quad (\nabla_X L)Y - (\nabla_Y L)X = \frac{3}{4(2n-1)}\Big[Y\Big((n-1)f_1+3f_2\Big)X - X\Big((n-1)f_1+3f_2\Big)Y\Big].$$

If $(n-1)f_1 + 3f_2$ is a constant, then $L$ is of Lie-Codazzi type.
Conversely, suppose $L$ is of Lie-Codazzi type. Then from (3.5), we have

$$(3.6)\qquad Y\big((n-1)f_1 + 3f_2\big)X + X\big((n-1)f_1 + 3f_3\big)Y = 0.$$

Contracting the above equation with $W$, we obtain

$$(Y\alpha)g(X, W) + (X\alpha)g(Y, W) = 0,$$

where $\alpha = (n-1)f_1 + 3f_2$.
Taking $Y = W = e_i$ and summing over $i = 1, 2, 3......n$, we get

$$(3.7)\qquad Y\Big((n-1)f_1 + 3f_2\Big) = 0,$$

for all vector fields $Y \in TM$.
Therefore $(n-1)f_1 + 3f_2$ is a constant.
Further if $(n-1)f_1 + 3f_2$ is a constant then contracting (3.5) with $W$, setting $Y = W = e_i$ , summation over $i$, $1 \le i \le n$, gives

$$(3.8)\qquad \nabla_X g(Le_i, e_i) - (divL)(X) = \frac{-3(n-1)}{4(2n-1)} X\Big((n-1)f_1 + 3f_2\Big).$$

This leads to

$$(divL)(X) = X\Big((traceL) + \frac{3(n-1)}{4(2n-1)}\big((n-1)f_1 + 3f_2\big)\Big). \tag{3.9}$$

From (3.9), we see that $L$ is Solenoidal ( i.e., $divL = 0$ ) if and only if $traceL$ is a constant. This completes the proof. □

In the following we prove that conformality leads to vanishing of $(n-1)f_1 + 3f_2$.

**Theorem 3.2.** *In an $n$-dimensional Kahlerian manifold, vanishing of conformal or $W_2$-curvature tensor gives vanishing of curvature tensor $R$. In particular in a conformally flat or $W_2$-flat generalized complex space form we have $(n-1)f_1 + 3f_2 = 0$.*

*Proof.* If $n$-dimensional Kahlerian manifold $M$ is $W_2$-flat then from (2.11), we get

$$R(X,Y,Z,W) = \frac{1}{n-1}\big[S(X,W)g(Y,Z) - S(Y,W)g(X,Z)\big] \tag{3.10}$$

Letting $X = JX$ and $Y = JY$ and using (2.3), we get

(3.11)

$$g(Y,Z)S(X,W) - g(X,Z)S(Y,W) = g(JY,Z)S(JX,W) - g(JX,Z)S(JY,W).$$

Letting $Y = Z = e_i$ in (3.11) and using (2.3) and (2.4), we get

$$S(Y,Z) = 0$$

which when substituted in (3.10) gives

$$R(X,Y,Z,W) = 0.$$

Thus vanishing of $W_2$-tensor in a Kahlerian manifold leads to vanishing of curvature tensor $R$ and all other curvatures in Kahlerian manifold.

If $M$ is conformally flat then from (2.10), we have

(3.12)

$$R(X,Y,Z,W) = \frac{1}{2n-1}\big[S(Y,Z)g(X,W) - S(X,Z)g(Y,W) + S(X,W)g(Y,Z) - g(X,Z)S(Y,W)\big] + \frac{r}{2n(2n-1)}\big[g(X,Z)g(Y,W) - g(Y,Z)g(X,W)\big].$$

Changing $X$ to $JX$ and $Y$ to $JY$ and in view of (2.3), we have

(3.13)

$$\big[S(Y,Z)g(X,W) - S(X,Z)g(Y,W) + S(X,W)g(Y,Z) - S(Y,W)g(X,Z) - S(JY,Z)g(JX,W) + S(JX,Z)g(JY,W) - S(JX,W)g(JY,Z) + g(JX,Z)S(JY,W)\big] = \frac{r}{2n}\big[g(JX,Z)g(TY,W) - g(JY,Z)g(JX,W) - g(X,Z)g(Y,W) + g(Y,Z)g(X,W)\big].$$

Letting $Y = Z = e_i$ in (3.13) and summing over $i = 1, 2....n$, gives

$$S(X,W) = \frac{-(n+2)}{2n(n-4)} r g(X,W). \tag{3.14}$$

Setting $X = W = e_i$ in (3.14), gives

$$r = 0. \tag{3.15}$$

Using (3.15) in (3.14) and (3.12), we get

$$R(X,Y,Z,W) = 0. \tag{3.16}$$

Thus the vanishing of conformal curvature tensor in a Kahlerian manifold $M$ vanishes curvature tensor $R$ and all other curvature tensors in $M$.

In particular, in a generalized complex space form $M(f_1, f_2)$ if the conformal curvature tensor or $W_2$ tensor vanishes then from (3.3), we get

$$(n-1)f_1 + 3f_2 = 0. \tag{3.17}$$

This completes the proof.

□

Combining the above theorems 3.1 and 3.2, we write the following corollary.

**Corollary 3.1.** *In a $W_2$-flat or conformally flat generalized complex space form, the Schouten tensor $L$ is Solenoidal if and only if trace $L$ is a constant.*

## 4. Locally symmetric and generalized recurrent generalised Complex space forms

Let $M(f_1, f_2)$ be a generalized complex space form and let $\{e_i\}_{i=1}^n$ be an orthonormal basis of the tangent space at each point of $M$.

**Theorem 4.1.** *In a locally symmetric generalized complex space form, $(n-1)f_1+3f_2$ is a constant.*

*Proof.* Differentiating (2.8) covariantly with respect to $W$, we have

$$\begin{aligned}(\nabla_W R)(X,Y)Z =&(Wf_1)\big[g(Y,Z)X - g(X,Z)Y\big]\\ &+ (Wf_2)\big[g(X,JZ)JY - g(Y,JZ)JX + 2g(X,JY)JZ\big].\end{aligned} \tag{4.1}$$

If $M(f_1, f_2)$ is locally symmetric then from (4.1), we obtain

$$\begin{aligned}&(Wf_1)\big[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)\big]\\ &+ (Wf_2)\big[g(X,JZ)g(JY,U) - g(Y,JZ)g(JX,U) + 2g(X,JY)g(JZ,U)\big] = 0.\end{aligned} \tag{4.2}$$

Letting $Z = W = e_i$ in (4.2), we obtain

$$(Yf_1)\big[g(X,U) - (Xf_1)g(Y,U) - (JXf_2)g(JY,U) + (JYf_2)g(JX,U) - 2(JUf_2)g(X,JY)\big] = 0. \tag{4.3}$$

Setting $X = U = e_i$ in (4.3), we get

$Y\big((n-1)f_1 + 3f_2\big) = 0$ for all vector fields $Y$ on $M$ i.e., $(n-1)f_1 + 3f_2$ is a constant. □

**Theorem 4.2.** *A generalized recurrent Kahlerian manifold reduces to recurrent Kahlerian manifold. Further in a generalized complex space form recurrent vector $A$ is* *an eigen vector of Ricci operator $Q$ corresponding to eigen value $r$ if and only if $(n-1)f_1+3f_2$ is a constant.*

*Proof.* Suppose $M(f_1,f_2)$ is a generalized recurrent generalized complex space form. From (2.13) and (2.5), we have

(4.4)
$$B(W)\big[g(Y,Z)g(X,U)-g(X,Z)g(Y,U)+g(Y,Z)g(X,U)+g(X,JZ)g(JY,U)\big]=0.$$

Letting $Y=U=e_i$ in (4.4) gives

$$(n-2)g(X,Z)B(W)=0.$$

or

$$B(W)=0.$$

Equation(2.13) reduces to

$$(\nabla_X R)(Y,Z)W=A(X)R(Y,Z)W. \tag{4.5}$$

i.e., $M(f_1,f_2)$ reduces to a recurrent generalized complex space form.
From (4.5), we have

$$(\nabla_X S)(Y,Z)=A(X)S(Y,Z). \tag{4.6}$$

It is well known that

$$(divR)(X,Y)Z=(\nabla_X S)(Y,Z)-(\nabla_Y S)(X,Z). \tag{4.7}$$

Differentiating (2.8) covariantly with respect to $W$, we have

$$\begin{aligned}(\nabla_X R)(Y,Z)W&=(Xf_1)\big[g(Z,W)Y-g(Y,W)Z\big]\\&\quad+(Xf_2)\big[g(Y,JW)JZ-g(Z,JW)JY+2g(Y,JZ)JW\big].\end{aligned} \tag{4.8}$$

Using (4.6) and (4.7) in (4.8), we obtain

$$\begin{aligned} A(X)S(Y,Z) - A(Y)S(X,Z) &= (Xf_1)g(Y,Z) - (Yf_1)g(X,Z) \\ &+ (JYf_2)g(X,JZ) - (JXf_2)g(Y,JZ) + 2(JZf_2)g(X,JY). \end{aligned} \tag{4.9}$$

Letting $X = Z = e_i$ in (4.9), we get

$$S(Y,\rho) - rA(Y) = -Y\big((n-1)f_1 + 3f_2\big), \tag{4.10}$$

which implies $Q\rho = r\rho$ if and only if $(n-1)f_1 + 3f_2$ is a constant.
This shows that $\rho$ in an eigen vector of $Q$ corresponding to eigen value $r$ if and only if $(n-1)f_1 + 3f_2$ is a constant. This completes the proof. □

The following corollary is immediate from theorem (3.1) and theorem (4.2).

**Corollary 4.1.** *If in a generalized recurrent generalized complex space form with recurrence vector $\rho$, Schouten tensor $L$ is codazzi type then $\rho$ is an eigen vector of Ricci operator $Q$ corresponding to eigen value $r$.*

## 5. Semi-symmetric and Pseudo-symmetric generalized Complex space forms

Let $M(f_1, f_2)$ be a generalized complex space form and $\{e_i\}$ be an orthonormal basis of the tangent space at each point of $M$. A generalized complex space form $M(f_1, f_2)$ is $J$-semi-symmetric if

$$(R(X,Y)\cdot J)Z = R(X,Y)JZ - JR(X,Y)Z = 0, \tag{5.1}$$

for any vector fields $X, Y$ and $Z \in TM$.

**Theorem 5.1.** *In a $J$-semi symmetric generalized complex space form with $f_1$ and $f_2$ as associated functions, we have $f_1 = f_2$.*

*Proof.* Letting $Z = JZ$ and contracting with $W$ in (2.8), we get

(5.2)
$$\begin{aligned}R(X,Y,JZ,W) =& f_1\big[g(Y,JZ)g(X,W) - g(X,JZ)g(Y,W)\big] + f_2\big[g(X,Z)g(Y,JW)\\ &+ g(Y,Z)g(JX,W) - 2g(X,JY)g(Z,W)\big].\end{aligned}$$

Applying $J$ to $R(X,Y)Z$ and contracting with $W$ in (2.8), we get

(5.3)
$$\begin{aligned}g(JR(X,Y,Z,W) =& f_1\big[g(Y,Z)g(JX,W) - g(X,Z)g(JY,W)\big] + f_2\big[g(JX,Z)g(Y,W)\\ &+ g(Y,JZ)g(X,W) - 2g(X,JY)g(Z,W)\big].\end{aligned}$$

If $M(f_1,f_2)$ is $J$-semi symmetric then from (5.1), (5.2) and (5.3), we get

$$\begin{aligned}&f_1\big[g(Y,JZ)g(X,W) - g(X,JZ)g(Y,W)\big]\\ &+ f_2\big[g(X,Z)g(Y,JW) + g(X,Z)g(JY,W)\\ &- g(Y,Z)g(JX,W) + g(Y,Z)g(JX,W)\\ &- 2g(X,JY)g(Z,W) - g(JX,Z)g(Y,W)\\ &- g(X,W)g(Y,JZ) + 2g(X,JY)g(Z,W)\big] = 0.\end{aligned} \tag{5.4}$$

Letting $Y = Z = e_i$ in (5.4), in view of $g(e_i, Je_i) = S(e_i, Je_i) = 0$, we obtain

$$(n-2)(f_1 - f_2)g(X,JW) = 0, \tag{5.5}$$

which implies $f_1 = f_2$ for $n \geq 3$.

□

The following corollary is immediate from theorem (3.1) and theorem (5.1).

**Corollary 5.1.** *A $J$-semi symmetric generalized complex space form $M(f_1,f_2)$ reduces to a complex space form provided the Schouten tensor $L$ is of codazzi type or $M(f_1,f_2)$ is locally symmetric.*

**Theorem 5.2.** *A generalized complex space form is always Ricci semi-symmetric and if it is Ricci generalized pseudo symmetric then it is Einstein.*

*Proof.* If $M(f_1, f_2)$ is Ricci semi-symmetric then from (3.1), it follows that

$$\begin{aligned}(R(X,Y)\cdot S) &= S(R(X,Y)Z,W) + S(Z,R(X,Y)W)\\ &= \big((n-1)f_1 + 3f_2\big)(g(R(X,Y)Z,W) + g(R(X,Y)W,Z)) = 0,\end{aligned} \tag{5.6}$$

since $R(X,Y,Z,W) = -R(X,Y,W,Z)$. Suppose $M(f_1, f_2)$ is psudo generalized Ricci semi symmetric. Then from (5.6), we have

$$L_S\big(g(Y,U)S(X,V) - g(X,U)S(Y,V) + g(Y,V)S(U,X) - g(X,V)S(U,Y)\big) = 0. \tag{5.7}$$

Letting $Y = U = e_i$ in (5.7) gives

$$S(X,V) = rg(X,V), \tag{5.8}$$

since $L_S \neq 0$. □

**Theorem 5.3.** *An n-dimensional Bochner flat, Ricci generalized pseudo symmetric Kahlerian manifold is a generalized complex space form.*

*Proof.* Suppose Kahlerian manifold $M$ is Ricci generalized pseudo symmetric. Then from (2.15), we have

$$R(X,Y)\cdot S = L_S Q(g,S), \tag{5.9}$$

which is equivalent to

(5.10)

$$\begin{aligned}S(R(X,Y)Z,U) + S(Z,R(X,Y)U) &= -L_S\big(g(X,Z)S(Y,U) - g(Y,Z)S(X,U)\\ &+ g(Y,U)S(Z,X) - g(X,U)S(Z,Y)\big).\end{aligned}$$

Letting $Z = JZ$ and $U = JU$ in (5.10), we get

$$\begin{aligned}S(R(X,Y)JZ,JU)+S(JZ,R(X,Y)JU)&=-L_S\big[g(X,JZ)S(Y,JU)\\&-g(Y,JZ)S(X,JU)+g(Y,JU)S(Z,JX)-g(JX,U)S(JZ,Y)\big].\end{aligned}\tag{5.11}$$

If $M$ is $J$-semi symmetric then (5.11) gives

$$\begin{aligned}S(JR(X,Y)Z,JU)+S(JZ,JR(X,Y)U)&=-L_S\big[g(X,JZ)S(Y,JU)\\&-g(Y,JZ)S(X,JU)+g(Y,JU)S(Z,JX)-g(JX,U)S(JZ,Y)\big].\end{aligned}\tag{5.12}$$

From (2.3), (5.10) and (5.12), we have

$$\begin{aligned}&L_S\big[g(X,Z)S(Y,U)-g(Y,Z)S(X,U)+g(Y,U)S(X,Z)\\&-g(X,U)S(Y,Z)-g(X,JZ)S(Y,JU)+g(Y,JZ)S(X,JU)\\&-g(Y,JU)S(JZ,X)+g(X,JU)S(JZ,Y)\big]=0.\end{aligned}\tag{5.13}$$

Letting $X = Z = e_i$ in (5.13), we get

$$L_S\big[(n-4)S(Y,U)+rg(Y,U)\big]=0,\tag{5.14}$$

which implies

$$S(Y,U)=-\frac{r}{n-4}g(Y,U),\tag{5.15}$$

since $L_S \neq 0$. If $M$ is Bochner flat then from (5.15) and (2.17), we get

$$\begin{aligned}R(X,Y,Z,U)&=f_1\big[g(X,Z)g(Y,U)-g(Y,Z)g(X,U)\big]+\\&f_2\big[g(JX,Z)g(JY,U)-g(Y,Z)g(JX,U)+2g(Y,JX)g(JZ,U)\big],\end{aligned}\tag{5.16}$$

where $f_1 = f_2 = \dfrac{5nr}{(2n+4)(n-4)(2n+2)}$. □

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