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ACCEPTANCE SAMPLING PLANS BASED ON TRUNCATED LIFE TESTS FOR THE MARSHALL-OLKIN INVERSE GAMMA DISTRIBUTION

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Abstract. An acceptance sampling plans (AS) for a truncated life test is developed, when the lifetime follows the Marshall-Olkin Inverse Gamma distribution (MOIG). The minimum sample size necessary to ensure the specified mean life is obtained. Additionally, the operating characteristic function values of the proposed sampling plans and producer's risk are provided. Using the proposed model, some tables are given and the results are illustrated by numerical examples. Finally, Numerical examples for our proposed method are illustrated as well as a real life application is demonstrated.

1. Introduction

Single sampling plans for acceptance or rejection of a lot play a crucial role as an assay procedure in statistical quality control. Many researches considered Acceptance Sampling (AS) in different research fields such as agriculture, industry, ecology, management, quality maintenance... etc. In the single sampling plan, products or items have variations even though they are produced by the same producer, same machine and under the same manufacturing conditions. Simply it can be said that

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the procedure will accept or reject some lots even though they may have the same qualities. The producer and the consumer are subject to risks due to the decision on the acceptance or rejection of lot of products based on sample results.

Consumer's risk of accepting bad lots (type-II error, β) and Producer's risk of rejecting good lots (type-I error, α) may be minimized to a certain level by increasing the sample size. But this will increase the cost of inspection. Therefore, we put a random sample on test and accept the entire lot if no more than c (AS number) failures occur during the experiment time. The lot is accepted if the specified life can be stablished with preassigned probability (P^*) specified by the consumer (e.g. [1], [19]). So, that P^* can be considered as a minimum confidence level with which a lot of true average life below ξ_0 is rejected, by the sampling plan.

Many attempts have been made to study the single AS plans based on truncated life tests. Such an example was by [15] while [30] discussed the issue for exponential distribution, particularly. [18] and [20] considered AS plans for Weibull and gamma distributions, respectively. Moreover, [21] addressed it for a half logistic distribution. Recently, [22] defined it for log-logistic distribution whereas [31] defined it for the inverse Rayleigh model. [6] applied the two-point approach to the designing of the acceptance sampling plans based on a truncated life test for various life distributions and log-logistic distributions. [8] considered the Pareto model of the second kind. [7], considered the Birnbaum Saunders model and later on [11] applied the generalized Birnbaum–Saunders distribution. And more of different distributions were considered by [9], [3], [2] and [5].

[6] dealt with proposed variables sampling plan for life testing in a continuous process under Weibull distribution. [4] studied a two points acceptance sampling method were used to draw a decision on accepting or rejecting a tested product. It is also, assumed that the life time product following a new distribution that formulated

based on Weibull and Pareto life time distributions that it is known as new Weibull-Pareto (NWP) distribution. [26] and [27] suggested time truncated chain sampling plans for Marshall-Olkin extended exponential and generalized Rayleigh distribution. [16] investigate properties of a new parametric distribution generated by Marshall and Olkin extended family of distributions based on the Lomax model. Also they showed that the proposed distribution can be expressed as a compound distribution with mixing exponential model.

On the other hand, [23] derived a new method of including an extra positive shape parameter to a given baseline model thus extending a new distribution. The Marshall-Olkin transformation provides a wide range of behaviors with respect to the baseline distribution ([28]). Adding parameters to a well-established distribution is a time-honored devicefor obtaining more flexible new families of distributions ([14]). Several new models have been proposed that are some way related to the weibull distribution which is a very popular distribution formodelling data in reliability, engineering and biological studies. Extendedforms of the weibull distribution and applications in the literature such as [32], [12], [13] and [29].

This article aims to introduce an acceptance sampling plan (ASP) based on truncated life tests when the lifetime of a product follows the Marshall-Olkin inverse gamma distribution (MOIG) with a known shape parameter.

The article can be organized as follows; in section 2, some structural properties of the Inverse Gamma distribution are provided. In Section 3, the proposed sampling plans are established for the Marshall–Olkin Inverse gamma under a truncated life test, along with the operating characteristic (OC) and some relevant tables are given. Illustration of the tables and example for test plan is given in section 4, and then in Section 5 we apply our sampling plan to a real-life application. Finally, in section 6 we provide a conclusion remark.

2. Inverse Gamma Distribution

The Inverted Gamma (IG) distribution is a two-parameter continuous exponential family and has positive support and thus, if a variable has a gamma distribution then its reciprocal follow IG distribution. The Inverted Gamma distribution remains marginally studied and used in practice. Mainly it applied for in Bayesian statistics whereas it can also be useful in many problems of diffraction theory and corrosion of new machines. Also, gamma distribution can be considered for modeling positive data.

A significant difference between these two distributions, is that IG mode is always positive nevertheless for gamma distribution which can be zero. This property makes the IG very attractive to distinguish some kind of positive activation from stochastic noise which usually modeled using a Gaussian distribution ([10]). Also, it is most often used as a conjugate prior distribution in Bayesian statistics. Some other applications and motivations for this model can be found in [24] while [17] gave a number of properties that are useful when considering IG distribution as a lifetime of a product.

An IG random variable T can be derived by transforming a random variable $Y \sim \text{Gamma}(\eta,\ \xi)$ with the multiplicative inverse, i.e., $T = Y^{-1}$. Thus, as the *pdf* of gamma distribution is given by

$$\mathcal{G}\left(y|\eta,\, \xi\right) = \frac{\xi^{\eta}}{\Gamma(\eta)} y^{\eta-1} \exp\left(-\xi y\right)\ , \quad y,\ \eta,\ \xi \in \mathbb{R}^{+},$$

the *pdf* of the IG distribution(denoted by, $\mathcal{IG}(\eta,\, \xi)$), the *cdf* $(F_{\mathcal{IG}}\left(t|\eta, \xi\right))$, the survivor function $(S_{\mathcal{IG}}\left(t|\eta, \xi\right))$, and the hazard function $(h_{\mathcal{IG}}\left(t|\eta, \xi\right))$ are respectively as follows:

$$\mathcal{IG}(t|\eta,\, \xi) = \frac{\xi^{\eta}}{\Gamma(\eta)} t^{-\eta-1} \exp\left(-\xi t^{-1}\right)\ ,$$

$$F_{\mathcal{IG}}\left(t|\eta, \xi\right) = \frac{\Gamma\left(\eta, \xi t^{-1}\right)}{\Gamma\left(\eta\right)},$$

$$S_{\mathcal{IG}}\left(t|\eta,\xi\right)=1-\frac{\Gamma\left(\eta,\xi t^{-1}\right)}{\Gamma\left(\eta\right)},$$

$$h_{\mathcal{IG}}\left(t|\eta,\xi\right)=\frac{\xi^{\eta}}{\gamma\left(\eta,\xi t^{-1}\right)}t^{-\eta-1}\exp\left(-\xi t^{-1}\right),$$

such that $t\in\mathbb{R}^{+}$, with a shape parameterη, and a scale parameter ξ, where both parameters belongs to the positive reals. Recall that $\Gamma(.)$ is the Euler gamma function, $\Gamma(.,.)$ is the upper incomplete gamma function, and $\gamma(.,.)$is the lower incomplete gamma function.

The r^{th} moment about the origin is given as

$$\mathbb{E}_{\mathcal{IG}}\left(T^{r}\right)=\frac{\xi^{r}\Gamma\left(\eta-r\right)}{\Gamma\left(\eta\right)},\quad \eta>r.$$

Therefore, the mean and the variance are
$\mathbb{E}_{\mathcal{IG}}\left(T\right)=\frac{\xi}{\eta-1},\ \eta>1$ and $\mathbb{E}_{\mathcal{IG}}(T-\mathbb{E}_{\mathcal{IG}}\left(T\right))^{2}=\frac{\xi^{2}}{(\eta-1)(\eta-2)^{2}},\ \eta>2.$

3. Marshal-Olikin Inverse Gamma Distribution

If $\overline{\mathcal{F}}(x)$ and $f(x)$denotes the survival function and probability density function (*pdf*), respectively, of a parent distribution, then the survival function of the Marshall–Olkin (MO) family of distributions is defined by

$$\mathcal{L}\left(x;\vartheta\right)=\frac{\vartheta\overline{\mathcal{F}}(x)}{1-\overline{\vartheta}\overline{\mathcal{F}}(x)},\quad x\in\mathbb{R},\,\vartheta\in\mathbb{R}^{+},\overline{\vartheta}=1-\vartheta. \tag{3.1}$$

Clearly, $\mathcal{L}\left(x;1\right)=\overline{\mathcal{F}}(x)$.

The density function corresponding to (3.1) is given by

$$\ell\left(x;\vartheta\right)=\vartheta f\left(x\right)\left(1-\overline{\vartheta\mathcal{F}}\left(x\right)\right)^{-2}. \tag{3.2}$$

Assume that the lifetime of a product follows the Marshall–Olkin family of distributions, then, by using transformation (3.1), the probability density function and cumulative distribution function of Marshall–Olkin inverse gamma (MOIG) distribution, respectively, are;

(3.3)

$$g_{\mathcal{MOIG}}\left(t|\Theta\right) = \nabla\left(\Theta\right) t^{-\eta-1}\exp\left(-\xi t^{-1}\right)\,\left(\nabla\left(\Theta\right)\xi^{-\eta}\vartheta^{-1}-(1-\vartheta)\Lambda\left(\eta,\xi\right)\right)^{-2},\ t\in\mathbb{R},\ \vartheta\in\mathbb{R}^{+}$$

and

$$G_{\mathcal{MOIG}}\left(t|\Theta\right) = 1+\frac{\vartheta\Lambda\left(\eta,\xi\right)}{\nabla\left(\Theta\right)\xi^{-\eta}\vartheta^{-1}-(1-\vartheta)\Lambda\left(\eta,\xi\right)}, \tag{3.4}$$

where,ϑ is the shape parameter,$\Theta = (\eta,\xi,\vartheta)$, $\nabla\left(\Theta\right) = \vartheta\xi^{\eta}\Gamma\left(\eta\right)$ and $\Lambda\left(\eta,\xi\right) = \Gamma\left(\eta\right)-\Gamma\left(\eta,\xi t^{-1}\right)$.

In this article, we consider the lifetime of submitted products follows a MOIG distribution with scale parameter ξ as defined in (3.3) and (3.4). The life test terminates at a preassigned time t_0 and notes the number of failures during the time interval $[0,\ t]$. Thus, our desired goal is to establish a specified average life with a given probability of at least P^* (probability of rejecting a bad lot). Then, the decision is to accept the specified average life occurs if and only if the number of observed failures at the end of the fixed time t does not exceed a given number c (reference value) which is called the acceptance number (maximum number of allowable bad items to accept the lot). The acceptance sampling plan under a truncated life test is to set up the minimum sample size n for this given acceptance number c such that the consumer's risk, the probability of accepting a bad lot, does not exceed $1-P^*$. Therefore, for a given P^*, the proposed acceptance sampling plan can be characterized by the triplet $(n,\ c,\ t/\xi_0)$, such that n is the number of units on test, c is the acceptance number, t is the maximum test duration and t/ξ_0 is the ratio where ξ_0 is the specified average life.

The authors would stress out, that one of the objectives of these experiments is to set a lower confidence limit on the average life. It is then desired to establish a specified average life with a given probability of at least P^*. The consumer's risk, i.e., the probability of accepting a bad lot not to exceed $1-P^*$, so that P^* is a minimum confidence level with which a lot of true average life below ξ_0 is rejected, by the sampling plan.

The binomial probability distribution theory can be used in this analysis by assuming the lot size can be seen as infinite. The problem is, for given values of P^* with support (0, 1), ξ_0 and c the smallest positive integer n is to be determined such that

(3.5) $$\sum_{k=0}^{c} \binom{n}{k} p_0^k (1-p_0)^{n-k} \leq 1 - P^*,$$

where,$p_0 = G_{\mathcal{MOIG}}(t;\ \xi_0)$ is given by (3.4) and it is the probability of a failure observed during the time t, which depends only on the ratio t/ξ_0, it is sufficient to specify this ratio for designing the experiment.If the number of observed failures within the time $t$is at most c, then from (3.5) we can confirm with probability P^*that $G_{\mathcal{MOIG}}(t;\xi) \leq G_{\mathcal{MOIG}}(t;\xi_0)$, which implies $\xi_0 \leq \xi$, as it is a monotonically increasing function of t/ξ_0.

The values of $t/\xi_0$and P^*presented in this work are the same with the corresponding values of many authors ([7],[22] and [20]). For $t/\xi_0 = 0.942$, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712, with P^* =0.75, 0.9, 0.95, 0.99 and c =0, 1, 2,...,10. The minimum sample sizes that satisfy above inequality based on the suggested acceptance sampling plan and the values stated above are presented in Table 1 for $\eta = 1$ and $\vartheta = 2$.

Table 1. Minimum sample size n necessary to assert the average life to exceed a given value ξ_o with probability $P^*$and the corresponding acceptance number c *for* $\eta = 1$,$\vartheta = 2$

		t/ξ_o							
P^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.972	4.712
0.75	0	12	6	5	4	3	2	2	2
	1	23	12	9	7	5	4	4	3

	t/ξ_o							
	2	34	18	13	10	8	6	6	5
	3	45	24	17	14	10	8	7	7
	4	55	29	21	17	12	10	9	8
	5	65	35	25	20	14	12	11	10
	6	75	40	29	23	17	14	12	12
	7	85	45	32	26	19	16	14	13
	8	94	51	36	29	21	18	16	15
	9	104	56	40	32	23	20	18	16
	10	114	61	44	35	26	22	19	18
0.90	0	20	10	7	6	4	3	3	3
	1	33	18	12	10	7	6	5	5
	2	46	24	17	14	10	8	7	6
	3	58	31	21	17	12	10	9	8
	4	69	37	26	21	15	12	11	10
	5	80	43	30	24	17	14	13	12
	6	91	48	34	27	20	16	14	13
	7	102	54	38	31	22	18	16	15
	8	113	60	42	34	24	20	18	17
	9	123	66	46	37	27	22	20	18
	10	134	71	50	40	29	24	21	20
0.95	0	25	13	9	7	5	4	3	3
	1	40	21	15	12	8	7	6	5
	2	54	28	20	16	11	9	8	7
	3	67	35	25	19	14	11	10	9
	4	79	42	29	23	16	14	12	11

	t/ξ_o							
	5	91	48	34	27	19	16	14	13
	6	102	54	38	30	22	18	16	14
	7	113	60	42	34	24	20	18	16
	8	125	66	47	37	27	22	19	18
	9	136	72	51	41	29	24	21	20
	10	147	78	55	44	31	26	23	21
0.99	0	39	20	14	11	7	6	5	5
	1	56	29	20	16	11	9	8	7
	2	71	37	26	20	14	12	10	9
	3	86	45	31	25	17	14	12	11
	4	99	52	36	29	20	16	14	13
	5	112	59	41	33	23	19	16	15
	6	125	66	46	36	26	21	18	17
	7	137	72	51	40	28	23	20	19
	8	149	79	55	44	31	25	22	20
	9	161	85	60	47	34	28	24	22
	10	173	92	64	51	36	30	26	24

The operating characteristic function (OC) of the sampling plan $(n,\ c,\ t/\xi_0)$is the probability of accepting a lot is given by

(3.6) $$L\left(p\right)=\sum_{k=0}^{c}\binom{n}{k}p^k(1-p)^{n-k},$$

where,$p=G_{\mathcal{MOIG}}(t_0;\xi)$, for $\eta=1$, considered as a function of ξ (the lot quality parameter), and values of the $L(p)$ is a function of ξ/ξ_0 for some selected sampling plan are given in Table 2.

Table 2. Operating characteristic values for the sampling plan $(n,\ c,\ t/\xi_o)$ for a given $P^*$when c=2 *for* $\eta = 1,\vartheta = 2$

	t/ξ_o							
P^*	n	t/ξ_o	2	4	6	8	10	12
0.75	34	0.628	0.96530	1	1	1	1	1
	18	0.942	0.89715	0.99972	1	1	1	1
	13	1.257	0.82380	0.99768	0.99998	1	1	1
	10	1.571	0.78602	0.99344	0.99985	1	1	1
	8	2.356	0.62265	0.96114	0.99675	0.99974	0.99998	1
	6	3.141	0.62755	0.94148	0.99151	0.99877	0.99982	0.99997
	6	3.972	0.47600	0.87022	0.97143	0.99380	0.99865	0.99971
	5	4.712	0.52618	0.87173	0.96675	0.99125	0.99766	0.99936
0.90	46	0.628	0.92674	0.99999	1	1	1	1
	24	0.942	0.80610	0.99932	1	1	1	1
	17	1.257	0.69775	0.99482	0.99995	1	1	1
	14	1.571	0.59542	0.98238	0.99955	1	1	1
	10	2.356	0.46020	0.92848	0.99345	0.9995	0.99996	1
	8	3.141	0.40504	0.87251	0.97894	0.9968	0.99951	0.99993
	7	3.972	0.35014	0.80936	0.95456	0.9897	0.99771	0.99949
	6	4.712	0.37030	0.79400	0.94146	0.9838	0.99554	0.99877
0.95	54	0.628	0.89392	0.99998	1	1	1	1
	28	0.942	0.73756	0.99893	1	1	1	1
	20	1.257	0.60040	0.99172	0.99992	1	1	1
	16	1.571	0.50390	0.97449	0.99931	0.99998	1	1
	11	2.356	0.38876	0.90879	0.99126	0.99926	0.9999	1
	9	3.141	0.31523	0.83096	0.97026	0.99528	0.9993	0.9999

	t/ξ_o							
	8	3.972	0.25025	0.74319	0.93387	0.98443	0.9965	0.9992
	7	4.712	0.25012	0.70913	0.90970	0.97378	0.9926	0.9979
0.99	71	0.628	0.81011	0.99997	1	1	1	1
	37	0.942	0.57861	0.99758	1	1	1	1
	26	1.257	0.42179	0.98282	0.99982	1	1	1
	20	1.571	0.34562	0.95395	0.99865	0.99997	1	1
	14	2.356	0.22195	0.83900	0.98241	0.99843	0.99987	1
	12	3.141	0.13578	0.69151	0.93492	0.98871	0.99820	0.9997
	10	3.972	0.11970	0.60628	0.88250	0.97005	0.99287	0.9984
	9	4.712	0.10438	0.53873	0.83091	0.94599	0.98378	0.9953

If the producer's risk is given and a sampling plan $(n,\ c,\ t/\xi_0)$ is adopted, one interesting question is what value of $\xi/\xi_0(\geq 1)$ will insure the producer's risk to be at least 0.95,Then ξ/ξ_0 is the smallest positive number for which p satisfied the inequality

$$\sum_{k=0}^{c} \binom{n}{k} p^k (1-p)^{n-k} \geq 0.95, \qquad (3.7)$$

where,$p = G_{\mathcal{MOIG}}\left(\frac{t}{\xi_0}\frac{\xi_0}{\xi}\right)$. For the given acceptance sampling plan $(n,\ c,\ t/\xi_0)$, at a specified confidence level P^*, the minimum values of ξ/ξ_o satisfying (3.7) are presented in Table 3 for $\eta = 1$ and $\vartheta = 2$.

Table 3. Minimum ratio of true mean life to specified mean life for the acceptability of a lot with producer's risk of 0.05 *for* $\eta = 1$,$\vartheta = 2$.

	t/ξ_o								
P^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.972	4.712
0.75	0	3	3.85	4.91	5.79	8.02	9.45	11.95	14.18
	1	2.19	2.67	3.2	3.6	4.6	5.43	6.86	6.76
	2	1.91	2.28	2.63	2.88	3.8	4.17	5.27	5.4

	t/ξ_o								
	3	1.77	2.07	2.34	2.63	3.17	3.55	3.98	4.72
	4	1.67	1.91	2.16	2.38	2.79	3.18	3.63	3.78
	5	1.6	1.83	2.04	2.22	2.53	2.93	3.39	3.61
	6	1.55	1.75	1.95	2.09	2.48	2.75	2.93	3.47
	7	1.51	1.69	1.84	2	2.32	2.61	2.83	3.05
	8	1.47	1.65	1.79	1.93	2.2	2.51	2.76	3.01
	9	1.45	1.61	1.75	1.87	2.1	2.42	2.69	2.72
	10	1.42	1.57	1.72	1.82	2.1	2.34	2.47	2.71
0.90	0	3.32	4.33	5.33	6.42	8.68	10.69	13.51	16.03
	1	2.42	3.05	3.56	4.16	5.4	6.71	7.76	9.2
	2	2.1	2.54	2.96	3.4	4.31	5.06	5.87	6.25
	3	1.93	2.31	2.6	2.92	3.58	4.23	4.94	5.32
	4	1.81	2.14	2.42	2.7	3.29	3.72	4.38	4.77
	5	1.73	2.02	2.26	2.49	2.96	3.38	4	4.39
	6	1.67	1.91	2.14	2.33	2.83	3.13	3.48	3.81
	7	1.62	1.85	2.05	2.26	2.64	2.94	3.3	3.65
	8	1.58	1.8	1.97	2.16	2.48	2.8	3.17	3.52
	9	1.55	1.76	1.91	2.08	2.44	2.68	3.06	3.2
	10	1.52	1.71	1.86	2.01	2.33	2.58	2.81	3.13
0.95	0	3.46	4.57	5.64	6.66	9.2	11.57	13.51	16.03
	1	2.54	3.2	3.84	4.45	5.71	7.2	8.48	9.2
	2	2.2	2.69	3.17	3.61	4.54	5.42	6.4	6.97
	3	2.02	2.42	2.81	3.09	3.94	4.51	5.34	5.86
	4	1.9	2.25	2.55	2.84	3.43	4.18	4.7	5.19
	5	1.81	2.12	2.41	2.66	3.21	3.77	4.27	4.74
	6	1.74	2.02	2.27	2.49	3.04	3.47	3.96	4.12
	7	1.68	1.94	2.17	2.39	2.82	3.24	3.72	3.92
	8	1.65	1.88	2.11	2.28	2.73	3.06	3.36	3.76
	9	1.61	1.83	2.04	2.22	2.59	2.92	3.23	3.62
	10	1.58	1.79	1.98	2.14	2.47	2.8	3.12	3.33

t/ξ_o									
0.99	0	3.74	4.98	6.19	7.36	9.98	12.83	15.5	18.39
	1	2.75	3.5	4.2	4.9	6.46	7.99	9.63	10.79
	2	2.38	2.95	3.49	3.95	5.1	6.32	7.27	8.14
	3	2.17	2.66	3.08	3.52	4.38	5.25	6.04	6.77
	4	2.04	2.45	2.82	3.19	3.94	4.58	5.28	5.93
	5	1.94	2.31	2.64	2.96	3.63	4.27	4.76	5.37
	6	1.86	2.2	2.5	2.76	3.41	3.92	4.39	4.96
	7	1.8	2.11	2.4	2.63	3.16	3.64	4.1	4.64
	8	1.75	2.05	2.29	2.53	3.03	3.42	3.87	4.19
	9	1.71	1.98	2.23	2.42	2.93	3.35	3.69	4.02
	10	1.68	1.94	2.15	2.36	2.78	3.2	3.54	3.87

4. Illustration and Example for Test Plan

Assume that the life distribution is a MOIG distribution and the experimenter is interested in showing that the true unknown average life is at least 1000 hours. Let the consumer's risk be set to $1 - P^* = 0.05$. It is desired to stop the experiment at $t = 942$ hours. Then, for an acceptance number$c = 2$, the required n is the entry in Table 1 is 28. If during 942 hours no more than 2 failures out of 28 are observed, then the experimenter can assert with a confidence level of 0.95 that the average life is at least 1000 hours.

For the sampling plan($n = 28,\ c = 2,\ t/\xi_o = 0.942$), the operating characteristic values from (3.6) are as follows

ξ/ξ_0	2	4	6	8	10	12
t/ξ_0	0.73756	0.99893	1	1	1	1

This implies that if the true mean life is twice the specified mean life($\xi/\xi_0 = 2$), the producer's risk is about 0.26, and when $\xi/\xi_0 = 4$, producer's risk is about 0.1%.

However, the producer's risk approaches zero the mean life is at least 10000 or $\xi/\xi_0 \geq 6$. (while it is about zero when the true mean life is 6 times the specified mean life.) Table 3 used to get the value of ξ/ξ_0 for various values of $c,\ t/\xi_0$ when the producer's risk may not exceed 0.05. For example, for $c = 2,\ t/\xi_0 = 0.942$, $P^* = 0.95$, the value of ξ/ξ_0 is 2.69. This means that the product should have an average life of 2.69 times the specified average life if 1000 hours in order that the product be accepted with probability 0.95.

5. Application

A numerical example using a real data set studied by Meeker and Escobar (1988), which gives the times of failure and running times for a sample of devices from a field-tracking study of a larger system. At a certain point in time, 30 units were installed in normal service conditions. Two causes of failure were observed for each unit that failed, the failure caused by an accumulation of randomly occurring damage from power-line voltage spikes during electric storms and failure caused by normal product wear. The times are: 275, 13, 47, 23, 181, 30, 65, 10,300, 173, 106, 300, 300, 212, 300, 300, 300, 2, 261, 293, 88, 247, 28, 143, 300, 23, 300, 80,245, 266.

The maximum likelihood estimators for the MOIG parameters are $\widehat{\eta}$ =0.6789, $\widehat{\xi}$ =0.2103 and $\widehat{\vartheta}$ =0.00542.

The following table gives Anderson-Darling (AD) statistic, Kolmogorov-Smirnov (KS) test and their p-values, alongside the value of Min (-log Likelihood).

Table 4. Goodness of fit measures

	Statistic	A-D	K-S statistic	Min (-log Likelihood)
p-value		2.2946	0.2386	189.9843
	p-value	0.0568	0.06561	

Based on the goodness of fit tests, there is no strong evidence against that the MOIG distribution provides a good fit to this data set.

Suppose that the specified mean life $\xi_0 = 10$ hours and testing time be $t = 12.57$ hours. Therefore, for acceptance number $c = 2$ with probability $P^* = 0.99$, the minimum sample size is found from Table 1 to be 26.

The lot will be accepted only if the number of failures before 12.57 hours is at most 2. Since there are two failures at 2 and 10 from the data set before the time hours, then we will accept the lot, asserting a mean life time 10 hours with probability 0.99 (with significance level $\alpha = 0.01$).

6. Conclusion

This study established the acceptance sampling plans for the Marshall-Olkin inverse gamma (MOIG) life time distribution based on truncated lifetime tests. Various values of the MOIG distribution parameters are considered and the necessary tables based on the suggested sampling plan are presented. Both artificial and real life data sets were demonstrated to show our findings.

In future work, developing sampling plans based on the MOIG to satisfy both consumer's risk and producer's risk might prove important.

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