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SOME SPECIAL FEATURES OF RELATIVE INTUITIONISTIC DYNAMICAL SYSTEMS

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Abstract. This study has investigated the properties of the relative intuitionistic dynamical systems, and a series of essential properties, including $(\alpha\beta, \langle\mu, \nu\rangle)-$hole, $\langle\mu, \nu\rangle_{\alpha\beta}-$minimal, and $\langle\mu, \nu\rangle_{\alpha\beta}-$transitive, are introduced and examined. In this paper, it has also been specified in which conditions the relative intuitionistic dynamical system is an invariant system. Finally, an example of the relative intuitionistic dynamical system is presented, and its topologic properties are analyzed.

1. Introduction

For many years, the mathematicians only used the two-valued logic till Zadeh introduced fuzzy logic [19]. In this logic, the membership function had a fundamental role, and a fuzzy set was identified by this function. Also, the membership degree of an element of fuzzy sets was considered as a number between zero and one. After introducing the fuzzy logic, mathematicians generalized most of the math concepts from a two-valued logic to a fuzzy logic [3, 10, 11]. Very soon, other fields of science found out that this new logic and the mathematical models relevant to it are a powerful and useful tool for them either in the theoretical domain or in application [13, 17, 21].

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In the real world, we have to handle situations involving uncertainty, imprecision, and vagueness. Inspired by the fuzzy set in which membership function has an uncontested role in its specification, Atanassov introduced the intuitionistic fuzzy set (IFS for short)[2]. IFS is a generalization of fuzzy set and this set completely specify by membership and non- membership functions. If X is a non-empty set; an IFS A of X is $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$, where $\mu_A : X \to [0,1]$ represents the membership function and $\upsilon_A : X \to [0,1]$ shows the non-membership function of every element x of X. Membership and non-membership of each element x of X to IFS A is along with a degree of hesitancy. The function $\pi_A : X \to [0,1]$ with the condition $\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$ is called a hesitancy function. The set of all intuitionistic fuzzy sets of X, is displayed with $IFS(X)$.
Intuitionistic fuzzy sets are a suitable model for describing those phenomena that we cannot talk definitely about, and there is always a degree of hesitancy in them. These phenomena have an essential role in areas such as multi-criteria decision making, logic programming, medical diagnostics, and pattern recognition. Ashraf et al. introduced a new bounded variation based on similarity measures between IFSs and showed that the proposed measure obtained better results for pattern recognition and hierarchical clustering problems [1]. Besides, IFS was used to solve complex decision-making problems, especially the multi-attribute group decision-making problems under uncertain circumstances. Based on the IFS group decision-making model and intuitionistic fuzzy entropy, a new decision-making support method for project delivery system selection was proposed in [14]. Another interesting application of IFSs is in medical diagnostics. MRI segmentation is essential for clinical study and diagnosis. Using IFSs, Zang et al. proposed a new formulation of the MRI segmentation problem and showed that their method outperforms other methods in both the clustering metrics and the computational efficiency [20]. The application of intuitionistic fuzzy set in image processing, segmentation, and retrieval is discussed in [4], where the author has

examined the intuitionistic fuzzy mathematics and unified the latest existing works in literature.

In recent decades, scientists have tried to provide mathematical models for describing the phenomena which often occur in a system. There is a change or movement during the time in most systems, which are technically called dynamical systems. In mathematics, the proposed model for a discrete-time dynamical system includes two sets, a set like M, called the state space, and another set consisting of maps $\varphi: T \times M \to M$, which explains the variation in per unit time (T is time set). Often, state space has a mathematical structure; the state space may be a topological space, algebraic structure, or a measure space. The study of dynamical systems and their features has been noticed by many mathematicians having various viewpoints [6,7, 9, 15, 18]

In 2015, considering IFSs, we introduced a new topology which was a topology in point of view of a relative intuitionistic observer [8]. We called this topology the relative intuitionistic topology.

If $A = \langle \mu_A, \nu_A \rangle$ is a relative intuitionistic observer of a non-empty set X, collection $\tau_{(\mu_A,\nu_A)}$ from subsets of $A = \langle \mu_A, \nu_A \rangle$ is a relative intuitionistic topology, if the following principles are true:

(a) $\chi_\emptyset, \langle \mu_A, \nu_A \rangle \in \tau_{(\mu_A,\nu_A)}$;

(b) $G_1 \cap G_2 \in \tau_{(\mu_A,\nu_A)}$, for every $G_1, G_2 \in \tau_{(\mu_A,\nu_A)}$;

(c) $\bigcup\limits_{i\in I} G_i \in \tau_{(\mu_A,\nu_A)}$, for every collection $\{G_i | i \in I\} \subseteq \tau_{(\mu_A,\nu_A)}$.

The pair $(X, \tau_{(\mu_A,\nu_A)})$ is called a relative intuitionistic topological (RIT in short) space and elements of $\tau_{(\mu_A,\nu_A)}$ are called $\langle \mu_A, \nu_A \rangle-$ open. Also, in this topology, $B = \langle \mu_B, \nu_B \rangle$ is called $\langle \mu_A, \nu_A \rangle$ -close if $B^C = \langle \nu_B, \mu_B \rangle$ is a member of $\tau_{(\mu_A,\nu_A)}$.

In addition, in [8], we defined the relative intuitionistic dynamical system (RID system in short), that RIT space was its state space. For determining the complexity and/or uncertainty of this system, we introduced the concept of topological entropy and proved that the introduced topological entropy is an invariant object under the conjugate relation.

To understand a dynamic system wholly and accurately, it is necessary to examine its orbits, holes, fixed points, and transitive points. In [16], the authors studied these properties for fuzzy dynamical systems. In the present study, we introduce and study orbits, fixed points, $(\alpha\beta, \langle\mu, \nu\rangle)-$hole, and $\langle\mu, \nu\rangle_{\alpha\beta}-$transitive point for RID systems and examine under what conditions the RID system is minimal. For this purpose, in the second section, definitions and concepts required for exploring RI dynamical systems are stated. In the third section, some interesting propositions, theorems, and an example are presented.

2. Definitions and concepts

In this section, definitions and concepts required for introducing and studing RI dynamical systems have been stated.

If $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | \, x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | \, x \in X\}$ are IF sets, then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, for every $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,

(c) $A^C = \{\langle x, \nu_A(x), \mu_A(x)\rangle | \, x \in X\}$,

(d) $A \cap B = \{\langle x, \inf\{\mu_A(x), \mu_B(x)\}, \sup\{\nu_A(x), \nu_B(x)\}\rangle | \, x \in X\}$,

(e) $A \cup B = \{\langle x, \sup\{\mu_A(x), \mu_B(x)\}, \inf\{\nu_A(x), \nu_B(x)\}\rangle | \, x \in X\}$.

IF sets $\chi_\emptyset = 0_\sim = \{\langle x, 0, 1\rangle | \, x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle | \, x \in X\}$ are empty and X sets respectively.

In this paper, $A = \langle\mu_A, \nu_A\rangle$ symbol is used instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | \, x \in X\}$ and the set which includes IF sets of X is shown with IFS(X).

Let $f : X \to Y$ be a map, μ_A, ν_A, μ_B and ν_B be IF sets. Then

$$f^{-1}(\mu_B)(x) = \mu_B(f(x)),$$

$$f^{-1}(\nu_B)(x) = \nu_B(f(x)),$$

$$f(\nu_A)(y) = \begin{cases} \sup\limits_{x\in f^{-1}(y)} \nu_A(x) & f^{-1}(y) \neq \emptyset \\ 0 & o.w \end{cases},$$

$$f(\mu_A)(y) = \begin{cases} \sup\limits_{x\in f^{-1}(y)} \mu_A(x) & f^{-1}(y) \neq \emptyset \\ 0 & o.w \end{cases}.$$

Definition 2.1. The IF set $A = \langle \mu_A, \nu_A \rangle$ is called a relative intuitionistic observer (RIO in short).

Definition 2.2. Let $(X, \tau_{(\mu_A,\nu_A)})$ be a RIT space. Collection $\{\langle \mu^\alpha, \nu^\alpha \rangle : \alpha \in I\}$ is a relative intuitionistic open (RIO in short) cover of $\langle \mu_A, \nu_A \rangle$, if $\underset{\alpha\in I}{\cup} \langle \mu^\alpha, \nu^\alpha \rangle = \langle \mu_A, \nu_A \rangle$. RIT space $(X, \tau_{(\mu_A,\nu_A)})$ is compact, if every RIO cover has a finite sub cover.

Definition 2.3. Let f be a mapping from a RITS $(X, \tau_{(\mu_A,\nu_A)})$ into a RITS $(Y, \tau_{(\mu_B,\nu_B)})$. Then f is said to be a relative intuitionistic continuous map(RIC map in short) if $f^{-1}(B) = \{x, \langle f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) \rangle : x \in X\} \cap \{x, \langle \mu_A(x), \nu_A(x) \rangle : x \in X\} \in \tau_{(\mu_A,\nu_A)}$ for every $B = \{y, \langle \mu_B(y), \nu_B(y) \rangle : y \in Y\} \in \tau_{(\mu_B,\nu_B)}$. f is a relative intuitionistic homeomorphism (RI homeomorphism in short) if f is bijection and both f and f^{-1} are RIC maps.
$(X, \tau_{(\mu_A,\nu_A)}, f)$ is a relative intuitionistic(RI in short) dynamical system where X is non-empty set, $\tau_{(\mu_A,\nu_A)}$ is a RIT space and f is a RIC map.

For more familiarity with features of RIT spaces, RIC mapping and RI dynamical systems refer to[8].

Definition 2.4. Let $(X, \tau_{(\mu,\nu)})$ be a RIT space, $\langle \mu_i, \nu_i \rangle \in \tau_{(\mu,\nu)}$ and $\langle \mu_i, \nu_i \rangle_{\alpha\beta} = \{x \in X : \mu_i(x) > \alpha, \nu_i(x) < \beta, \alpha + \beta \leq 1\}$. If $\underset{i \in I}{\cup} \langle \mu_i, \nu_i \rangle_{\alpha\beta} = \langle \underset{i \in I}{\cup} \langle \mu_i, \nu_i \rangle \rangle_{\alpha\beta}$, for any collection $\{\langle \mu_i, \nu_i \rangle \in \tau_{(\mu,\nu)} : i \in I\}$, then set $(\tau_{(\mu,\nu)})_{\alpha\beta} = \{\langle \mu_i, \nu_i \rangle_{\alpha\beta} : \langle \mu_i, \nu_i \rangle \in \tau_{(\mu,\nu)}\}$ is called $(\alpha\beta, \langle \mu, \nu \rangle)-$topology.

We say that X is $(\alpha\beta, \langle \mu, \nu \rangle)-$topology if $(\langle \mu, \nu \rangle_{\alpha\beta}, (\tau_{(\mu,\nu)})_{\alpha\beta})$ is an $(\alpha\beta, \langle \mu, \nu \rangle)-$topology. $\langle \mu_i, \nu_i \rangle_{\alpha\beta}$ is an $(\alpha\beta, \langle \mu, \nu \rangle)-$open if $\langle \mu_i, \nu_i \rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta}$ and also collection $\{\langle \mu_i, \nu_i \rangle_{\alpha\beta} : \langle \mu_i, \nu_i \rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta}, i = 1, ..., n\}$ is an open cover if $\overset{n}{\underset{i=1}{\cup}} \langle \mu_i, \nu_i \rangle_{\alpha\beta} = \langle \mu, \nu \rangle_{\alpha\beta}$.

Definition 2.5. Let X and Y be $(\alpha\beta, \langle \mu_A, \nu_A \rangle)$ and $(\alpha\beta, \langle \mu_B, \nu_B \rangle)-$ topology spaces, respectively. $\varphi : X \to Y$ is said to be an $\alpha\beta$- relative continuous map if $\langle \mu_i \circ \varphi, \nu_i \circ \varphi \rangle_{\alpha\beta} \cap \langle \mu_A, \nu_A \rangle_{\alpha\beta} = \varphi^{-1}(\langle \mu_i, \nu_i \rangle_{\alpha\beta}) \cap \langle \mu_A, \nu_A \rangle_{\alpha\beta} \in (\tau_{(\mu_A,\nu_A)})_{\alpha\beta}$ for every $\langle \mu_i, \nu_i \rangle_{\alpha\beta} \in (\tau_{(\mu_B,\nu_B)})_{\alpha\beta}$.
φ is $\alpha\beta$- relative homeomorphism if φ is bijection and both of φ and φ^{-1} are $\alpha\beta$-relative continuous map.

Definition 2.6. X is called $(\alpha\beta, \langle \mu, \nu \rangle)-$ Hausdorff space if $(\langle \mu, \nu \rangle_{\alpha\beta}, (\tau_{(\mu,\nu)})_{\alpha\beta})$ is a Hausdorff space, and is said to be a $(\alpha\beta, \langle \mu, \nu \rangle)-$ compact if $(\langle \mu, \nu \rangle_{\alpha\beta}, (\tau_{(\mu,\nu)})_{\alpha\beta})$ is a compact space.

If X is $(\alpha\beta, \langle \mu, \nu \rangle)-$ compact space and $\varphi : X \to Y$ is an $\alpha\beta$- relative continuous map. Then $(X, \tau_{(\mu,\nu)}, \varphi)$ is named a RI dynamical system with level (α, β).

Definition 2.7. Let $(X, \tau_{(\mu,\nu)}, f)$ be a RI dynamical system. The set $\{p, f(p), f^2(p), ...\}$ is named orbit of point P. We show the orbit of point P with $O^f_{\langle \mu \times \nu \rangle}(p)$ and point P is a fixed point if $Card(O^f_{\langle \mu \times \nu \rangle}(p)) = 1$. In addition subset D of X is invariant if $f(D) \subseteq D$.

Definition 2.8. RI dynamical system $(X, \tau_{(\mu,\nu)}, \varphi)$ is called minimal on $\langle \mu, \nu \rangle_{\alpha\beta}$, if the following two conditions are hold:

(a)$\varphi(\langle\mu,\nu\rangle_{\alpha\beta}) \subseteq \langle\mu,\nu\rangle_{\alpha\beta}$;
(b) for every $x \in \langle\mu,\nu\rangle_{\alpha\beta}$, orbit $\{\varphi^n(x) : n = 0, 1, 2, ...\}$ is a dence subset of $\langle\mu,\nu\rangle_{\alpha\beta}$ which considering topoloy for $\langle\mu,\nu\rangle_{\alpha\beta}$ is $(\tau_{(\mu,\nu)})_{\alpha\beta}$.

RI minimal dynamic system on $\langle\mu,\nu\rangle_{\alpha\beta}$ is briefly called $\langle\mu,\nu\rangle_{\alpha\beta}-$minimal.

Definition 2.9. Let $(X, (\tau_{(\mu,\nu)})_{\alpha\beta}, \varphi)$ be a RI dynamical system with level (α, β). Subset E of $\langle\mu,\nu\rangle_{\alpha\beta}$ is $\langle\mu,\nu\rangle_{\alpha\beta}-$minimal if the following two conditions are hold:
(a) $\varphi(E) \subseteq E$;
(b)$\overline{\{\varphi^n(x) : n \in N \cup \{0\}\}} = E$, for any $x \in E$.

Definition 2.10. Point $x \in X$ is called $\langle\mu,\nu\rangle_{\alpha\beta}-$transitive point, if $O^{\varphi}_{\langle\mu,\nu\rangle}(x) \cap \langle\mu,\nu\rangle_{\alpha\beta}$ is a dence subset of $\langle\mu,\nu\rangle_{\alpha\beta}$ and $\varphi : X \to X$ is called $\langle\mu,\nu\rangle_{\alpha\beta}-$transitive, if X has a $\langle\mu,\nu\rangle_{\alpha\beta}-$transitive point.

Definition 2.11. Let $(X, (\tau_{(\mu,\nu)})_{\alpha\beta}, \varphi)$ be a RI dynamical system with level (α, β). Fix point $p \in X$ is $(\alpha\beta, \langle\mu,\nu\rangle)-$hole, if there is $\langle\mu,\nu\rangle_{\alpha\beta}-$open $\langle\lambda,\gamma\rangle_{\alpha\beta}$ such that $\{p\} = \bigcap\limits_{n=0}^{\infty} \varphi^n(\langle\lambda,\gamma\rangle_{\alpha\beta})$ which $\varphi^0(\langle\lambda,\gamma\rangle_{\alpha\beta}) = \langle\lambda,\gamma\rangle_{\alpha\beta}$.

3. Result

In this section, first some properties of IF sets are stated and then the topological features of the introduced dynamical systems are studied. Finally, an example of a RI dynamic system with level (α, β) is given.

Theorem 3.1. *Let $(X, \tau_{(\mu,\nu)})$ be a RIT space, $\langle\lambda_i,\gamma_i\rangle$, $\langle\mu,\nu\rangle$, A_i $(i \in I)$, A and $\langle\lambda_j,\gamma_j\rangle$ be IF sets of X. Then*
(a)$\varphi^{-1}(\bigcup\limits_j A_j) = \bigcup\limits_j \varphi^{-1}(A_j)$, $\varphi^{-1}(\bigcap\limits_j A_j) = \bigcap\limits_j \varphi^{-1}(A_j)$ *and* $\varphi(\bigcup\limits_i A_i) = \bigcup\limits_i \varphi(A_i)$,
(b)$\varphi^{-1}(\langle\mu,\nu\rangle_{\alpha\beta}) = \langle\varphi^{-1}\langle\mu,\nu\rangle\rangle_{\alpha\beta}$,
(C)$\varphi^{-1}(\langle\lambda_i,\gamma_i\rangle_{\alpha\beta} \cup \langle\lambda_j,\gamma_j\rangle_{\alpha\beta}) = \varphi^{-1}(\langle\lambda_i,\gamma_i\rangle_{\alpha\beta}) \cup \varphi^{-1}(\langle\lambda_j,\gamma_j\rangle_{\alpha\beta})$,
(d) $\varphi^{-1}(\langle\lambda_i,\gamma_i\rangle_{\alpha\beta} \cap \langle\lambda_j,\gamma_j\rangle_{\alpha\beta}) = \varphi^{-1}(\langle\lambda_i,\gamma_i\rangle_{\alpha\beta}) \cap \varphi^{-1}(\langle\lambda_j,\gamma_j\rangle_{\alpha\beta})$,

(e) $\varphi^{-1}(\langle\lambda,\gamma\rangle^C) = (\varphi^{-1}\langle\lambda,\gamma\rangle)^C$,

(f)$\varphi\left(\langle\lambda_i,\gamma_i\rangle_{\alpha\beta} \cap \langle\lambda_j,\gamma_j\rangle_{\alpha\beta}\right) \subseteq \varphi\left(\langle\lambda_i,\gamma_i\rangle_{\alpha\beta}\right) \cap \varphi\left(\langle\lambda_j,\gamma_j\rangle_{\alpha\beta}\right)$,

(g) $\varphi(\langle\lambda_i,\gamma_i\rangle_{\alpha\beta} \cup \langle\lambda_j,\gamma_j\rangle_{\alpha\beta}) \subseteq \varphi(\langle\lambda_i,\gamma_i\rangle_{\alpha\beta}) \cup \varphi(\langle\lambda_j,\gamma_j\rangle_{\alpha\beta})$.

Proof. a) The proof can be found in [5].

b) $\varphi^{-1}(\langle\mu,\nu\rangle_{\alpha\beta}) = \langle\mu\circ\varphi,\nu\circ\varphi\rangle_{\alpha\beta} = \langle\varphi^{-1}\langle\mu,\nu\rangle\rangle_{\alpha\beta}$.

c)

$$x \in \varphi^{-1}(\langle\lambda_i,\gamma_i\rangle_{\alpha\beta}) \cup \varphi^{-1}(\langle\lambda_j,\gamma_j\rangle_{\alpha\beta})$$
$$\Leftrightarrow x \in (\langle\lambda_i\circ\varphi,\gamma_i\circ\varphi\rangle_{\alpha\beta}) \cup (\langle\lambda_j\circ\varphi,\gamma_j\circ\varphi\rangle_{\alpha\beta})$$
$$\Leftrightarrow \lambda_i\circ\varphi\,(x) > \alpha,\, \gamma_i\circ\varphi\,(x) < \beta$$
$$or\,\lambda_j{}^\circ\varphi\,(x) > \alpha,\, \gamma_j{}^\circ\varphi\,(x) < \beta \Leftrightarrow \varphi\,(x)\ \in \langle\lambda_i,\gamma_i\rangle_{\alpha\beta}$$
$$or\,\varphi\,(x)\ \in \langle\lambda_j,\gamma_j\rangle_{\alpha\beta} \Leftrightarrow \varphi\,(x)\ \in (\langle\lambda_i,\gamma_i\rangle_{\alpha\beta} \cup \langle\lambda_j,\gamma_j\rangle_{\alpha\beta})$$
$$\Leftrightarrow x \in \varphi^{-1}(\langle\lambda_i,\gamma_i\rangle_{\alpha\beta} \cup \langle\lambda_j,\gamma_j\rangle_{\alpha\beta}).$$

d)The proof is simillar to the previouse part.

e)

$$\varphi^{-1}(\langle\lambda,\gamma\rangle^C)\,(x) = \langle\lambda,\gamma\rangle^C\varphi\,(x)\ = \langle\gamma,\lambda\rangle\varphi\,(x)\ = \langle\gamma\circ\varphi,\lambda\circ\varphi\rangle\,(x)$$
$$= (\langle\lambda\circ\varphi,\gamma\circ\varphi\rangle\,(x))^C\ =\ (\varphi^{-1}\langle\lambda,\gamma\rangle\,(x))^C.$$

f) If $y\ \in\ \varphi(\langle\lambda_i,\gamma_i\rangle_{\alpha\beta}\cap\langle\lambda_j,\gamma_j\rangle_{\alpha\beta})$ then there exists $x\ \in \langle\lambda_i,\gamma_i\rangle_{\alpha\beta}\cap\langle\lambda_j,\gamma_j\rangle_{\alpha\beta}$ such that $y\ =\ \varphi\,(x)\,,\ \lambda_i\,(x)\,,\lambda_j\ (x)\ >\ \alpha$ and $\gamma_i\,(x)\,,\ \gamma_j\,(x) <\ \beta$. This means $y\ \in \varphi(\langle\lambda_i,\gamma_i\rangle_{\alpha\beta})\cap \varphi(\langle\lambda_j,\gamma_j\rangle_{\alpha\beta})$.

g) The proof is simillar to the previouse part. □

Lemma 3.1. $(\langle\mu,\nu\rangle_{\alpha\beta},(\tau_{(\mu,\nu)})_{\alpha\beta})$ *is a topological space.*

Proof. The proof can be found in [12]. □

Theorem 3.2. *If* $(X,\tau_{(\mu,\nu)},\varphi)$ *is a RI dynamical system, then* $(X\times X,\tau_{(\mu\,\times\,\mu\,,\nu\,\times\,\nu)},\varphi\times\varphi)$ *is a RI dynamical system which* $\tau_{(\mu\,\times\,\mu\,,\nu\,\times\,\nu)}\ =\ \left\{\langle\lambda\ \times\ \eta,\gamma\ \times\ \kappa\rangle\ :\ \langle\lambda\,,\gamma\,\rangle,\,\langle\,\eta,\,\kappa\rangle\ \in\ \tau_{(\mu\ ,\nu\,)}\,\right\},\langle\lambda\ \times\ \eta,\gamma\ \times\ \kappa\rangle\,(x,\,y) = \langle\lambda\,,\gamma\,\rangle\,(x)$ *and* $\varphi\times\ \varphi\,(x,\,y) = \varphi\,(x)$.

Proof. $(X\times X, \tau_{(\mu\times\mu,\nu\times\nu)}, \varphi\times\varphi)$ is a RI dynamical system, because;

a) $\chi_\emptyset \times \chi_\emptyset = \chi_\emptyset \in \tau_{(\mu\times\mu,\nu\times\nu)}$ and $\langle \mu\times\mu, \nu\times\nu\rangle \in \tau_{(\mu\times\mu,\nu\times\nu)}$.

b) If $\langle\lambda_1\times\lambda_2, \gamma_1\times\gamma_2\rangle$ and $\langle\eta_1\times\eta_2, \kappa_1\times\kappa_2\rangle$ are belong to $\tau_{(\mu\times\mu,\nu\times\nu)}$, then $\langle\lambda_1\times\lambda_2, \gamma_1\times\gamma_2\rangle \cap \langle\eta_1\times\eta_2, \kappa_1\times\kappa_2\rangle(x,y) = \langle\lambda_1,\gamma_1\rangle\cap\langle\eta_1,\kappa_1\rangle(x)$. Since $(X, \tau_{(\mu,\nu)}, \varphi)$ is a RI dynamical system so there exists $\langle\lambda, \gamma\rangle \in \tau_{(\mu,\nu)}$ such that $\langle\lambda_1,\gamma_1\rangle\cap\langle\eta_1,\kappa_1\rangle(x) = \langle\lambda, \gamma\rangle(x) = \langle\lambda\times\lambda, \gamma\times\gamma\rangle(x, y) \in \tau_{(\mu\times\mu,\nu\times\nu)}$.

c) Suppose that $\{\langle\lambda_i\times\zeta_i, \gamma_i\times\kappa_i\rangle : i\in I\} \subseteq \tau_{(\mu\times\mu,\nu\times\nu)}$, then $\bigcup_{i\in I}\langle\lambda_i\times\zeta_i, \gamma_i\times\kappa_i\rangle(x,y) = \bigcup_{i\in I}\langle\lambda_i,\gamma_i\rangle(x) = \langle\lambda,\eta\rangle(x) = \langle\lambda\times\lambda, \eta\times\eta\rangle(x,y)$ which $\langle\lambda,\eta\rangle\in\tau_{(\mu,\nu)}$.

By a), b) and c), $(X\times X, \tau_{(\mu\times\mu,\nu\times\nu)}, \varphi\times\varphi)$ is a RI topological space. Now we prove that $\varphi\times\varphi$ is a $\langle\mu\times\mu, \nu\times\nu\rangle-$ continous map.

If $\langle\lambda\times\eta, \gamma\times\kappa\rangle \in \tau_{(\mu\times\mu,\nu\times\nu)}$ then for every $(x,y)\in X\times X$, we have;

$(\varphi\times\varphi)^{-1}\langle\lambda\times\eta, \gamma\times\kappa\rangle\cap\langle\mu\times\mu, \nu\times\nu\rangle(x,y) = \varphi^{-1}\langle\lambda,\gamma\rangle\cap\langle\mu,\nu\rangle(x) = \langle\lambda',\eta'\rangle(x) = \langle\lambda'\times\lambda', \eta'\times\eta'\rangle(x,y)$ which $\langle\lambda',\eta'\rangle\in\tau_{(\mu,\nu)}$. Therefore $(\varphi\times\varphi)^{-1}\langle\lambda\times\eta, \gamma\times\kappa\rangle\cap\langle\mu\times\mu, \nu\times\nu\rangle \in \tau_{(\mu\times\mu,\nu\times\nu)}$ and this implies $\varphi\times\varphi$ is a $\langle\mu\times\mu, \nu\times\nu\rangle-$ continous map. □

Theorem 3.3. *If $\varphi : Y \to X$ is a map and X is an $(\alpha\beta, \langle\mu,\nu\rangle)-$topological space. Then*

a) *Y is an $(\alpha\beta, \langle\mu,\nu\rangle)-$topological space;*

b) *φ is an $\alpha\beta-$ relative continous map;*

c) *if X is an $(\alpha\beta, \langle\mu,\nu\rangle)-$Hausdorff space, then Y is $(\alpha\beta, \langle\mu\circ\varphi, \nu\circ\varphi\rangle)$ - Hausdorff space.*

Proof. The proof can be found in [12]. □

Theorem 3.4. *If $(\alpha\beta, \langle\mu,\nu\rangle)-$hole p belong to $\langle\lambda,\gamma\rangle_{\alpha\beta}$, which $\langle\lambda,\gamma\rangle_{\alpha\beta}$ is $\langle\mu,\nu\rangle_{\alpha\beta}-$ open, then there is no other $(\alpha\beta, \langle\mu,\nu\rangle)-$hole belong to $\langle\lambda,\gamma\rangle_{\alpha\beta}$.*

Proof. Suppose that q is $(\alpha\beta,\langle\mu,\nu\rangle)-$hole p belong to $\langle\lambda,\gamma\rangle_{\alpha\beta}$, which $p\neq q$. Since q is a fix point, so for every $n\in N\cup\{0\}$ we have $\varphi^n(q)=q$ which means $q\in\bigcap_{n=0}^{\infty}\varphi^n(\langle\lambda,\gamma\rangle_{\alpha\beta})$. By definition of $(\alpha\beta,\langle\mu,\nu\rangle)-$hole, we conclude that $q=p$. □

Theorem 3.5. *Let $(X,\tau_{(\mu,\nu)},\varphi)$ be a RI dynamical system and $\alpha\in[0,1]$. The following statements are equivalent;*

a) *φ is $\langle\mu,\nu\rangle_{\alpha\beta}-$minimal;*

b) *let $\varphi(\langle\mu,\nu\rangle_{\alpha\beta})\subseteq\langle\mu,\nu\rangle_{\alpha\beta}$. If C is a closed subset of $(\tau_{(\mu,\nu)})_{\alpha\beta}$ such that $\varphi(C)\subseteq C$, then $C=\langle\mu,\nu\rangle_{\alpha\beta}$ or $C=\emptyset$;*

c) *if $\varphi(\langle\mu,\nu\rangle_{\alpha\beta})\subseteq\langle\mu,\nu\rangle_{\alpha\beta}$, $\emptyset\neq P\in(\tau_{(\mu,\nu)})_{\alpha\beta}$ and φ is an one to one map. Then $\langle\mu,\nu\rangle_{\alpha\beta}=\bigcup_{n=0}^{\infty}P_n$, which $P_0=P$ and $P_n=\varphi^{-n}(P)\cap(\bigcap_{i=0}^{n-1}\varphi^{-i}\langle\mu,\nu\rangle_{\alpha\beta})$.*

Proof. $a\Rightarrow b$

Let C be a close set of $(\tau_{(\mu,\nu)})_{\alpha\beta}$, $C\neq\emptyset$ and $y\in C$. Since $\varphi(C)\subseteq C$ so $\{\varphi^n(y):n=0,1,2,...\}\subseteq C$. Also φ is $\langle\mu,\nu\rangle_{\alpha\beta}-$minimal, therefore $\langle\mu,\nu\rangle_{\alpha\beta}=\overline{\{\varphi^n(y):n\in N\cup\{0\}\}}\subseteq\bar{C}=C$ and this implies $\langle\mu,\nu\rangle_{\alpha\beta}=C$.

$b\Rightarrow c$ Suppose that $P_n=\varphi^{-n}(P)\cap(\bigcap_{i=0}^{n-1}\varphi^{-i}\langle\mu,\nu\rangle_{\alpha\beta})$, $P_0=P$ and $M=\bigcup_{n=0}^{\infty}P_n$. The definition of RI topology and proposition 3.1, imply that M is $\langle\mu,\nu\rangle_{\alpha\beta}-$ open set and $D=M^C$ is $\langle\mu,\nu\rangle_{\alpha\beta}-$ close set.

If $y\in\varphi(D)$, then there is $x\in D$ such that $y=\varphi(x)$. Since $x\in D$ so $x\notin M$ and $x\notin P_n$ for any $n\in N\cup\{0\}$. Therefore $x\notin\varphi^{-n}(P)$ or $x\notin\bigcap_{i=0}^{n-1}\varphi^{-i}\langle\mu,\nu\rangle_{\alpha\beta}$.

Let $y\notin D$ so, $y\in\varphi^{-n}(P)$, for every $n\in N\cup\{0\}$ and $y\in\bigcap_{i=0}^{n-1}\varphi^{-i}\langle\mu,\nu\rangle_{\alpha\beta}$. φ is an one to one map, so $x=\varphi^{-1}(y)\in\varphi^{-n-1}(P)$ and $x=\varphi^{-1}(y)\in\bigcap_{i=0}^{n-1}\varphi^{-i-1}\langle\mu,\nu\rangle_{\alpha\beta}$ and these imply that $x=\varphi^{-1}(y)\in M$ which is a contradiction with $x\in D$. Therefore $y\in D$ which means $\varphi(D)\subseteq D$. Since $M\neq\emptyset$, by part b) $D=\emptyset$ and $\langle\mu,\nu\rangle_{\alpha\beta}=\bigcup_{n=0}^{\infty}P_n$.

$c\Rightarrow a$

Suppose that $P=\{\varphi^n(x):n\in N\cup\{0\}\}$, $x,y\in\langle\mu,\nu\rangle_{\alpha\beta}$ and $O\in(\tau_{(\mu,\nu)})_{\alpha\beta}$ such that $y\in O$. we have $\langle\mu,\nu\rangle_{\alpha\beta}=\bigcup_{n=0}^{\infty}O_n$ which $O_0=O$ and $O_n=\varphi^{-n}(O)\cap(\bigcap_{i=0}^{n-1}\varphi^{-i}\langle\mu,\nu\rangle_{\alpha\beta})$.

Therefore there is $m \in N \cup \{0\}$ such that $x \in O_m$. Since $\varphi^m(O_m) \subseteq O$ so $\varphi^n(x) \in O$ and $\bar{P} = \langle \mu, \nu \rangle_{\alpha\beta}$. □

Theorem 3.6. *Let* $(X, (\tau_{(\mu,\nu)})_{\alpha\beta}, \varphi)$ *be a RI dynamical system with the level* (α, β) , $\varphi(\langle \mu, \nu \rangle_{\alpha\beta}) \subseteq \langle \mu, \nu \rangle_{\alpha\beta}$ *and* $\langle \mu, \nu \rangle_{\alpha\beta} \neq \emptyset$, *then* φ *has a* $\langle \mu, \nu \rangle_{\alpha\beta}-$*minimal set.*

Proof. Suppose that $M = \{D \subseteq \langle \mu, \nu \rangle_{\alpha\beta} : \varphi(D) \subseteq D,\ D\, is\, a\, \langle \mu, \nu \rangle_{\alpha\beta} - close\, set\}$
If $\{C_k\}_{k\in I}$ is a decreasing sequence of non- empty subsets of M, such that $C_{k+1} \subseteq C_k$, which means that $\{C_k\}_{k\in I}$ has a total order, then based on Cantor's intersection theorem, we have $\underset{k\in I}{\cap} C_k \neq \emptyset$. $\underset{k\in I}{\cap} C_k$, is the lower bound for subsets of $\{C_k\}_{k\in I}$ and also $\underset{k\in I}{\cap} C_k$ is a $\langle \mu, \nu \rangle_{\alpha\beta}-$close set and $\varphi(\underset{k\in I}{\cap} C_k) \subseteq \underset{k\in I}{\cap} \varphi(C_k) \subseteq \underset{k\in I}{\cap} C_k$. Therefore, by Zorn's Lemma, M has a minimal set. Suppose that D is a minimal set of M, $x \in D$ and $K = \{\varphi^n(x) : n \in N \cup \{0\}\}$. Now, we prove that $\varphi(\bar{K}) \subseteq \bar{K}$. Suppose that $y \in \varphi(\bar{K})$ and $x \in \bar{K}$, so that $y = \varphi(x)$. If P is $\langle \mu, \nu \rangle_{\alpha\beta}-$ open set including $y = \varphi(x)$, then $\varphi^{-1}(P) \cap \langle \mu, \nu \rangle_{\alpha\beta}$ is $\langle \mu, \nu \rangle_{\alpha\beta}-$open set including x.
Since $x \in \bar{K}$, therefore there is $t \in K$, such that $t \in \varphi^{-1}(P) \cap \langle \mu, \nu \rangle_{\alpha\beta}$. $t \in k$ so there is $m \in N \cup \{0\}$ suchthat $t = \phi^m(x)$ and $\varphi(t) = \phi^{m+1}(x)$. On the other hand, $\phi^{m+1}(x) = \varphi(t) \in \varphi(\varphi^{-1}(P) \cap \langle \mu, \nu \rangle_{\alpha\beta}) \subseteq P$ therefore, $y \in \bar{K}$. It is proved that $\varphi(\bar{K}) \subseteq \bar{K}$ and on the other hand, $\bar{K}$ is $\langle \mu, \nu \rangle_{\alpha\beta}$-close set, then $\bar{K} \in M$. D is a minimal set and $\bar{K} \in M$, therefore, $D = \bar{K}$ and we conclude that D is a $\langle \mu, \nu \rangle_{\alpha\beta}-$ minimal set. □

Theorem 3.7. *Let* $(X, (\tau_{(\mu,\nu)})_{\alpha\beta}, \varphi)$ *be a compact RI dynamical system and* $\langle \mu, \nu \rangle_{\alpha\beta}-$ *minimal, Y a Hausdorff space and* $\psi : \langle \mu, \nu \rangle_{\alpha\beta} \to Y$ *be a continuous map such that* $\psi \circ \varphi = \psi$ *, then* ψ *is an invarient function.*

Proof. suppose $q \in \langle \mu, \nu \rangle_{\alpha\beta}$. Since $\psi \circ \varphi = \psi$, then $\psi \circ \varphi^n = \psi$. And this implies that $\psi(O^{\varphi}_{\langle \mu,\nu \rangle}(q)) = \psi(q)$. ψ is a continuous function and $(X, (\tau_{(\mu,\nu)})_{\alpha\beta}, \varphi)$ is a compact RI dynamical system which result in $\psi(\overline{O^{\varphi}_{\langle \mu,\nu \rangle}(q)})$ is compact and since Y is Haudroff space , then $\psi(\overline{O^{\varphi}_{\langle\mu,\nu\rangle}(q)})$ is a close set. On the other hand φ is $\langle\mu,\nu\rangle_{\alpha\beta}-$minimal, then $\langle\mu,\nu\rangle_{\alpha\beta} = \overline{O^{\varphi}_{\langle\mu,\nu\rangle}(q)}$. And this gives $\psi(\langle\mu,\nu\rangle_{\alpha\beta}) = \psi(\overline{O^{\varphi}_{\langle\mu,\nu\rangle}(q)}) = \overline{\psi(O^{\varphi}_{\langle\mu,\nu\rangle(q)})} = \psi(q)$. □

Theorem 3.8. *Let $\varphi : X \to X$ be $\langle\mu,\nu\rangle$- homeomorphism and $\langle\mu,\nu\rangle_{\alpha\beta}-$ transitive, then:*

a) *if U is a non- empty set and $\langle\mu,\nu\rangle_{\alpha\beta}-$ open, such that $\varphi(U) = U$, then U is a dense subset of $\langle\mu,\nu\rangle_{\alpha\beta}$,*

b) *if W and V are non- empty sets and $\langle\mu,\nu\rangle_{\alpha\beta}-$ open, then there is $n \in Z$, such that $\varphi^n(V) \cap W \neq \emptyset$.*

Proof. a) suppose x is a $\langle\mu,\nu\rangle_{\alpha\beta}-$ transitive point. Then, $O_{\alpha\beta}(x) = \{x, \varphi(x), \varphi^2(x), ...\} \cap \langle\mu,\nu\rangle_{\alpha\beta}$ is dense subset of $\langle\mu,\nu\rangle_{\alpha\beta}$. U is a non empty and $\langle\mu,\nu\rangle_{\alpha\beta}-$ open, therefore $n \in N$ exists, such that $\varphi^n(x) \in U$ and $x \in \varphi^{-n}(U)$. $\varphi : X \to X$ is $\langle\mu,\nu\rangle$- homeomorphism. Therefore, for each $k \in N$, we have $\varphi^k(x) \in \varphi^k(\varphi^{-n}(U)) \subseteq U$ and this gives $\langle\mu,\nu\rangle_{\alpha\beta} = \overline{O_{\alpha\beta}(x)} \subseteq \bar{U}$, so U is a dense subset of $\langle\mu,\nu\rangle_{\alpha\beta}$.
b)$\varphi : X \to X$ is $\langle\mu,\nu\rangle$- homeomorphism. Then $U = \bigcup_{n\in Z} \varphi^n(V)$is non-empty $\langle\mu,\nu\rangle_{\alpha\beta}-$ - open and $\varphi(U) = U$. Therefore, according to part a), U is a dense subset of $\langle\mu,\nu\rangle_{\alpha\beta}$ and this gives $U \cap W \neq \emptyset$, which means that exists $n \in Z$, such that $\varphi^n(V) \cap W \neq \emptyset$. □

Example 3.1. *Let $\mu, \nu, \lambda_i, \gamma_i$ be functions of R to $[0, 1]$ and φ be a function of R to R, such that*

$$\mu(x) = \begin{cases} \frac{1}{2} & x \in [0,\infty) - Z \\ 0 & o.w \end{cases}$$

$$\nu(x) = \begin{cases} \frac{1}{3} & x \in [0,\infty) - Z \\ 1 & o.w \end{cases}$$

$$\lambda_i(x) = \begin{cases} \frac{1}{2^i} & x \in (i, i+1) \\ 0 & o.w \end{cases}$$

$$and \qquad \gamma_i(x) = \begin{cases} \frac{1}{3^i} & x \in (i, i+1) \\ 1 & o.w \end{cases}.$$

Suppose that $\tau_{(\mu,\nu)}$ *is* $\langle \mu, \nu \rangle-$*topology, generated by* $\{\langle \mu, \nu \rangle, \langle \lambda_i, \gamma_i \rangle : i \in N \cup \{0\}\}$*, and also* φ *is a function with* $\varphi(x) = x + 1$.

Let $0 \leq \alpha < \frac{1}{2}$ *and* $\frac{1}{3} < \beta \leq 1$ *such that* $\alpha + \beta \leq 1$*. We have*

$$\varphi(\langle \mu, \nu \rangle_{\alpha\beta}) = \{\varphi(x) : \mu(x) > \alpha, \nu(x) < \beta\} = \{x+1 : x \in R^+ - Z\} = R^+ - Z \subseteq \langle \mu, \nu \rangle_{\alpha\beta}.$$

suppose that $x \in \langle \mu, \nu \rangle_{\alpha\beta}$ *and* $l \in N \cup \{0\}$ *such that* $x \in (l, l+1)$*. We will prove that orbit* $\{\varphi^n(x) : n = 0, 1, 2, ...\}$ *is a dence subset of* $\langle \mu, \nu \rangle_{\alpha\beta}$.

Let $y \in \langle \mu, \nu \rangle_{\alpha\beta}$ *and* $\langle \lambda, \gamma \rangle_{\alpha\beta} \in (\tau_{(\mu,\nu)})_{\alpha\beta}$ *such that* $y \in \langle \lambda, \gamma \rangle_{\alpha\beta}$. $\langle \lambda, \gamma \rangle_{\alpha\beta} \subseteq \bigcup_{i \in I} \langle \lambda_i, \gamma_i \rangle_{\alpha\beta}$ *and also* $\langle \lambda_i, \gamma_i \rangle_{\alpha\beta} \cap \langle \lambda_j, \gamma_j \rangle_{\alpha\beta} = \chi_\emptyset$ *for any* $i \neq j$ *so, there is* $k \in I$ *such that* $y \in \langle \lambda_k, \gamma_k \rangle_{\alpha\beta}$ *which means* $\lambda_k(y) > \alpha$ *and* $\gamma_k(y) < \beta$*. Therefore* $\frac{1}{2^k} > \alpha$, $y \in (k, k+1)$ *and* $\frac{1}{3^k} < \beta$*. Let* $n = k - l$. $\varphi^n(x) = x + n \in (k, k+1)$ *and this implies* $\varphi^n(x) \in \langle \lambda_k, \gamma_k \rangle_{\alpha\beta} \subseteq \langle \lambda, \gamma \rangle_{\alpha\beta}$.

4. Concluding Remarks

Man's recognition of his surrounding phenomena has always been under evolution. Mathematics as a strong instrument for stating and explaining these phenomena has been also growing according to human need. Intuitionistic fuzzy logic is a new logic which has been introduced in recent years for better and more realistic study of some biological, physical and behavioral phenomena. Providing mathematic model for dynamical systems based on this logic is necessary. As yet, no suitable model has been provided for describing phenomena which precaution them is dependent to observer views. In this study, topological properties of relative intuitionistic dynamical systems in point of view relative intuitionistic observer have been examined. The systems studied in this paper, are proper theoretical models for studying systems which have a main role in multi-criteria decision making, logical programming, medical diagnostics and pattern recognition.

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