

ALGEBRAIC MODELS IN APPLIED RESEARCH

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ABSTRACT:

Mathematical models can be used in several sciences as a tool to organize the research in such a way that the results could be transferred into other sciences. Here we go on with a method how to organise some results, which do not seem to have any relation with mathematics in a strictly algebraic way. The topic of algebra used is called H_v -structures, the largest class of hyperstructures. Our examples are taken from recent research in linguistics, more specifically on sociolinguistics. We believe that this game with mathematics will stimulate the interest of both researchers and students; moreover it is within the frame of Interdisciplinary Approach, one of the most recent teaching attitudes.

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1. INTRODUCTION

In this paper we get on with our proposed method for modelling in research by using the special algebraic domains called hyperstructures. This method is motivated by research in linguistics and this is where we apply this modelling. Our point is that, during the research and teaching process, it is interesting to see if the results could be possibly formulated in an algebraic domain. Such a process could be applied in many ways bringing to light interesting aspects connecting pure research with applications in the classroom environment.

We deal with hyperstructures called H_v -structures introduced by T. Vougiouklis, in 1990 [5], which satisfy the *weak axioms* where the non-empty intersection replaces the equality. This topic is growing rapidly as one can see in the site: aha.eled.duth.gr.

2. H_v -STRUCTURES

We recall some basic definitions mainly from [6]:

In a set H equipped with a hyperoperation $\cdot : H \times H \rightarrow \mathcal{P}(H) - \{\emptyset\}$, we abbreviate by

WASS the *weak associativity*: $(xy)z \cap x(yz) \neq \emptyset, \forall x, y, z \in H$ and by

COW the *weak commutativity*: $xy \cap yx \neq \emptyset, \forall x, y \in H$.

The hyperstructure (H, \cdot) is called H_v -semigroup if it is WASS and it is called H_v -group if it is reproductive H_v -semigroup, i.e. $xH = Hx = H, \forall x \in H$.

The hyperstructure $(R, +, \cdot)$ is called H_v -ring if $(+)$ and (\cdot) are WASS, the reproduction axiom is valid for $(+)$ and (\cdot) is *weak distributive* with respect to $(+)$:

$$x(y+z) \cap (xy+xz) \neq \emptyset, (x+y)z \cap (xz+yz) \neq \emptyset, \forall x, y, z \in R.$$

For more definitions such as H_v -vector spaces, H_v -algebras or H_v -Lie algebras and applications on H_v -structures, see the books [6],[2],[3], review papers as [4] and related papers as [1], [7].

The fundamental relations β^* , γ^* and ε^* are defined, in H_v -groups, H_v -rings and H_v -vector spaces, respectively, as the smallest equivalences so that the quotient would be group, ring and vector space, respectively [6]. A way to find the fundamental classes is given by analogous theorems to the following [5],[6],[7]:

Theorem 1. Let (H, \cdot) be a H_v -group and denote by U the set of all finite products of elements of H . We define the relation β in H by $x\beta y$ iff $\{x, y\} \subset u$ where $u \in U$. Then the fundamental relation β^* is the transitive closure of β .

An element is called *single* if its fundamental class is singleton.

The fundamental relations are used for general definitions. Thus, in order to define the H_v -field the γ^* is used: A H_v -ring $(R, +, \cdot)$ is called *H_v -field* if R/γ^* is a field. In the sequence the *H_v -vector space* is defined [7],[10].

Let (H, \cdot) , $(H, *)$ be H_v -semigroups. (\cdot) is called *smaller* than $(*)$, and $(*)$ *greater* than (\cdot) , iff there exists an

$$f \in \text{Aut}(H, *) \text{ such that } xy \subset f(x*y), \forall x, y \in H.$$

Then we write $\cdot \leq^*$ and we say that $(H, *)$ *contains* (H, \cdot) . If (H, \cdot) is a structure then it is called *basic structure* and $(H, *)$ is called *H_b -structure*.

Theorem 2 [6]. Greater hyperoperations of the ones which are *WASS* or *COW* are also *WASS* or *COW* respectively.

Definitions [8],[9],[10]. Let (H, \cdot) be a hypergroupoid. We *remove* $h \in H$, if we take the restriction of (\cdot) in $H - \{h\}$. $\underline{h} \in H$ *absorbs* $h \in H$ if it replaces h so h does not appear in the structure. $\underline{h} \in H$ *merges* with $h \in H$, if the product of any $x \in H$ by \underline{h} is the union of the results of x with both h , \underline{h} , and consider h and \underline{h} as one class with representative \underline{h} .

Therefore one can add or remove elements in H_v -structures and the obtained H_v -structures have new and old properties.

The H_v -structures are used in Representation Theory. Representations of H_v -groups can be considered either by generalized permutations or by H_v -matrices [6]. The representations by generalized permutations can be achieved by using left or right translations. The single elements are playing a crucial role.

Definitions [6]. Let (H, \cdot) be H_v -group, then the *powers* of an element $x \in H$, using the circle hyperoperation (\otimes) , i.e. take the union of all hyperproducts putting the parentheses on all possible ways, are defined as follows:

$$x^1 = \{x\}, \quad x^2 = x \otimes x, \dots, \quad x^s = x \otimes \dots \otimes x, \dots$$

The H_v -group (H, \cdot) is called *cyclic* with finite *period* s , the minimum one, with respect to the *generator* $h \in H$, if

$$H = h^1 \cup h^2 \cup \dots \cup h^s$$

If all generators have the same period, then H is *cyclic with period*. If there exists $h \in H$ and s , the minimum one, such that $H = h^s$, then H is a *single-power cyclic*, h is a *generator with single-power period* s . There is no any analogous definition in the classical theory, moreover one can define the infinite cyclicity as well.

3. THETA HYPERSTRUCTURES

In our modelling we need the following: In [11] a hyperoperation in a groupoid with a map f on it, is defined, which is denoted by *theta* ∂ .

Definitions 3. Let H be a set equipped with n operations (or hyperoperations) $\otimes_1, \otimes_2, \dots, \otimes_n$ and a map (or multivalued) $f: H \rightarrow H$, then n hyperoperations $\partial_1, \partial_2, \dots, \partial_n$ on H can be defined, called *theta-operations* by putting

$$x\partial_i y = \{f(x)\otimes_i y, x\otimes_i f(y)\}, \quad \forall x, y \in H \text{ and } i \in \{1, 2, \dots, n\}$$

or in case where \otimes_i is hyperoperation or f is multivalued map we have

$$x\partial_i y = (f(x)\otimes_i y) \cup (x\otimes_i f(y)), \quad \forall x, y \in H \text{ and } i \in \{1, 2, \dots, n\}$$

If \otimes_i is associative then ∂_i is WASS.

Let (G, \cdot) be groupoid, $f_i: G \rightarrow G$, $i \in I$, be a set of maps, then the *union* of the $f_i(x)$ is

$$f_{\cup}: G \rightarrow \mathbf{P}(G) \text{ such that } f_{\cup}(x) = \{f_i(x) \mid i \in I\}$$

The *union theta-operation* (∂) on G is obtained if we consider the $f_{\cup}(x)$. A special case is the union of a map with the identity: $f_{\cup} \equiv f \cup (\text{id})$, so $f_{\cup}(x) = \{x, f(x)\}$, $\forall x \in G$, is called *b-theta-operation*. We denote by (∂) the b-theta-operation, so we have

$$x\partial y = \{xy, f(x) \cdot y, x \cdot f(y)\}, \quad \forall x, y \in G.$$

Motivation for the definition of the theta-operation is the map *derivative* where only the multiplication of functions can be used. Therefore, in these terms, for two functions $s(x)$, $t(x)$, we have $s\partial t = \{s't, st'\}$ where $(')$ denotes the derivative.

Properties 4 [11]. If (G, \cdot) is a semigroup then:

- (a) For every f , the hyperoperation (∂) is WASS.
- (b) For every f , the b-theta-operation (∂) is WASS.
- (c) If f is homomorphism and projection, i.e. $f^2 = f$, then (∂) is associative.

Proof. (a) For all x, y, z in G we have

$$\begin{aligned} (x\partial y)\partial z &= \{f(f(x) \cdot y) \cdot z, f(x) \cdot y \cdot f(z), f(x \cdot f(y)) \cdot z, x \cdot f(y) \cdot f(z)\} \\ x\partial(y\partial z) &= \{f(x) \cdot f(y) \cdot z, x \cdot f(f(y) \cdot z), f(x) \cdot y \cdot f(z), x \cdot f(y \cdot f(z))\} \end{aligned}$$

Therefore $(x\partial y)\partial z \cap x\partial(y\partial z) = \{f(x) \cdot y \cdot f(z)\} \neq \emptyset$, so (∂) is WASS.

(b) and (c) are proved similarly. ■

Properties 5. Reproductivity. If (\cdot) is reproductivity then (∂) is also reproductivity:

$$\begin{aligned} x\partial G &= \cup_{g \in G} \{f(x) \cdot g, x \cdot f(g)\} = \cup_{g \in G} \{f(x) \cdot g\} = G, \\ G\partial x &= \cup_{g \in G} \{f(g) \cdot x, g \cdot f(x)\} = \cup_{g \in G} \{g \cdot f(x)\} = G \end{aligned}$$

Commutativity. If (\cdot) is commutative then (∂) is commutative, if (\cdot) is COW then (∂) is COW.

Unit elements: u is a right unit element if $x\partial u = \{f(x) \cdot u, x \cdot f(u)\} \ni x$. So $f(u) = e$, where e is a unit in (G, \cdot) . The elements of the kernel of f , are the units of (G, ∂) .

Inverse elements: let (G, \cdot) is a monoid with unit e and u be a unit in (G, ∂) , then $f(u) = e$. For given x , the element x' is an inverse with respect to u , if

$$x\partial x' = \{f(x) \cdot x', x \cdot f(x')\} \ni u \text{ and } x'\partial x = \{f(x') \cdot x, x' \cdot f(x)\} \ni u.$$

So, $x' = (f(x))^{-1}u$ and $x' = u(f(x))^{-1}$, are the right and left inverses, respectively. We have two-sided inverses iff $f(x)u = uf(x)$.

Similar properties for multivalued maps are obtained.

Finally, since (∂) is WASS and the reproductivity is valid, we have the following

Proposition 6 [11]. Let (G, \cdot) be group then, for all $f: G \rightarrow G$, the (G, ∂) is a H_v -group.

4. THE SOCIOLINGUISTIC EXPERIMENT

Now we present an experiment on linguistics on which we apply our modelling.

Introduction

The application of the suggested method of modelling was tested in a sociolinguistic experiment conducted in the Greek Department of Democritus University of Thrace, in spring, 2004. We are actually investigating a very interesting aspect of sociolinguistic research, namely the language of young people. More specifically, we want to find out which are some of the most popular and commonly used expressions among students, mainly of the Greek Department but also by other students of different departments, in Komotini [13], during the actual period of the experiment, as well.

Taking into account the fact that about 200 students – mainly female and few male- had to attend the obligatory subject of *Introduction to General Linguistics* during their first year of study, we should focus on two important points: first, all students were still unaware of linguistic research and what exactly or vaguely was sought in the specific experiment-consequently unsuspecting and ideal for this type of research; and second, there was the lure of a possible bonus, i.e. a rewarding higher score in the exams; and this is fair enough, as in any activity of this type there is always a form of reward to motivate the participants. This happy coincidence gave us the opportunity to choose amongst the most interested students and to create a group of willing to learn and conscious subjects who would also act as researchers themselves – although they did not know it at that point. The final selection was made on the basis of an expressed interest and motivation, followed by an introduction made by the tutor roughly explaining the methodology and pointing out the most important parameters. Fairly enough, many students were discouraged at this point and lost interest.

Method

Subjects

A total of 46 students finally participated, selected in the above described procedure and were allocated into nine groups – *teams* - for the needs of present study-of four members each and two teams of five, a total of eleven. All participating students were rewarded with an extra mark on a ten-range scale, provided they would have achieved a pass, i.e. a score higher than 5 out of 10. For example, if they would achieve a 5 out of 10, they would be given 6, if 7, they would be given 8, if 9 out of 10, they would be given 9, a fact that was highly appreciated. We call the subjects *informants-researchers* as they carry both these two properties in the specific piece of research.

Procedure, materials and tasks

The procedure and the tasks to be accomplished fall in the following four-step process:

— 1st step: the teams scattered all over the amphitheatre so as to occupy a certain area well away from the other teams and privacy to be secured. Every single member, i.e.

every informant-researcher, of each team had to decide all by her/himself about the 25 most commonly used expressions by young students of her/his environment and write them down in an order of frequency, starting with the items of the highest frequency, of course in her/his own, personal, subjective opinion.

— 2nd step: after having completed their personal lists, the members of each team, still isolated from the other ten teams, met together, chose a secretary and wrote down a list of every expression each of them had recorded when working alone. The possibilities we could have here ranged from 25 -word lists, if everybody had written the same words, to 100-word lists, if every participants had written totally different words. Both cases were extreme and they did not finally occur, as one should expect.

— 3rd step: still trying not to be overheard or overseen by the other teams, the members of each team have to decide whether the list they have come up with is really representative of the most commonly used expressions by young people of their environment, as well as if the order of frequency corresponds to the reality as they perceive it. This is a very demanding and time consuming process, as they have to conduct a lot of discussion and use arguments supporting their opinions, for example, why their personal classifications vary so much or why the divergence from the final classification is so great, etc. Needless to say, the role of the secretary becomes more demanding and consequently more important in the whole process; furthermore the teacher should go around and support the secretaries without interfering in the discussion, though.

— 4th step: the final task of our informants-researchers was to find five different students from Komotini and ask them to name the five most commonly used expressions by young people. Extra attention should be paid so as that the informants our researchers had chosen had not participated in the experiment before and that they should answer as spontaneously as possible.

Discussion

We finally came up with eleven different classifications compiled by the eleven different teams. In order to exploit as many as possible examples, in present piece of research, we take the first five items from each classification. We do so because we feel that these specific items are the ones most likely to appear after a process of objectification. Hence we numbered these specific items and we have the following results:

$$\begin{aligned} & \mathbf{t}_1(1,2,3,4,5), \quad \mathbf{t}_2(6,7,3,8,9), \quad \mathbf{t}_3(6,10,11,3,12), \quad \mathbf{t}_4(6,13,14,15,16), \\ & \mathbf{t}_5(17,6,13,4,16), \quad \mathbf{t}_6(6,10,11,17,7), \quad \mathbf{t}_7(6,18,16,19,20), \quad \mathbf{t}_8(21,17,1,22,23), \\ & \mathbf{t}_9(6,23,14,8,24), \quad \mathbf{t}_{10}(6,25,26,27,28), \quad \mathbf{t}_{11}(6,29,2,30,20), \end{aligned}$$

where the numbers in parenthesis indicates the ordered expressions.

5. THE MATHEMATICAL MODEL

We present now some examples on our modelling using results of the above experiment from [14] in order to see the variety of the topic by using only theta operations.

Example 7. Let us reduce the number of teams in order to see the models with not a lot of calculations. We exclude teams \mathbf{t}_1 , \mathbf{t}_5 and \mathbf{t}_8 because they do not have the most common expression no 6. Then we exclude \mathbf{t}_{11} because it has no common expression with the other teams and, finally, we exclude teams \mathbf{t}_7 and \mathbf{t}_{10} because they have less

connection with the remaining. Therefore, we renumber the remaining teams into the model of Z_5 as follows

$$\underline{1} \equiv \mathbf{t}_2, \quad \underline{2} \equiv \mathbf{t}_3, \quad \underline{3} \equiv \mathbf{t}_4, \quad \underline{4} \equiv \mathbf{t}_6, \quad \underline{0} \equiv \mathbf{t}_9.$$

So, if we exclude the most common expression we have the following:

$$\underline{0} (23,14,8,24), \underline{1} (7,3,8,9), \underline{2} (10,11,3,12), \underline{3} (13,14,15,16), \underline{4} (10,11,17,7).$$

Let us define the map f as follows:

$$f(\underline{i}) = \{ \underline{j} \mid \text{the element (team) } \underline{j} \neq \underline{i} \text{ and has at least one common expression with } \underline{i} \}$$

More precisely, in our example we have

$$f(\underline{0}) = \{\underline{1}, \underline{3}\}, \quad f(\underline{1}) = \{\underline{0}, \underline{2}, \underline{4}\}, \quad f(\underline{2}) = \{\underline{1}, \underline{4}\}, \quad f(\underline{3}) = \{\underline{0}\}, \quad f(\underline{4}) = \{\underline{1}, \underline{2}\}.$$

Therefore, for example, we have

$$\underline{2} \partial \underline{3} = (f(\underline{2}) + \underline{3}) \cup (\underline{2} + f(\underline{3})) = (\{\underline{1}, \underline{4}\} + \underline{3}) \cup (\underline{2} + \{\underline{0}\}) = \{\underline{2}, \underline{4}\},$$

and so on. Taking into account that $(+)$ is commutative then (∂) is also commutative we obtain the following table

∂	<u>0</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
<u>0</u>	<u>1,3</u>	<u>0,2,4</u>	<u>0,1,3,4</u>	<u>0,1,4</u>	<u>0,1,2</u>
<u>1</u>	<u>0,2,4</u>	<u>0,1,3</u>	<u>0,1,2,4</u>	<u>0,1,2,3</u>	<u>1,2,3,4</u>
<u>2</u>	<u>0,1,3,4</u>	<u>0,1,2,4</u>	<u>1,3</u>	<u>2,4</u>	<u>0,3,4</u>
<u>3</u>	<u>0,1,4</u>	<u>0,1,2,3</u>	<u>2,4</u>	<u>3</u>	<u>0,4</u>
<u>4</u>	<u>0,1,2</u>	<u>1,2,3,4</u>	<u>0,3,4</u>	<u>0,4</u>	<u>0,1</u>

From this table we obtain that the hyperstructure (Z_5, ∂) is an abelian H_v -group where the element 1 is a two-sided unit. The (Z_5, ∂) is cyclic where the elements 0,1,2 and 4 are generators with period 3. The (Z_5, ∂) is a single-power cyclic where the elements 0,1,2 and 4 are generators with period 4, 3, 4 and 3 respectively. Finally, the element 3 is an idempotent.

Example 8 [14]. In order to specify the teams under modelling one can use the experience of the researcher on the way the teams worked.

Different correspondence in Z_8 gives other hyperstructures, sometimes more interesting in hyperstructure theory. For example, we can exclude the teams 1 and 8, since they do not include the most common expression and then the team 10 because this team do not have any common expressions with the others. The remaining 8 teams have the following four, except the common expression:

$$\mathbf{t}_2(7,3,8,9), \quad \mathbf{t}_3(10,11,3,12), \quad \mathbf{t}_4(13,14,15,16), \quad \mathbf{t}_5(17,13,4,16), \\ \mathbf{t}_6(10,11,17,7), \quad \mathbf{t}_7(18,16,19,20), \quad \mathbf{t}_9(23,14,8,24), \quad \mathbf{t}_{11}(29,2,30,20).$$

We define the following algorithm to obtain the map:

$$f(\mathbf{t}_i) = \{ \mathbf{t}_j \mid \mathbf{t}_j \neq \mathbf{t}_i \text{ and has the first expression of } \mathbf{t}_i \}.$$

If this set is empty then we take the second expression of \mathbf{t}_i and so on}

Therefore we have

$$f(\mathbf{t}_2)=\{\mathbf{t}_6\}, \quad f(\mathbf{t}_3)=\{\mathbf{t}_6\}, \quad f(\mathbf{t}_4)=\{\mathbf{t}_5\}, \quad f(\mathbf{t}_5)=\{\mathbf{t}_6\}, \\ f(\mathbf{t}_6)=\{\mathbf{t}_3\}, \quad f(\mathbf{t}_7)=\{\mathbf{t}_4, \mathbf{t}_5\}, \quad f(\mathbf{t}_9)=\{\mathbf{t}_4\}, \quad f(\mathbf{t}_{11})=\{\mathbf{t}_7\}.$$

The theta operation (∂) is more interesting if there exists a unit element, we can select, for example, \mathbf{t}_6 to be the element $\underline{0}$ of (\mathbf{Z}_8, ∂) , because it appears as a singleton in the above f three times.

Finally, we renumber the teams in the model of \mathbf{Z}_8 as follows

$$\underline{0} \equiv \mathbf{t}_6, \quad \underline{1} \equiv \mathbf{t}_2, \quad \underline{2} \equiv \mathbf{t}_3, \quad \underline{3} \equiv \mathbf{t}_4, \quad \underline{4} \equiv \mathbf{t}_5, \quad \underline{5} \equiv \mathbf{t}_7, \quad \underline{6} \equiv \mathbf{t}_9, \quad \underline{7} \equiv \mathbf{t}_{11}.$$

Or in the new enumeration in \mathbf{Z}_8 we have

$$\underline{0} (10,11,17,7), \quad \underline{1} (7,3,8,9), \quad \underline{2} (10,11,3,12), \quad \underline{3} (13,14,15,16), \\ \underline{4} (17,13,4,16), \quad \underline{5} (18,16,19,20), \quad \underline{6} (23,14,8,24), \quad \underline{7} (29,2,30,20),$$

Therefore, we have the following:

$$f(\underline{0}) = \{\underline{2}\}, \quad f(\underline{1}) = \{\underline{0}\}, \quad f(\underline{2}) = \{\underline{0}\}, \quad f(\underline{3}) = \{\underline{4}\}, \\ f(\underline{4}) = \{\underline{0}\}, \quad f(\underline{5}) = \{\underline{3}, \underline{4}\}, \quad f(\underline{6}) = \{\underline{3}\}, \quad f(\underline{7}) = \{\underline{5}\}.$$

and the following table is obtained

∂	<u>0</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>
<u>0</u>	<u>2</u>	<u>0,3</u>	<u>0,4</u>	<u>4,5</u>	<u>0,6</u>	<u>3,4,7</u>	<u>0,3</u>	<u>1,5</u>
<u>1</u>	<u>0,3</u>	<u>1</u>	<u>1,2</u>	<u>3,5</u>	<u>1,4</u>	<u>4,5</u>	<u>4,6</u>	<u>1,7</u>
<u>2</u>	<u>0,4</u>	<u>1,2</u>	<u>2</u>	<u>3,6</u>	<u>2,4</u>	<u>5,6</u>	<u>5,6</u>	<u>7</u>
<u>3</u>	<u>4,5</u>	<u>3,5</u>	<u>3,6</u>	<u>7</u>	<u>0,3</u>	<u>1,6,7</u>	<u>2,6</u>	<u>0,3</u>
<u>4</u>	<u>0,6</u>	<u>1,4</u>	<u>2,4</u>	<u>0,3</u>	<u>4</u>	<u>0,5,7</u>	<u>6,7</u>	<u>1,7</u>
<u>5</u>	<u>3,4,7</u>	<u>4,5</u>	<u>5,6</u>	<u>1,6,7</u>	<u>0,5,7</u>	<u>0,1</u>	<u>0,1,2</u>	<u>2,3</u>
<u>6</u>	<u>0,3</u>	<u>4,6</u>	<u>5,6</u>	<u>2,6</u>	<u>6,7</u>	<u>0,1,2</u>	<u>1</u>	<u>2,3</u>
<u>7</u>	<u>1,5</u>	<u>1,7</u>	<u>7</u>	<u>0,3</u>	<u>1,7</u>	<u>2,3</u>	<u>2,3</u>	<u>4</u>

From this table we obtain that the hyperstructure (\mathbf{Z}_8, ∂) is an abelian H_v -group where there are three unit elements, the elements 1, 2, and 4. We can obtain, after some calculations, that the (\mathbf{Z}_8, ∂) is cyclic where the elements 0, 3, 5 and 6 are generators with period 8, 6, 4 and 7, respectively. The (\mathbf{Z}_8, ∂) is a single-power cyclic where the elements 0, 3, 5 and 6 are generators with period 8, 6, 5 and 7, respectively.

6. THE SECOND EXPERIMENT

Two years later we applied the lists obtained from our experiment to a new group of informants. This time we chose academics, all doing teaching and research in the Department of Greek, aged ± 40 , mainly females and a few males. Their task was to put a tick next to each phrase of the list they themselves also used in production. The process was actually very quick and spontaneous in order to achieve a good degree of objectivity. They all enjoyed the test, some asked to take it with them to answer taking

their time but they were not allowed to. Certain phrases are defined as ‘not used’ if the number of the choices is according to the Golden Ratio either in the normal ratio ≈ 0.618 , or the compliment ≈ 0.382 .

Example 9. For the second experiment using the *Golden Ratio*, we obtain that the expressions used by the young people but not by the older- still young enough- are the expressions 7,9,13,14,16,18,19,21,24,28. The teams which have at least one of the above expressions are

$$t_2(7,9), t_4(13,14,16), t_5(13,16), t_6(7), t_7(16,18,19), t_8(21), t_9(14,24), t_{10}(28).$$

The teams t_8, t_{10} do not have any connection with the rest, therefore in $(Z_6,+)$ we have

$$\underline{0} \equiv t_2(7,9), \underline{1} \equiv t_4(13,14,16), \underline{2} \equiv t_5(13,16), \underline{3} \equiv t_6(7), \underline{4} \equiv t_7(16,18,19), \underline{5} \equiv t_9(14,24).$$

Let us define the map f as follows:

$$f(i) = \{ j \mid \text{the team } j \neq i \text{ and has at least one common expression with } i \}$$

Thus

$$f(\underline{0}) = \{\underline{3}\}, f(\underline{1}) = \{\underline{2}, \underline{4}, \underline{5}\}, f(\underline{2}) = \{\underline{1}, \underline{4}\}, f(\underline{3}) = \{\underline{0}\}, f(\underline{4}) = \{\underline{1}, \underline{2}\}, f(\underline{5}) = \{\underline{1}\}.$$

Therefore, for example, we have

$$\underline{2} \oplus \underline{4} = (f(\underline{2}) + \underline{4}) \cup (\underline{2} + f(\underline{4})) = (\{\underline{1}, \underline{4}\} + \underline{4}) \cup (\underline{2} + \{\underline{1}, \underline{2}\}) = \{\underline{2}, \underline{3}, \underline{4}, \underline{5}\},$$

and so on. Therefore, we obtain the following table

∂	<u>0</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
<u>0</u>	<u>3</u>	<u>2,4,5</u>	<u>1,4,5</u>	<u>0</u>	<u>1,2</u>	<u>1,2</u>
<u>1</u>	<u>2,4,5</u>	<u>0,3,5</u>	<u>0,1,2,4,5</u>	<u>1,2,5</u>	<u>0,2,3</u>	<u>1,2,3,4</u>
<u>2</u>	<u>1,4,5</u>	<u>0,1,2,4,5</u>	<u>0,3</u>	<u>1,2,4</u>	<u>2,3,4,5</u>	<u>0,3</u>
<u>3</u>	<u>0</u>	<u>1,2,5</u>	<u>1,2,4</u>	<u>3</u>	<u>4,5</u>	<u>4,5</u>
<u>4</u>	<u>1,2</u>	<u>0,2,3</u>	<u>2,3,4,5</u>	<u>4,5</u>	<u>0,5</u>	<u>0,1,5</u>
<u>5</u>	<u>1,2</u>	<u>1,2,3,4</u>	<u>0,3</u>	<u>4,5</u>	<u>0,1,5</u>	<u>0</u>

From the above table, after some calculations, we obtain the following:

- The element 3 is the only one unit element.
- The set $\{\underline{0}, \underline{3}\}$ is a subgroup of the ∂ -group (Z_6, ∂) .
- The (Z_6, ∂) is a cyclic H_V -group where the elements 1, 2, 4, 5 are generators with period, respectively 3, 3, 4, 5 and single-power cyclic where the elements 1, 2, 4, 5 are single-power generators with period 4, 4, 4, 5, respectively.

Example 10. Using the second experiment again we consider the expressions according to the compliment of the *golden ratio* on the answers. The expressions used by the teams are

$$t_1(\text{non}), t_2(6,7,9), t_3(6,10), t_4(6,13,14,16), t_5(13,16), t_6(6,7,10), \\ t_7(6,16,18,19,20), t_8(21), t_9(6,14,24), t_{10}(6,28), t_{11}(6,20).$$

Now, we do not consider the most common expression (6), and consequently teams t_8 and t_{10} are excluded. Then we rename the teams as follows:

$$\underline{0} \equiv \mathbf{t}_2(7,9), \quad \underline{1} \equiv \mathbf{t}_3(10), \quad \underline{2} \equiv \mathbf{t}_4(13,14,16), \quad \underline{3} \equiv \mathbf{t}_5(13,16),$$

$$\underline{4} \equiv \mathbf{t}_6(7,10), \quad \underline{5} \equiv \mathbf{t}_7(16,18,19,20), \quad \underline{6} \equiv \mathbf{t}_9(14,24), \quad \underline{7} \equiv \mathbf{t}_{11}(20).$$

Let us define the map f as follows:

$$f(i) = \{ j \mid \text{the team } j \neq i \text{ and has at least one common expression with } i \}$$

Thus

$$f(\underline{0}) = \{\underline{4}\}, \quad f(\underline{1}) = \{\underline{4}\}, \quad f(\underline{2}) = \{\underline{3}, \underline{5}, \underline{6}\}, \quad f(\underline{3}) = \{\underline{2}, \underline{5}\},$$

$$f(\underline{4}) = \{\underline{0}, \underline{1}\}, \quad f(\underline{5}) = \{\underline{2}, \underline{3}, \underline{7}\}, \quad f(\underline{6}) = \{\underline{2}\}, \quad f(\underline{7}) = \{\underline{5}\}.$$

Then we obtain the following table

∂	<u>0</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>
<u>0</u>	4	4,5	3,5,6	2,5,7	0,1	1,2,3,7	2	3,5
<u>1</u>	4,5	5	4,6,7	3,6,7	0,1,2	0,1,3,4	2,3	3,6
<u>2</u>	3,5,6	4,6,7	0,5,7	0,1,4,6,7	1,2,3,7	0,1,2,3,4,6	1,3,4	2,4,5,7
<u>3</u>	2,5,7	3,6,7	0,1,4,6,7	0,5	1,3,4,6	2,5,6,7	0,3,5	0,1,4
<u>4</u>	0,1	0,1,2	1,2,3,7	1,3,4,6	4,5	3,5,6,7	6,7	0,1,7
<u>5</u>	1,2,3,7	0,1,3,4	0,1,2,3,4,6	0,5,6,7	3,5,6,7	0,4,7	0,1,5,7	1,2,6
<u>6</u>	2	2,3	1,3,4	0,3,5	6,7	0,1,5,7	0	1,3
<u>7</u>	3,5	3,6	2,4,5,7	0,1,4	0,1,7	1,2,6	1,3	4

From the above table, we obtain the following:

- (a) The element 4 is the only one unit element.
- (b) The (\mathbf{Z}_8, ∂) is a cyclic H_v -group where all its elements: 0, 1, 2, 3, 4, 5, 6, 7, are generators with period, respectively 6, 4, 3, 4, 5, 3, 5, 5 and single-power cyclic, except the element 0, with period 5, 3, 4, 5, 4, 7, 6, respectively.

7. CONCLUSIONS

The above modelling provides a game between hyperstructures and applications, between H_v -structures and linguistics. This modelling gives on the one side new H_v -structures and on the other side it provides an organizing devise on applied research which is expected to result in boosting the interest of both the teachers and learners in the classroom environment.

On the algebraic point of view, on hyperstructures, we can see, for example that there are H_v -groups, which have unit elements, which are generators (see example 9).

It is of great interest, from the language teaching or linguistic research point of view, to examine whether the special elements such as the units, the generators or properties as associativity, cyclicity are due to the special action of the teams and the relations they have between them. Therefore, one can see why the elements 0 and 3 form a subgroup or what the special behaviour of the teams \mathbf{t}_2 and \mathbf{t}_6 is; nevertheless such an issue concerns mainly sociolinguistic research.

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