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AN EXTENSION OF MULHOLLAND'S INEQUALITY

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Abstract. By introducing multi-parameters and using the way of weight coefficients and Hadamard's inequality, a more accurate extension of Mulholland's inequality and the equivalent form are considered.

1. Introduction

If $p > 1, \frac{1}{p} + \frac{1}{q} = 1, a_m, b_n \geq 0, 0 < \sum_{m=2}^{\infty} m^{p-1} a_m^p < \infty$ and $0 < \sum_{n=2}^{\infty} n^{p-1} b_n^q < \infty$, then we have the following Mulholland's inequality [1]:

$$\sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{\ln mn} < \frac{\pi}{\sin(\pi/p)} \{\sum_{m=2}^{\infty} m^{p-1} a_m^p\}^{\frac{1}{p}} \{\sum_{n=2}^{\infty} n^{q-1} b_n^q\}^{\frac{1}{q}}, \tag{1.1}$$

where the constant factor $\frac{\pi}{\sin(\pi/p)}$ is the best possible. In 2007, by assuming that $\alpha \geq e^{7/6}, 2 - \min\{p, q\} < \lambda \leq 2$, Yang [2] gave a more accurate Hilbert-type inequality similar to (1.1) as:

$$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{a_m b_n}{\ln^{\lambda} \alpha mn} < k_{\lambda}(p) \{\sum_{m=1}^{\infty} \frac{m^{p-1} a_m^p}{\ln^{\lambda-1} \sqrt{\alpha} m}\}^{\frac{1}{p}} \{\sum_{n=1}^{\infty} \frac{n^{q-1} b_n^q}{\ln^{\lambda-1} \sqrt{\alpha} n}\}^{\frac{1}{q}}, \tag{1.2}$$

where $k_{\lambda}(p) = B(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q})$ is the best possible and $B(u, v) = \int_0^{\infty} \frac{t^{u-1} dt}{(1+t)^{u+v}}$ $(u, v > 0)$ is the Beta function(cf. [3]). There are some publishing results about more

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accurate Hilbert-type inequalities in [4], [5], [6] and [7]. In this paper, by introducing multi-parameters and using the way of weight coefficients and Hadamard's inequality, a more accurate extension of (1.1) and the equivalent form are given. That is

Theorem 1.1. *If* $p, r > 1, \frac{1}{p} + \frac{1}{q} = \frac{1}{r} + \frac{1}{s} = 1, 0 < \gamma \leq 1, 0 < \gamma\lambda \leq \min\{r, s\}$, $\beta, \alpha \geq -\frac{1}{2}, a_m, b_n \geq 0$, *such that*

$$0 < \sum_{m=2}^{\infty} \frac{(m+\alpha)^{p-1} a_m^p}{[\ln(m+\alpha)]^{p(\frac{\gamma\lambda}{r}-\frac{1}{q})}} < \infty, 0 < \sum_{n=2}^{\infty} \frac{(n+\beta)^{q-1} b_n^q}{[\ln(n+\beta)]^{q(\frac{\gamma\lambda}{s}-\frac{1}{p})}} < \infty,$$

then we have the following equivalent inequalities:

$$\begin{aligned} I &: = \sum_{n=2}^{\infty}\sum_{m=2}^{\infty} \frac{a_m b_n}{[\ln^{\gamma}(m+\alpha) + \ln^{\gamma}(n+\beta)]^{\lambda}} < \frac{1}{\gamma} B(\frac{\lambda}{r}, \frac{\lambda}{s}) \\ &\times\{\sum_{m=2}^{\infty} \frac{(m+\alpha)^{p-1} a_m^p}{[\ln(m+\alpha)]^{p(\frac{\gamma\lambda}{r}-\frac{1}{q})}}\}^{\frac{1}{p}} \{\sum_{n=2}^{\infty} \frac{(n+\beta)^{q-1} b_n^q}{[\ln(n+\beta)]^{q(\frac{\gamma\lambda}{s}-\frac{1}{p})}}\}^{\frac{1}{q}}, \end{aligned} \tag{1.3}$$

$$\begin{aligned} J &: = \sum_{n=2}^{\infty} \frac{\ln(n+\beta)]^{(p\frac{\gamma\lambda}{s}-1)}}{n+\beta} \left[\sum_{m=2}^{\infty} \frac{a_m}{[\ln^{\gamma}(m+\alpha) + \ln^{\gamma}(n+\beta)]^{\lambda}}\right]^p \\ &< \left[\frac{1}{\gamma} B(\frac{\lambda}{r}, \frac{\lambda}{s})\right]^p \sum_{m=2}^{\infty} \frac{(m+\alpha)^{p-1}}{[\ln(m+\alpha)]^{p(\frac{\gamma\lambda}{r}-\frac{1}{q})}} a_m^p, \end{aligned} \tag{1.4}$$

where the constant factors $\frac{1}{\gamma} B(\frac{\lambda}{r}, \frac{\lambda}{s})$ *and* $\left[\frac{1}{\gamma} B(\frac{\lambda}{r}, \frac{\lambda}{s})\right]^p$ *are the best possible.*

Remark 1. For $\gamma = \lambda = 1, \alpha = \beta = 0$, $r = q, s = p$ in (1.3), we have (1.1). Hence inequality (1.3) is a more accurate extension of (1.1).

2. **Some Lemmas**

Lemma 2.1. *If* $r > 1, \frac{1}{r} + \frac{1}{s} = 1, 0 < \gamma \leq 1, 0 < \gamma\lambda \leq \min\{r, s\}, \beta, \alpha \geq -\frac{1}{2}$, *define the following weight coefficient:*

$$\omega_m(s) := \sum_{n=2}^{\infty} \frac{[\ln(m+\alpha)]^{\gamma\lambda/r}[\ln(n+\beta)]^{(\gamma\lambda/s)-1}}{[\ln^{\gamma}(m+\alpha) + \ln^{\gamma}(n+\beta)]^{\lambda}(n+\beta)}, \tag{2.1}$$

$$\varpi_n(r) := \sum_{m=2}^{\infty} \frac{[\ln(n+\beta)]^{\gamma\lambda/s}[\ln(m+\alpha)]^{(\gamma\lambda/r)-1}}{[\ln^{\gamma}(m+\alpha) + \ln^{\gamma}(n+\beta)]^{\lambda}(m+\alpha)} (m, n \in \mathbf{N}\backslash\{1\}). \tag{2.2}$$

Then we have inequalities $\varpi_n(r) < \frac{1}{\gamma}B(\frac{\lambda}{r}, \frac{\lambda}{s})$ *and*

$$0 < \frac{1}{\gamma}B(\frac{\lambda}{r}, \frac{\lambda}{s})[1 - O([\frac{1}{\ln(m+\alpha)}]^{\frac{\gamma\lambda}{s}})] < \omega_m(s) < \frac{1}{\gamma}B(\frac{\lambda}{r}, \frac{\lambda}{s}). \tag{2.3}$$

Proof. Define the function $f(y)$ as

$$f(y) := \frac{[\ln(m+\alpha)]^{\gamma\lambda/r}[\ln(y+\beta)]^{(\gamma\lambda/s)-1}}{[\ln^{\gamma}(m+\alpha) + \ln^{\gamma}(y+\beta)]^{\lambda}(y+\beta)} (y > \frac{3}{2}).$$

Setting $u = [\frac{\ln(y+\beta)}{\ln(m+\alpha)}]^{\gamma}$, since $\beta \geq -\frac{1}{2}$, we find

$$\int_{\frac{3}{2}}^{\infty} f(y)dy = \int_{[\frac{\ln(\frac{3}{2}+\beta)}{\ln(m+\alpha)}]^{\gamma}}^{\infty} \frac{u^{(\lambda/s)-1}du}{\gamma(1+u)^{\lambda}} \leq \frac{1}{\gamma}\int_{0}^{\infty} \frac{u^{(\lambda/s)-1}du}{(1+u)^{\lambda}} = \frac{1}{\gamma}B(\frac{\lambda}{r}, \frac{\lambda}{s}). \tag{2.4}$$

It is obvious that $f'(y) < 0, f''(y) > 0 (y > \frac{3}{2})$. By Hadamard's inequality (cf.[8]), it follows that $f(n) < \int_{n-\frac{1}{2}}^{n+\frac{1}{2}} f(y)dy (n \in \mathbf{N}\backslash\{1\})$ and then

$$\omega_m(s) = \sum_{n=2}^{\infty} f(n) < \int_{\frac{3}{2}}^{\infty} f(y)dy \leq \frac{1}{\gamma}B(\frac{\lambda}{r}, \frac{\lambda}{s}). \tag{2.5}$$

By the same way, $\varpi_n(r) < \frac{1}{\gamma}B(\frac{\lambda}{r}, \frac{\lambda}{s})$. Since $f(y)$ is strictly decreasing, then

$$\begin{aligned}
\omega_m(s) &= \sum_{n=2}^{\infty} f(n) > \sum_{n=2}^{\infty} \int_n^{n+1} f(y)dy = \int_2^{\infty} f(y)dy \\
&= \frac{1}{\gamma} \int_{[\frac{\ln(2+\beta)}{\ln(m+\alpha)}]^{\gamma}}^{\infty} \frac{u^{(\lambda/s)-1}du}{(1+u)^{\lambda}} = \frac{1}{\gamma} B(\frac{\lambda}{r}, \frac{\lambda}{s})[1-\theta_m(s)] > 0, \quad (2.6)\\
0 &< \theta_m(s) := [B(\frac{\lambda}{r}, \frac{\lambda}{s})]^{-1} \int_0^{[\frac{\ln(2+\beta)}{\ln(m+\alpha)}]^{\gamma}} \frac{u^{(\lambda/s)-1}du}{(1+u)^{\lambda}} \\
&\le \frac{1}{B(\frac{\lambda}{r}, \frac{\lambda}{s})} \int_0^{[\frac{\ln(2+\beta)}{\ln(m+\alpha)}]^{\gamma}} u^{\frac{\lambda}{s}-1} du = \frac{s}{\lambda B(\frac{\lambda}{r}, \frac{\lambda}{s})} [\frac{\ln(2+\beta)}{\ln(m+\alpha)}]^{\frac{\gamma\lambda}{s}}, \quad (2.7)
\end{aligned}$$

and then $\theta_m(s) = O([\frac{1}{\ln(m+\alpha)}]^{\frac{\gamma\lambda}{s}})$. □

Lemma 2.2. *As the assumption of Lemma 1, if* $p > 1$, $\frac{1}{p} + \frac{1}{q} = 1, a_m, b_n \ge 0$, *$J$ is indicated by (1.4), then*

$$J \le \left[\frac{1}{\gamma} B(\frac{\lambda}{r}, \frac{\lambda}{s})\right]^{p-1} \sum_{m=2}^{\infty} \omega_m(s) \frac{(m+\alpha)^{p-1}}{[\ln(m+\alpha)]^{p(\frac{\gamma\lambda}{r}-\frac{1}{q})}} a_m^p. \quad (2.8)$$

Proof. By Hölder's inequality (cf. [8]) and Lemma 2.1, we find

$$\left[\sum_{m=2}^{\infty} \frac{a_m}{[\ln^{\gamma}(m+\alpha) + \ln^{\gamma}(n+\beta)]^{\lambda}}\right]^p = \left\{\sum_{m=2}^{\infty} \frac{1}{[\ln^{\gamma}(m+\alpha) + \ln^{\gamma}(n+\beta)]^{\lambda}}\right.$$

$$\left.\times \left[\frac{[\ln(m+\alpha)]^{(1-\frac{\gamma\lambda}{r})/q} (m+\alpha)^{\frac{1}{q}} a_m}{[\ln(n+\beta)]^{(1-\frac{\gamma\lambda}{s})/p} (n+\beta)^{\frac{1}{p}}}\right] \left[\frac{[\ln(n+\beta)]^{(1-\frac{\gamma\lambda}{s})/p} (n+\beta)^{\frac{1}{p}}}{[\ln(m+\alpha)]^{(1-\frac{\gamma\lambda}{r})/q} (m+\alpha)^{\frac{1}{q}}}\right]\right\}^p$$

$$\begin{aligned}
&\leq \sum_{m=2}^{\infty} \frac{a_m^p}{[\ln^\gamma(m+\alpha)+\ln^\gamma(n+\beta)]^\lambda} \frac{[\ln(m+\alpha)]^{(1-\frac{\gamma\lambda}{r})p/q}(m+\alpha)^{\frac{p}{q}}}{[\ln(n+\beta)]^{(1-\frac{\gamma\lambda}{s})}(n+\beta)} \\
&\quad \times \left\{ \sum_{m=2}^{\infty} \frac{1}{[\ln^\gamma(m+\alpha)+\ln^\gamma(n+\beta)]^\lambda} \frac{[\ln(n+\beta)]^{(1-\frac{\gamma\lambda}{s})\frac{q}{p}}(n+\beta)^{\frac{q}{p}}}{[\ln(m+\alpha)]^{(1-\frac{\gamma\lambda}{r})}(m+\alpha)} \right\}^{p-1} \\
&\leq \frac{(n+\beta)}{[\ln(m+\alpha)]^{(p\frac{\gamma\lambda}{s}-1)}}[\frac{1}{\gamma}B(\frac{\lambda}{r},\frac{\lambda}{s})]^{p-1} \\
&\quad \times \sum_{m=2}^{\infty} \frac{(m+\alpha)^{p-1}a_m^p}{[\ln^\gamma(m+\alpha)+\ln^\gamma(n+\beta)]^\lambda} \frac{[\ln(m+\alpha)]^{(1-\frac{\gamma\lambda}{r})(p-1)}}{[\ln(n+\beta)]^{(1-\frac{\gamma\lambda}{s})}(n+\beta)},
\end{aligned}$$

$$\begin{aligned}
J &\leq [\frac{1}{\gamma}B(\frac{\lambda}{r},\frac{\lambda}{s})]^{p-1} \\
&\quad \times \sum_{m=2}^{\infty}\sum_{n=2}^{\infty} \frac{(m+\alpha)^{p-1}a_m^p}{[\ln^\gamma(m+\alpha)+\ln^\gamma(n+\beta)]^\lambda} \frac{[\ln(m+\alpha)]^{(1-\frac{\gamma\lambda}{r})(p-1)}}{[\ln(n+\beta)]^{(1-\frac{\gamma\lambda}{s})}(n+\beta)} \\
&= \left[\frac{1}{\gamma}B(\frac{\lambda}{r},\frac{\lambda}{s})\right]^{p-1} \left\{ \sum_{m=2}^{\infty} \omega_m(s) \frac{(m+\alpha)^{p-1}a_m^p}{[\ln(m+\alpha)]^{p(\frac{\gamma\lambda}{r}-\frac{1}{q})}} \right\}.
\end{aligned}$$

Hence the lemma is proved. □

3. **Proof of Theorem 1.1**

Proof of Theorem 1.1. In view of (2.8) and (2.3), we have (1.4). By Hölder's inequality, we find

$$\begin{aligned}
I &= \sum_{n=2}^{\infty} \left[\frac{[\ln(n+\beta)]^{(\frac{-1}{p}+\frac{\gamma\lambda}{s})}}{(n+\beta)^{1/p}} \sum_{m=2}^{\infty} \frac{a_m}{[\ln^\gamma(m+\alpha)+\ln^\gamma(n+\beta)]^\lambda} \right] \\
&\quad \times \left[\frac{(n+\beta)^{1/p} b_n}{[\ln(n+\beta)]^{(\frac{-1}{p}+\frac{\gamma\lambda}{s})}} \right] \leq J^{\frac{1}{p}} \{ \sum_{n=2}^{\infty} \frac{(n+\beta)^{q-1}}{[\ln(n+\beta)]^{q(\frac{\gamma\lambda}{s}-\frac{1}{p})}} b_n^q \}^{\frac{1}{q}}. \qquad (3.1)
\end{aligned}$$

By (1.4), we have (1.3). On the other hand, suppose that (1.3) is valid. Setting

$$b_n := \frac{\ln(n+\beta)]^{(p\frac{\gamma\lambda}{s}-1)}}{n+\beta} \left[\sum_{m=2}^{\infty} \frac{a_m}{[\ln^\gamma(m+\alpha)+\ln^\gamma(n+\beta)]^\lambda} \right]^{p-1},$$

then $J=\sum_{n=2}^{\infty}\frac{(n+\beta)^{q-1}}{[\ln(n+\beta)]^{q(\frac{\gamma\lambda}{s}-\frac{1}{p})}}b_n^q$. By (2.8), $J<\infty$. If $J=0$, then (1.4) is naturally valid; if $J>0$, then by (1.3), we obtain

$$0<\sum_{n=2}^{\infty}\frac{(n+\beta)^{q-1}}{[\ln(n+\beta)]^{q(\frac{\gamma\lambda}{s}-\frac{1}{p})}}b_n^q=J=I<\frac{1}{\gamma}B(\frac{\lambda}{r},\frac{\lambda}{s})$$
$$\times\{\sum_{m=2}^{\infty}\frac{(m+\alpha)^{p-1}}{[\ln(m+\alpha)]^{p(\frac{\gamma\lambda}{r}-\frac{1}{q})}}a_m^p\}^{\frac{1}{p}}\{\sum_{n=2}^{\infty}\frac{(n+\beta)^{q-1}}{[\ln(n+\beta)]^{q(\frac{\gamma\lambda}{s}-\frac{1}{p})}}b_n^q\}^{\frac{1}{q}}<\infty,$$
$$\sum_{n=2}^{\infty}\frac{(n+\beta)^{q-1}b_n^q}{[\ln(n+\beta)]^{q(\frac{\gamma\lambda}{s}-\frac{1}{p})}}=J<[\frac{1}{\gamma}B(\frac{\lambda}{r},\frac{\lambda}{s})]^p\sum_{m=2}^{\infty}\frac{(m+\alpha)^{p-1}a_m^p}{[\ln(m+\alpha)]^{p(\frac{\gamma\lambda}{r}-\frac{1}{q})}}.$$

Hence (1.4) is valid, which is equivalent to (1.3). Without loss of generality, suppose that $\alpha\le\beta$. For $0<\varepsilon<q\frac{\gamma\lambda}{s}$, setting $\widetilde{a}_m,\widetilde{b}_n$ as: $\widetilde{a}_m=\frac{[\ln(m+\alpha)]^{\frac{\gamma\lambda}{r}-\frac{\varepsilon}{p}-1}}{m+\alpha}$, $\widetilde{b}_n=\frac{[\ln(n+\beta)]^{\frac{\gamma\lambda}{s}-\frac{\varepsilon}{q}-1}}{n+\beta}$ and $S=(\frac{1}{s}-\frac{\varepsilon}{q\gamma\lambda})^{-1}>0, R=(\frac{1}{r}+\frac{\varepsilon}{q\gamma\lambda})^{-1}>1$, we have by (2.3),

$$\begin{aligned}\widetilde{I}\ :&=\sum_{n=2}^{\infty}\sum_{m=2}^{\infty}\frac{\widetilde{a}_m\widetilde{b}_n}{[\ln^{\gamma}(m+\alpha)+\ln^{\gamma}(n+\beta)]^{\lambda}}=\sum_{m=2}^{\infty}\frac{\omega_m(S)}{[\ln(m+\alpha)]^{1+\varepsilon}(m+\alpha)}\\
&\ge\frac{1}{\gamma}B(\frac{\lambda}{R},\frac{\lambda}{S})\sum_{m=2}^{\infty}\frac{1}{[\ln(m+\alpha)]^{1+\varepsilon}(m+\alpha)}[1-O(\frac{1}{[\ln(m+\alpha)]^{\gamma\lambda/S}})]\\
&=\frac{1}{\gamma}B(\frac{\lambda}{R},\frac{\lambda}{S})[\sum_{m=2}^{\infty}\frac{[\ln(m+\alpha)]^{-1-\varepsilon}}{(m+\alpha)}-\sum_{m=2}^{\infty}O(\frac{[\ln(m+\alpha)]^{-1-(\gamma\lambda/S)-\varepsilon}}{(m+\alpha)})]\\
&=\frac{1}{\gamma}B(\frac{\lambda}{R},\frac{\lambda}{S})\sum_{m=2}^{\infty}\frac{[\ln(m+\alpha)]^{-1-\varepsilon}}{(m+\alpha)}\end{aligned}$$
$$\times\{1-[\sum_{m=2}^{\infty}\frac{[\ln(m+\alpha)]^{-1-\varepsilon}}{(m+\alpha)}]^{-1}\sum_{m=2}^{\infty}O(\frac{[\ln(m+\alpha)]^{-1-(\gamma\lambda/S)-\varepsilon}}{(m+\alpha)})\}. \tag{3.2}$$
$$\widetilde{H}\ :=\{\sum_{m=2}^{\infty}\frac{(m+\alpha)^{p-1}\widetilde{a}_m^p}{[\ln(m+\alpha)]^{p(\frac{\gamma\lambda}{r}-\frac{1}{q})}}\}^{\frac{1}{p}}\{\sum_{n=2}^{\infty}\frac{(n+\beta)^{q-1}\widetilde{b}_n^q}{[\ln(n+\beta)]^{q(\frac{\gamma\lambda}{s}-\frac{1}{p})}}\}^{\frac{1}{q}}$$

$$=\{\sum_{m=2}^{\infty}\frac{[\ln(m+\alpha)]^{-\varepsilon-1}}{m+\alpha}\}^{\frac{1}{p}}\{\sum_{n=2}^{\infty}\frac{[\ln(n+\beta)]^{-\varepsilon-1}}{n+\beta}\}^{\frac{1}{q}}\le\sum_{m=2}^{\infty}\frac{[\ln(m+\alpha)]^{-\varepsilon-1}}{m+\alpha}. \tag{3.3}$$

If there exists a constant $0 < k \leq \frac{1}{\gamma}B(\frac{\lambda}{r}, \frac{\lambda}{s})$, such that (1.3) is valid as we replace $\frac{1}{\gamma}B(\frac{\lambda}{r}, \frac{\lambda}{s})$ by k, then $\widetilde{I} < k\widetilde{H}$, and by (3.2) and (3.3), it follows that

$$\frac{1}{\gamma}B(\frac{\lambda}{R}, \frac{\lambda}{S})\{1 - [\sum_{m=2}^{\infty} \frac{[\ln(m+\alpha)]^{-1-\varepsilon}}{(m+\alpha)}]^{-1} \sum_{m=2}^{\infty} O(\frac{[\ln(m+\alpha)]^{-1-\frac{\gamma\lambda}{S}-\varepsilon}}{(m+\alpha)})\} < k.$$

Then for $\varepsilon \to 0^+$, $\frac{1}{\gamma}B(\frac{\lambda}{r}, \frac{\lambda}{s}) \leq k$. Therefore $k = \frac{1}{\gamma}B(\frac{\lambda}{r}, \frac{\lambda}{s})$ is the best value of (1.3). The constant $[\frac{1}{\gamma}B(\frac{\lambda}{r}, \frac{\lambda}{s})]^p$ in (1.4) is the best possible, otherwise, we get a contradiction by (3.1) that the constant in (1.3) is not the best value. □

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