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SUB COMPATIBLE AND AND SUB SEQUENTIAL CONTINUOUS MAPS IN NON-ARCHIMEDEAN MENGER PM-SPACE

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ABSTRACT: The aim of this paper is to establish some fixed point results by using the new concepts of sub compatibility and sub sequential continuity in non Archimedean Menger PM-spaces (Briefly, N. A. Menger PM-spaces).

1. INTRODUCTION

In 1942 K. Menger [9] introduced the notion of probabilistic metric spaces (briefly, PM-space) as a generalization of metric space. Such a probabilistic generalization of metric spaces appears to be well adapted for the investigation of physical quantities and physiological thresholds. It is also of fundamental importance in probabilistic functional analysis.

In 1975, Istratescu and Crivat [22] first studied the non-Archimedean PM-space. They presented some basic topological preliminaries of N. A. PM-space and later on Istratescu [19], [20], [21] proved some fixed point results on mappings on N. A. Menger PM-space by generalizing the results of Sehgal and Bharucha-Reid [23]. Achari [8] generalized the results of Istratescu and studied some fixed points of qausi-contraction type mappings in non-Archimedean PM - space.

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Recently Bouhadjera and Thobie [5] introduced the concept of sub compatibility and sub sequential continuity in metric spaces. Since then various mathematicians have extended the concept of sub compatibility and sub sequential continuity in certain spaces like fuzzy metric spaces, 2 metric spaces and intuitionistic fuzzy metric spaces etc.

In the present paper we introduce the concept of sub compatibility and sub sequential continuity in N. A. Menger PM-space. Our results extend and generalize several known results in the literature.

2. Preliminaries

Definition 2.1: Let X be any non-empty set and D be the set of all left continuous distribution functions. An ordered pair (X, F) is said to be non-Archimedean probabilistic metric space (briefly N. A. PM-space) if F is a mapping from $X \times X$ into D satisfying the following conditions where the value of F at $(x, y) \in X \times X$ is represented by $F_{x,y}$ or $F(x, y)$ for all $x, y \in X$ such that

i) $F(x, y; t) = 1$ for all $t > 0$ if and only if $x = y$

ii) $F(x, y; t) = F(y, x; t)$

iii) $F(x, y; 0) = 0$

iv) If $F(x, y; t_1) = F(y, z; t_2) = 1$, then $F(x, z; \max\{t_1, t_2\}) = 1$

Definition 2.2: A t-norm is a function $\Delta : [0,1] \times [0,1] \to [0,1]$ which is associative, commutative, non decreasing in each coordinate and $\Delta(a,1) = a$ for all $a \in [0,1]$.

Definition 2.3: A non-.Archimedean Menger PM-space is an ordered triplet (X, F, Δ), where Δ is a t-norm and (X, F) is a N.A. PM-space satisfying the following condition;

$$F(x, z; \max\{t_1, t_2\}) \geq \Delta(F(x, y; t_1), F(y, z; t_2)) \text{ for all } x, y, z \in X, t_1, t_2 \geq 0$$

For details of topological preliminaries on non-Archimedean Menger PM-spaces, we refer to [22].

Definition 2.4: A N. A. Menger PM- space (X, F, Δ) is said to be of type $(C)_g$ if there exists a $g \in \Omega$ such that

$$g(F(x,z;t)) \le g(F(x,y;t)) + g(F(y,z;t)) \text{ for all } x,y,z \in X, t \ge 0$$

Where $\Omega = \{g / g:[0,1] \to [0,\infty)$ is continuous, strictly decreasing $g(1) = 0$ and $g(0) < \infty\}$.

Definition 2.5: A N. A. Menger PM-space (X, F, Δ) is said to be of type $(D)_g$ if there exists a $g \in \Omega$ such that $g(\Delta(t_1, t_2)) \le g(t_1) + g(t_2)\ \forall t_1, t_2 \in [0,1]$.

Remark 1

i) If N. A. Menger PM-space is of type $(D)_g$ then (X, F, Δ) is of type $(C)_g$.

ii) If (X, F, Δ) is N. A. Menger PM-space and $\Delta \ge \Delta(r,s) = \max(r+s-1,1)$,

then (X, F, Δ) is of type $(D)_g$ for $g \in \Omega$ and $g(t) = 1-t$.

Through out this paper let (X, F, Δ) be a complete N.A. Menger PM-space with a continuous strictly increasing t-norm Δ.

Let $\phi:[0,\infty) \to [0,\infty)$ be a function satisfying the condition (Φ);

$(\Phi)\phi$ is semi upper continuous from right and $\phi(t) < t$ *for* $t > 0$.

Definition 2.6: A sequence $\{x_n\}$ in N. A. Menger PM-space (X, F, Δ) converges to x, if and only if for each $\varepsilon > 0$, $\lambda > 0$ there exists $M(\varepsilon, \lambda)$ such that

$g(F(x_n, x; \varepsilon)) < g(1-\lambda)\ \forall\ n,\ n > M$.

Definition 2.7: A sequence $\{x_n\}$ in N. A. Menger PM-space is Cauchy sequence if and only if for each $\varepsilon > 0, \lambda > 0$ there exists an integer $M(\varepsilon, \lambda)$ such that $g(F(x_n, x_{n+p}; \varepsilon)) < g(1-\lambda) \forall n,\ n \ge M$ *and* $p \ge 1$.

Lemma1. If a function $\phi:[0,\infty)\rightarrow[0,\infty)$ satisfies the condition (Φ) then we get:

i) For all t > 0, $\lim_{n\rightarrow\infty}\phi^{n}(t)=0$, where $\phi^{n}(t)$ is the nth iteration of ϕ (t).

ii) If {tn} is a non decreasing sequence of real numbers and tn+1 ≤ ϕ (tn), n = 1, 2, … then $\lim_{n\rightarrow\infty}t_{n}=0$. In particular, if t ≤ ϕ(t), ∀ t ≥0 then t = 0.

Example1 ([11]). Let X be any set with at least two elements. If we define

$$F(x,x;t)=1 \text{ for all } x\in X,\ t>0 \text{ and } F(x,y;t)=\begin{Bmatrix} 0\ ,\ t\le 1 \\ 1\ ,\ t>1 \end{Bmatrix} \text{ when } x,y\in X, x\neq y$$

then, (X,F,Δ) is N. A. Menger PM-space with $\Delta(a,b)=\min(a,b)$ or $(a.b)$.

Example 2 ([11]). Let $X=R$ be the set of real numbers equipped with metric defined as

$$d(x,y)=|x-y| \text{ and set } F(x,y;t)=\frac{t}{t+d(x,y)}$$

Then (X,F,Δ) is N. A. Menger PM-space with Δ as continuous t-norm satisfying $\Delta(r,s)=\min(r,s)$ *or* $(r.s)$.

Definition 2.7: Two self maps A and B of a N. A. Menger PM-space (X,F,Δ) are said to be weakly compatible if $At=Bt$ for some $t\in X$ implies that $ABt=BAt$.

It is well known fact that compatible maps are weak compatible but the converse is not true.

Definition 2.8: Two self maps A and B of a set X are said to be owc if and only if there is a point $x\in X$ which is a coincidence point of A and B at which A and B commute. i.e., there exists a point $x\in X$ such that $Ax=Bx$ and $ABx=BAx$.

Definition 2.9: Two self maps A and B of a N. A. Menegr PM-space (X,F,Δ) are said sub compatible if and only if there exists a sequence $\{x_n\}$ in X such that

$\lim_n Ax_n = \lim_n Bx_n = z$, $z \in X$ and which satisfy $\lim_n g\left(F\left(\left(ABx_n, BAx_n; t\right)\right)\right) = 0$ for all $t > 0$.

Obviously two occasionally weakly compatible maps are sub compatible maps, however the converse is not true in general as shown in the following example.

Example 3. ([11]). Let $X = R$ be the set of real numbers equipped with metric defined as

$$d(x, y) = \ |x - y| \ \textit{and} \quad \text{set } F(x, y; t) = \frac{t}{t + d(x,y)}$$

Then (X, F, Δ) is N. A. Menger PM-space with Δ as continuous t-norm satisfying $\Delta(r,s) = \min(r,s)$ *or* $(r.s)$.

Define the maps $A, B : X \to X$ by setting

$$Ax = \begin{cases} x^2, x < 1 \\ 2x - 1, x \geq 1 \end{cases}, \quad Bx = \begin{cases} 3x - 2, x < 1 \\ x + 3, x \geq 1 \end{cases}$$

Define a sequence $x_n = 1 - \dfrac{1}{n}$, then $Ax_n = \left(1 - \dfrac{1}{n}\right)^2 \to 1$

$$Bx_n = 3\left(1 - \frac{1}{n}\right) - 2 = 1 - \frac{3}{n} \to 1$$

$$ABx_n = A\left(1 - \frac{3}{n}\right) = \left(1 - \frac{3}{n}\right)^2 = 1 + \frac{9}{n^2} - \frac{6}{n} \qquad \text{and}$$

$$BAx_n = B\left(1 - \frac{1}{n}\right)^2 = 3\left(1 - \frac{1}{n}\right)^2 - 2 = 3\left[\left(1 + \left(\frac{1}{n}\right)^2 - \frac{2}{n}\right)\right] - 2 = 1 + \left(\frac{1}{n}\right)^2 - \frac{6}{n} \qquad \text{and}$$

$\lim_n g(F(ABx_n, BAx_n; t)) \to 0$, where g is a function defined in Def. 2.4.

Thus, A and B are sub compatible but A and B are not owc maps as, $A(4) = 7 = B(4)$ and $AB(4) = A(7) = 13 \neq BA(4) = 10$.

It is also interesting to see the following one way implication.

Commuting $\Rightarrow$ Weakly commuting $\Rightarrow$ Compatibility $\Rightarrow$ Weak Compatibility $\Rightarrow$ Occasionally Weak Compatibility $\Rightarrow$ Sub Compatibility.

Now, we aim at our second objective which is to introduce a new notion called sub sequential continuity in N. A. Menger PM-space (X, F, Δ) by weakening the concept of reciprocal continuity introduced by Pant [18].

Definition 2.10: Two self maps A and S of a N. A. Menegr PM-space (X, F, Δ) are called reciprocal continuous if $\lim_n ASx_n = At$ and $\lim_n SAx_n = St$ for some $t \in X$ whenever $\{x_n\}$ is a sequence in X such that $\lim_n Ax_n = \lim_n Sx_n = t \in X$.

Definition 2.11: Two self maps A and B of a N. A. Menegr PM-space (X, F, Δ) are said to be sub sequentially continuous if and only if there exists a sequence $\{x_n\}$ in X such that $\lim_n Ax_n = \lim_n Bx_n = t$ for some $t \in X$ and satisfy $\lim_n ABx_n = At$ and $\lim_n BAx_n = Bt$.

Remark 2. If A and B are both continuous or reciprocally continuous then they are obviously sub sequentially continuous.

The next example shows that there exist sub sequentially continuous pairs of maps which are neither continuous nor reciprocally continuous.

Example 4. ([11]). Let $X = R$ be the set of real numbers equipped with metric defined as

$$d(x, y) = |x - y| \quad and \quad \text{set } F(x, y; t) = \frac{t}{t+d(x,y)}$$

Then (X, F, Δ) is N. A. Menger PM- space with Δ as continuous t-norm satisfying $\Delta(r,s) = \min(r,s)$ *or* $(r.s)$.

Define $A, B : X \to X$ as;

$$Ax = \begin{Bmatrix} 2, x < 3 \\ x, x \geq 3 \end{Bmatrix} \quad , \quad Bx = \begin{Bmatrix} 2x-4, x \leq 3 \\ 3, x > 3 \end{Bmatrix}$$

Consider a sequence $x_n = 3 + \frac{1}{n}$, then $Ax_n = \left(3 + \frac{1}{n}\right) \to 3$, $Bx_n = 3$

$BAx_n = B\left(3 + \frac{1}{n}\right) = 3 \neq B(3) = 2$.

Thus A and B are not reciprocally continuous but if we consider a sequence $x_n = 3 - \frac{1}{n}$, then $Ax_n = 2$, $Bx_n = 2\left(3 - \frac{1}{n}\right) - 4 = \left(2 - \frac{2}{n}\right) \to 2$

$ABx_n = A\left(2 - \frac{2}{n}\right) = 2 = A(2)$, $BAx_n = B(2) = 0 = B(2)$.

Therefore, A and B are sub sequentially continuous.

3. Results and Discussion

Now, we prove our main result.

Theorem 1. Let A, B, S and T be four self maps of a N. A. Menger PM-space (X, F, Δ). If the pairs (A, S) and (B, T) are sub compatible and sub sequentially continuous, then

(i) A and S have a coincidence point,

(ii) B and T have a coincidence point.

Further, If (1.1)

$$g\left(F\left(Ax,By;t\right)\right) \leq \phi\left[\max\left\{\begin{matrix} g\left(F\left(Sx,Ty;t\right)\right), g\left(F\left(Ax,Sx;t\right)\right), g\left(F\left(By,Ty;t\right)\right) \\ g\left(F\left(Sx,By;t\right)\right), g\left(F\left(Ty,Ax;t\right)\right) \end{matrix}\right\}\right]$$

for all $x, y \in X$, t > 0 , where $\phi \in \Phi$ such that $\phi:[0,\infty) \to [0,\infty)$. Then A, B, S and T have a unique common fixed point in X.

Proof. Since the pairs (A,S) and (B,T) are sub compatible and sub sequentially continuous, therefore, there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_n Ax_n = \lim_n Sx_n = u$ for some $u \in X$ and which satisfy

$\lim_n g\left(F\left(ASx_n, SAx_n;t\right)\right) = g\left(F\left(Au,Su;t\right)\right) = 0$,

$\lim_n By_n = \lim_n Ty_n = v$ For some $v \in X$ and which satisfy

$\lim_n g\left(F\left(BTy_n, TBy_n;t\right)\right) = g(F(Bv,Tv;t)) = 0$.

Therefore, $Au = Su$ and $Bv = Tv$. i.e., u is the coincidence point of A and S and v is a coincidence point of B and T.

Now, using (1.1) for $x = x_n$ and $y = y_n$, we get

$$g\left(F\left(Ax_n,By_n;t\right)\right) \leq \phi\left[\max\left\{\begin{matrix} g\left(F\left(Sx_n,Ty_n;t\right)\right), g\left(F\left(Ax_n,Sx_n;t\right)\right), g\left(F\left(By_n,Ty_n;t\right)\right) \\ g\left(F\left(Sx_n,By_n;t\right)\right), g\left(F\left(Ty_n,Ax_n;t\right)\right) \end{matrix}\right\}\right]$$

Letting $n \to \infty$,

$$g\left(F\left(u,v;t\right)\right) \leq \phi\left[\max\left\{g\left(F\left(u,v;t\right)\right), 0, 0, g\left(F\left(u,v;t\right)\right), g\left(F\left(u,v;t\right)\right)\right\}\right]$$

i.e., $g\left(F\left(u,v;t\right)\right) \leq \phi\left[g\left(F\left(u,v;t\right)\right)\right] < g\left(F\left(u,v;t\right)\right)$, a contradiction.

Hence $u = v$.

Again using (1.1) for $x = u$, $y = y_n$, we obtain

$$g\left(F\left(Au,By_n;t\right)\right) \leq \phi\left[\max\left\{\begin{matrix} g\left(F\left(Su,Ty_n;t\right)\right), g\left(F\left(Au,Su;t\right)\right), g\left(F\left(By_n,Ty_n;t\right)\right) \\ g\left(F\left(Su,By_n;t\right)\right), g\left(F\left(Ty_n,Au;t\right)\right) \end{matrix}\right\}\right]$$

Letting $n \to \infty$,

$$g\left(F\left(Au,v;t\right)\right) \leq \phi\left[\max\left\{\begin{matrix} g\left(F\left(Su,v;t\right)\right), g\left(F\left(Au,Su;t\right)\right), g\left(F\left(v,v;t\right)\right) \\ g\left(F\left(Su,v;t\right)\right), g\left(F\left(v,Au;t\right)\right) \end{matrix}\right\}\right]$$

$$g\left(F\left(Au,v;t\right)\right) \leq \phi\left[\max\left\{g\left(F\left(Su,v;t\right)\right),0,0,\ g\left(F\left(Su,v;t\right)\right), g\left(F\left(v,Au;t\right)\right)\right\}\right]$$

i.e. $g\left(F\left(Au,v;t\right)\right)=\phi\left[g\left(F\left(Au,v;t\right)\right)\right]<g\left(F\left(Au,v;t\right)\right)$, which yields $Au=v=u$.

Therefore, $u=v$ is a common fixed point of A, B, S and T.

For uniqueness, let $w \neq u$ be another fixed point of A, B, S and T. Then from (1.1), we have

$$g\left(F\left(Au,Bw;t\right)\right) \leq \phi\left[\max\left\{\begin{matrix} g\left(F\left(Su,Tw;t\right)\right), g\left(F\left(Au,Su;t\right)\right), g\left(F\left(Bw,Tw;t\right)\right) \\ g\left(F\left(Su,Bw;t\right)\right), g\left(F\left(Tw,Au;t\right)\right) \end{matrix}\right\}\right]$$

$$= \phi\left[\max\left\{g\left(F\left(Au,Bw;t\right)\right),0,0,g\left(F\left(Au,Bw;t\right)\right),g\left(F\left(Au,Bw;t\right)\right)\right\}\right]$$

$= \phi\left[g\left(F\left(Au,Bw;t\right)\right)\right]<g\left(F\left(Au,Bw;t\right)\right)$ which yields $w=u$ and hence the theorem.

If we put $A=B$ and $S=T$, in above theorem , we get the following result.

Corollary 1. Let A and S be self maps of a N. A. Menger PM-space (X,F,Δ) such that the pairs (A,S) is sub compatible and sub sequentially continuous, then:

(i) A and S have a coincidence point,

Further, If (1.1)

$$g\left(F\left(Ax,Ay;t\right)\right) \leq \phi\left[\max\left\{\begin{matrix} g\left(F\left(Sx,Sy;t\right)\right), g\left(F\left(Ax,Sx;t\right)\right), g\left(F\left(Ay,Sy;t\right)\right) \\ g\left(F\left(Sx,Ay;t\right)\right), g\left(F\left(Sy,Ax;t\right)\right) \end{matrix}\right\}\right]$$

for all $x, y \in X$, t > 0 , where $\phi \in \Phi$. Then A and S have a unique common fixed point in X.

If we put $S = T$, in above corollary, we get the following result.

Corollary 2. Let A, B and S be self maps of a N. A. Menger PM-space (X, F, Δ). Suppose that the pairs (A, S) and (B, S) are sub compatible and sub sequentially continuous, then

(i) A and S have a coincidence point.

(ii) B and S have a coincidence point.

Further, If (1.1)

$$g\left(F\left(Ax, By; t\right)\right) \leq \phi\left[\max\left\{\begin{array}{l} g\left(F\left(Sx, Sy; t\right)\right), g\left(F\left(Ax, Sx; t\right)\right), g\left(F\left(By, Sy; t\right)\right) \\ g\left(F\left(Sx, By; t\right)\right), g\left(F\left(Sy, Ax; t\right)\right) \end{array}\right\}\right]$$

for all $x, y \in X$, t > 0 , where $\phi \in \Phi$. Then A, B and S have a unique common fixed point in X.

Now, we furnish our theorem with example.

Example 5([11]). Let $X = R$ be the set of real numbers equipped with metric defined as

$$d(x, y) = |x - y| \ \textit{and} \quad \text{set } F(x, y, t) = \frac{t}{t + d(x,y)}$$

Then (X, F, Δ) is N. A. Menger PM-space with Δ as continuous t-norm satisfying $\Delta(r, s) = \min(r, s)$ *or* $(r.s)$.

Define the maps A, B, S and $T : X \to X$ as

$$A(x) = \begin{Bmatrix} x, x \le 1 \\ 3x+1, x>1 \end{Bmatrix} \quad , \quad S(x) = \begin{Bmatrix} 2x-1, x \le 1 \\ 5x-1, x>1 \end{Bmatrix}$$

$$B(x) = \begin{Bmatrix} 3-2x, x \le 1 \\ 3, x>1 \end{Bmatrix} \quad , \quad T(x) = \begin{Bmatrix} 3x-2, x \le 1 \\ 2, x>1 \end{Bmatrix}$$

Define $\phi:[0,\infty)\to[0,\infty)$ as $\phi(t)=\sqrt{t}$ *and* $g:[0,1]\to[0,\infty)$ is continuous, strictly decreasing and $g(1)=0$ and $g(0)<\infty$ }.

Consider the sequences $\{x_n\}=\{y_n\}=1-\dfrac{1}{n}$.

Then, clearly Ax_n, Bx_n, Sx_n and $Tx_n \to 1$.

$AS(x_n) = A\left(1-\dfrac{1}{n}\right)=\left(1-\dfrac{1}{n}\right)\to 1 = A(1)$ and $SA(x_n)=S\left(1-\dfrac{1}{n}\right)=\left(1-\dfrac{1}{n}\right)\to 1 = S(1)$

Thus (A,S) is sub compatible and sub sequentially continuous.

Again, $BT(x_n)=B\left(1-\dfrac{3}{n}\right)=3-2\left(1-\dfrac{3}{n}\right)=\left(1+\dfrac{6}{n}\right)\to 1 = B(1)$

$TB(x_n)=T\left(1+\dfrac{2}{n}\right)=3\left(1+\dfrac{2}{n}\right)-2=\left(1+\dfrac{6}{n}\right)\to 1 = T(1)$,

which shows that (B,T) is sub compatible and sub sequentially continuous.

Also the condition (1.1) of our theorem is satisfied and '1' is unique common fixed point of A, B, S and T.

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