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A DECOMPOSITION OF (μ,λ)-CONTINUITY IN GENERALIZED TOPOLOGICAL SPACES

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Abstract. : In this paper, we introduce and study the notions of $w_{(\mu,\lambda)}$ - $\mathcal{H}$ - continuity and $w^*_{(\mu,\lambda)}$ - $\mathcal{H}$ -continuity in generalized topological spaces. Also, we prove that $f:(X,\mu)\rightarrow(Y,\lambda,\mathcal{H})$ is (μ,λ) -continuous if and only if it is $w_{(\mu,\lambda)}$ - $\mathcal{H}$ -continuous and $w^*_{(\mu,\lambda)}$ - $\mathcal{H}$ -continuous.

1. Introduction and Preliminaries

In 2002, Csaszar[2] introduced the notions of generalized topology and generalized continuity. Let X be a nonempty set and μ be a collection of subsets of X. Then μ is called a *generalized topology* (briefly GT) on X iff $\emptyset\in\mu$ and the union of an arbitrary class of elements of μ always belong to μ. We call the pair (X,μ) be a *generalized topological space* (briefly GTS) on X. Let μ be a GT in X. The elements of μ are said to be μ *-open*, their complements are μ *-closed*. We consider the largest μ-open subset of $A\subset X$ and denote it by $i_\mu(A)$ and the smallest μ-closed superset of A and denoted it by $c_\mu(A)$. A subset A of X is μ *-pre-open* [3] (resp. μ-semi-open [3]), if $A\subset i_\mu c_\mu(A)$ (resp. $A\subset c_\mu i_\mu(A)$). A

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generalized topological space (X,μ) is said to be μ-*regular* [8] if for each μ-closed set F of X not containing x, there exist disjoint μ-open sets U and V such that $x \in U$ and $F \subseteq V$. A function $f:(X,\mu)\to(Y,\lambda)$ is said to be (μ,λ)-continuous [2] (resp. (π,λ)- *continuous* [7]), iff $U \in \lambda$ implies that $f^{-1}(U)$ is μ-open (resp. μ-pre-open) in (X,μ). A function $f:(X,\mu)\to(Y,\lambda)$ is said to be *weakly* (μ,λ) - *continuous* [6], if for each $x \in X$ and each λ-open neighbourhood V of $f(x)$, there exist a μ-open neighbourhood U of x such that $f(U) \subseteq c_\lambda(V)$. A nonempty family $\mathcal{H}$ of subsets of X is said to be a *hereditary class* [4], if $A \in \mathcal{H}$ and $B \subset A$, then $B \in \mathcal{H}$. Given a generalized topological space (X,μ) with a hereditary class $\mathcal{H}$, for each $A \subseteq X$, $A^*(\mathcal{H},\mu)=\{x \in X : A \cap V \notin \mathcal{H}$ for every $V \in \mu$ such that $x \in V\}$ [4]. If $c^*_\mu(A) = A \cup A^*(\mathcal{H},\mu)$ for every subset A of X, then $\mu^* = \{A \subset X : X - A = c^*_\mu(X-A)\}$ is a GT, μ^* is finer than μ [[4], Theorem 3.6]. A subset A of $(X,\mu,\mathcal{H})$ is said to be pre-$\mathcal{H}$-open [6], if $A \subset i_\mu c^*_\mu(A)$. A hereditary class $\mathcal{H}$ is μ-codense [4], iff $\mu \cap \mathcal{H} = \{\emptyset\}$. A hereditary class $\mathcal{H}$ is strongly μ-codense [4], iff $M, M' \in \mu$, $M \cap M' \in \mathcal{H}$ implies $M \cap M' = \emptyset$.

Definition 1.1. [2] *A function* $f:(X,\mu)\to(Y,\lambda)$ *is said to be* $\theta(\mu,\lambda)$*-continuous at* x *if for each* λ*-open neighbourhood* V *of* $f(x)$*, there is a* μ*-open neighbourhood* U *of* x *such that* $f(c_\mu(U)) \subseteq c_\lambda(V)$.

Lemma 1.2. [[4], Proposition 2.8] *Let* $(X,\mu,\mathcal{H})$ *be a hereditary generalized topological space. Then* $A \in \mu$ *implies* $A \subseteq A^*$ *iff* $\mathcal{H}$ *is strongly* μ*-codense.*

Lemma 1.3. [4] *Let* $(X,\mu,\mathcal{H})$ *be a hereditary generalized topological space and* A, B *be subsets of* X*. Then the following properties are hold:*

(1) *If* $A \subset B$*, then* $A^* \subset B^*$,
(2) $A^* = c_\mu(A^*) \subseteq c_\mu(A)$,
(3) $(A^*)^* \subseteq A^*$.

Lemma 1.4. [[4], Proposition 3.7] *Let $(X,\mu,\mathcal{H})$ be a hereditary generalized topological space and $A \subset X$. Then the following statements are equivalent.*

(1) $A \subset A^*$,

(2) $A^* = c^*_\mu(A)$,

(3) $A^* = c_\mu(A)$.

Lemma 1.5. [[6], Theorem 3.2] *Let (X,μ) and (Y,λ) be generalized topological spaces. Then $f:(X,\mu)\to(Y,\lambda)$ is (μ,λ) continuous iff for each $x\in X$ and each λ-open set V containing $f(x)$, there exist a μ-open set U containing x such that $f(U)\subseteq V$.*

Lemma 1.6. [[8], Theorem 4.3] *Let (X,μ) be a generalized topological space. If X is μ-regular, then for each $x\in X$ and each $U\in\mu$ containing x, there exists $V\in\mu$ such that $x\in V\subseteq c_\mu(V)\subseteq U$.*

2. WEAKLY (μ,λ)- $\mathcal{H}$-CONTINUITY AND WEAK* (μ,λ)- $\mathcal{H}$-CONTINUITY

Definition 2.1. *A function $f:(X,\mu)\to(Y,\lambda,\mathcal{H})$ is said to be* weakly (μ,λ)-$\mathcal{H}$-continuous *(briefly $w_{(\mu,\lambda)}$-$\mathcal{H}$-c), if for each $x\in X$ and each λ-open neighbourhood V of $f(x)$, there exist a μ-open neighbourhood U of x such that $f(U)\subset c^*_\lambda(V)$.*

Remark 2.2. *Every weakly (μ,λ)-$\mathcal{H}$-continuous function is weakly (μ,λ)-continuous but the converse is need not be true.*

Example 2.3. *Let $X=Y=\{a,b,c,d\}$, $\mu=\{\emptyset,\{a,b\},\{c,d\},X\}$, $\lambda=\{\emptyset,\{a,c,d\},\{b,c,d\},Y\}$, and $\mathcal{H}=\{\emptyset,\{a\},\{b\},\{c\},\{d\},\{a,c\},\{c,d\}\}$. The identity function $f:(X,\mu)\to(Y,\lambda,\mathcal{H})$ is weakly (μ,λ)-continuous but not weakly (μ,λ)- $\mathcal{H}$-continuous.*

(i) Let $a \in X$. Then $V = \{a, c, d\}$ and Y are the λ-open sets containing $f(a)$ in (Y, λ). There exist a μ-open set $U = \{a, b\}$ containing a in (X, μ) such that $f(U) \subset c_\lambda(V) = Y$.

(ii) Let $b \in X$. Then $V = \{b, c, d\}$ and Y are the λ-open sets containing $f(b)$ in (Y, λ). There exist a μ-open set $U = \{a, b\}$ containing b in (X, μ) such that $f(U) \subset c_\lambda(V) = Y$.

(iii) Let $c \in X$. Then $V_1 = \{a, c, d\}$, $V_2 = \{b, c, d\}$ and Y are the λ-open sets containing $f(c)$ in (Y, λ). There exist a μ-open set $U = \{c, d\}$ containing c in (X, μ) such that $f(U) \subset c_\lambda(V) = Y$, where V be a λ-open set containing $f(c)$.

(iv) Let $d \in X$. Then $V_1 = \{a, c, d\}$, $V_2 = \{b, c, d\}$ and Y are the λ-open sets containing $f(d)$ in (Y, λ). There exist a μ-open set $U = \{c, d\}$ containing d in (X, μ) such that $f(U) \subset c_\lambda(V) = Y$, where V be a λ-open set containing $f(d)$.

By (i), (ii), (iii), and (iv), f is weakly (μ, λ)-continuous. On the other hand, consider the λ-open set $V = \{a, c, d\}$ in (Y, λ). Now, $\{a, c, d\}^* = \{a\}$ and so $c^*_\lambda(\{a, c, d\}) = \{a, c, d\}$. Note that the μ-open subsets of (X, μ) containing a are $U = \{a, b\}$ and X. Further $f(U) = U \nsubseteq c^*_\lambda(V)$ and $f(X) = Y \nsubseteq c^*_\lambda(V)$. Therefore f is not $w_{(\mu,\lambda)}$-$\mathcal{H}$-c.

Theorem 2.4. *A function* $f : (X, \mu) \to (Y, \lambda, \mathcal{H})$ *is weakly* (μ, λ)-$\mathcal{H}$*-continuous if and only if for each* λ*-open set* $V \subset Y$, $f^{-1}(V) \subset i_\mu(f^{-1}(c^*_\lambda(V)))$.

Proof. Let V be any λ-open set of Y and $x \in f^{-1}(V)$. Since f is $w_{(\mu,\lambda)}$-$\mathcal{H}$-c, there exists a μ-open set U such that $x \in U$ and $f(U) \subset c^*_\lambda(V)$. Hence $x \in U \subset f^{-1}(c^*_\lambda(V))$ and $x \in i_\mu(f^{-1}(c^*_\lambda(V)))$. Therefore, we obtain $f^{-1}(V) \subset i_\mu(f^{-1}(c^*_\lambda(V)))$. Conversely, let $x \in X$ and V be a λ-open set of Y containing $f(x)$. Then $x \in f^{-1}(V) \subset i_\mu(f^{-1}(c^*_\lambda(V)))$. Let $U = i_\mu(f^{-1}(c^*_\lambda(V)))$, then $f(U) = f(i_\mu(f^{-1}(c^*_\lambda(V)))) \subset f(f^{-1}(c^*_\lambda(V))) \subset c^*_\lambda(V)$. This shows that f is $w_{(\mu,\lambda)}$- $\mathcal{H}$-c.

Theorem 2.5. *Let $(Y,\lambda,\mathcal{H})$ be a hereditary generalized topological space, where $\mathcal{H}$ is strongly λ-codence. Then the following are equivalent:*

(a) $f:(X,\mu)\to(Y,\lambda,\mathcal{H})$ *is weakly (μ,λ)-$\mathcal{H}$-continuous,*

(b) *For every λ-semi-open set V in Y, there exist a λ-open set W in Y such that $W\subset V$ and $f^{-1}(W)\subset i_\mu(f^{-1}(V^*))$,*

(c) $f^{-1}(W)\subset i_\mu(f^{-1}(W^*))$ *for every λ-open set V in Y.*

Proof. $(a) \Rightarrow (b)$. Assume that f is weakly (μ,λ)-$\mathcal{H}$-continuous and V is λ-semi-open in (Y,λ). Since V is λ-semi-open in (Y,λ), there exist a λ-open set W in (Y,λ) such that $W\subset V\subset c_\lambda(W)$. Since $\mathcal{H}$ is strongly μ-codense, $W^* = c_\lambda(W) = c^*_\lambda(W)$ by Lemmas 1.2 and 1.3. Therefore, $W\subset V\subset W^*$ so that $W^* = V^* = c^*_\lambda(W)$. By Theorem 2.4, $f^{-1}(W)\subset i_\mu(f^{-1}(c^*_\lambda(W))) = i_\mu(f^{-1}(V^*))$, which proves (b).

$(b)\Rightarrow(c)$. Let V be λ-semi-open in (Y,λ), there exist a λ-open set W in (Y,λ) such that $W\subset V$ and $f^{-1}(W)\subset i_\mu(f^{-1}(V^*))$. The set W be λ-open in (Y,λ), then $f^{-1}(W)\subset i_\mu(f^{-1}(W^*))$.

$(c)\Rightarrow(a)$. Let V be λ-open set in (Y,λ). Then $f^{-1}(V)\subset i_\mu(f^{-1}(V^*))$. Since $\mathcal{H}$ is strongly μ-codense, then $f^{-1}(V)\subset i_\mu(f^{-1}(c^*_\lambda(V)))$. By Theorem 2.4
$f:(X,\mu)\to(Y,\lambda,\mathcal{H})$ is weakly (μ,λ)-$\mathcal{H}$-continuous.

Theorem 2.6. *If $(Y,\lambda,\mathcal{H})$ is a hereditary generalized topological space such that $\mathcal{H}$ is strongly λ-codense and $f:(X,\mu)\to(Y,\lambda,\mathcal{H})$ is $w_{(\mu,\lambda)}$-$\mathcal{H}$-c, then $c_\mu(f^{-1}(V))\subset f^{-1}(c^*_\lambda(V)) = f^{-1}(V^*)$ for every λ-open set V in Y.*

Proof. Let $x \in c_\mu(f^{-1}(V))$. Assume that $x \notin f^{-1}(c^*_\lambda(V))$. By Lemmas 1.2 and 1.3, we have $x \notin f^{-1}(V^*)$ and $f(x) \notin V^* = c_\lambda(V)$. Therefore, there exist a λ-open set W containing $f(x)$ such that $W \cap V = \emptyset$ which implies that $c_\lambda(W) \cap V = \emptyset$ and so $c^*_\lambda(W) \cap V = \emptyset$. Since f is weakly (μ, λ)-$\mathcal{H}$-c, there is a μ-open set U containing x in X such that $f(U) \subset c^*_\lambda(W)$ and so $f(U) \cap V = \emptyset$. Now $x \in c_\mu(f^{-1}(V))$ implies that $U \cap f^{-1}(V) \neq \emptyset$ and so $f(U) \cap V \neq \emptyset$, a contradiction, which completes the proof.

If $\mathcal{H} = \emptyset$, in the above Theorem 2.6, we have the following Corollary.

Corolary 2.7. [[6], Theorem 3.5]. *If (X, μ) and (Y, λ) are generalized topological spaces and $f : (X, \mu) \to (Y, \lambda)$ is weakly (μ, λ)-continuous, then $c_\mu(f^{-1}(V)) \subset f^{-1}(c_\lambda(V))$, for every λ-open set V in (Y, λ).*

Definition 2.8. *A hereditary generalized topological space $(X, \mu, \mathcal{H})$ is $R_\mu\mathcal{H}$-space if, for each $x \in X$ and each μ-open neighbourhood U of x, there exist a μ-open neighbourhood V of x such that $x \in V \subset c^*_\mu(V) \subset U$.*

Theorem 2.9. *Let $(Y, \lambda, \mathcal{H})$ be a $R_\mu\mathcal{H}$-space. Then $f : (X, \mu) \to (Y, \lambda, \mathcal{H})$ is $w_{(\mu,\lambda)}$-$\mathcal{H}$-c if and only if f is (μ, λ)-continuous.*

Proof. Let $x \in X$ and V be a λ-open set of Y containing $f(x)$. Since Y is a $R_\mu\mathcal{H}$-space, there exist a λ-open set W of Y such that $f(x) \in W \subset c^*_\lambda(W) \subset V$. Let f be $w_{(\mu,\lambda)}$-$\mathcal{H}$-c, there exist a μ-open set U such that $x \in U$ and $f(U) \subset c^*_\lambda(W) \subset V$. This implies f is (μ, λ)-continuous. Conversely, let $x \in X$ and V be any λ-open set of Y containing $f(x)$. Since f is (μ, λ)-continuous, there exist a μ-open set U containing x such that $f(U) \subset V \subset c^*_\lambda(V)$, hence f is $w_{(\mu,\lambda)}$-$\mathcal{H}$-c.

Theorem 2.10. *If $(Y,\lambda,\mathcal{H})$ is a hereditary generalized topological space and $f:(X,\mu)\rightarrow(Y,\lambda,\mathcal{H})$ is a (π,λ)- continuous mapping such that $c_\mu(f^{-1}(V))\subset f^{-1}(c^*_\lambda(V))$ for every λ-open set V in Y, then f is $w_{(\mu,\lambda)}$-$\mathcal{H}$-c.*

Proof. Let $x\in X$ and V be a λ-open set in Y containing $f(x)$. By hypothesis $c_\mu(f^{-1}(V))\subset f^{-1}(c^*_\lambda(V))$. Since f is (π,λ)-continuous, $f^{-1}(V)$ is μ-pre-open in X and so $x\in f^{-1}(V)\subset i_\mu(c_\mu(f^{-1}(V)))$. Which implies there exist a μ-open set such that $x\in U\subset c_\mu(f^{-1}(V))\subset f^{-1}(c^*_\lambda(V))$, that is $U\subset f^{-1}(c^*_\lambda(V))$, $f(U)\subset c^*_\lambda(V)$ which implies that f is $w_{(\mu,\lambda)}$-$\mathcal{H}$-c.

Theorem 2.11. *If $(Y,\lambda,\mathcal{H})$ is a hereditary generalized topological space such that $\mathcal{H}$ is strongly λ-codense and $f:(X,\mu)\rightarrow(Y,\lambda,\mathcal{H})$ is μ-pre-continuous, then f is $w_{(\mu,\lambda)}$-$\mathcal{H}$-c if and only if $c_\mu(f^{-1}(V))\subset f^{-1}(c^*_\lambda(V))=f^{-1}(V^*)$ for every λ-open set V in Y.*

Proof. Follows from Theorem 2.6 and 2.10.

If $f:(X,\mu,\mathcal{H})\rightarrow(Y,\lambda)$ is any mapping. Then $f(\mathcal{H})=\{f(H):H\in\mathcal{H}\}$ is a hereditary class on (Y,λ).

Theorem 2.12. *If $f:(X,\mu,\mathcal{H})\rightarrow(Y,\lambda,\mathcal{J})$ is a weakly (μ,λ)-$\mathcal{J}$-continuous where $\mathcal{J}=f(\mathcal{H})$ is strongly λ-codense, then $c^*_\mu(f^{-1}(V))\subset f^{-1}(c^*_\lambda(V))=f^{-1}(V^*)$ for every λ-open set V in Y.*

Proof. Let $x\in c^*_\mu(f^{-1}(V))$. Assume that $x\notin f^{-1}(c^*_\lambda(V))$. That is $x\notin f^{-1}(V^*(\mathcal{J}))$ implies that $f(x)\notin V^*(\mathcal{J})=c_\lambda(V)$, since $\mathcal{J}$ is strongly λ-codense. Therefore, there exist a λ-open set W containing $f(x)$ in Y such that $W\cap V=\emptyset$. Since V is λ-open, $c_\lambda(W)\cap V=\emptyset$ and so $c^*_\lambda(W)\cap V=\emptyset$. Since f is weakly (μ,λ)-$\mathcal{J}$-continuous, there exist a μ-open set U in X containing x such that $f(U)\subset c^*_\lambda(W)$

and so $f(U) \cap V = \emptyset$. Now, $x \in c^*_\mu(f^{-1}(V))$ implies that $x \in c_\mu(f^{-1}(V))$ which implies that $f^{-1}(V) \cap U \neq \emptyset$ and so $V \cap f(U) \neq \emptyset$, a contradiction. This completes the proof.

Definition 2.13. *A function $f : (X, \mu, \mathcal{H}) \to (Y, \lambda)$ is said to be* pre-$\mathcal{H}$-continuous*, if for each λ-open V in Y, $f^{-1}(V)$ is pre-$\mathcal{H}$-open in $(X, \mu, \mathcal{H})$.*

Theorem 2.14. *If $f : (X, \mu, \mathcal{H}) \to (Y, \lambda, \mathcal{J})$ is a pre-$\mathcal{H}$-continuous mapping where $\mathcal{J} = f(\mathcal{H})$ and $c^*_\mu(f^{-1}(V)) \subset f^{-1}(c^*_\lambda(V))$ for every λ-open set V in Y, then f is weakly (μ, λ)-$\mathcal{J}$-continuous.*

Proof. Let $x \in X$ and V be a λ-open set in Y containing $f(x)$. By hypothesis, $c^*_\mu(f^{-1}(V)) \subset f^{-1}(c^*_\lambda(V))$. Since f is pre-$\mathcal{H}$-continuous, $f^{-1}(V)$ is pre-$\mathcal{H}$-open in X and so $f^{-1}(V) \subset i_\mu(c^*_\mu(f^{-1}(V)))$. Since $x \in f^{-1}(V) \subset i_\mu(c^*_\mu(f^{-1}(V)))$, there exist a μ-open set U containing x such that $x \in U \subset c^*_\mu(f^{-1}(V)) \subset f^{-1}(c^*_\lambda(V))$ and so $f(U) \subset c^*_\lambda(V)$ which is implies that f is weakly (μ, λ)-$\mathcal{J}$-continuous.

Definition 2.15. *A function $f : (X, \mu) \to (Y, \lambda, \mathcal{H})$ is said to be* weak* (μ, λ)-$\mathcal{H}$-continuous *(briefly $w^*_{(\mu,\lambda)}$-$\mathcal{H}$-c), if for each λ-open set V in Y, $f^{-1}(f^*_r(V))$ is μ-closed in (X, μ), where $f^*_r(V) = V^* - i_\lambda(V)$ is λ-closed in $(Y, \lambda, \mathcal{H})$.*

Theorem 2.16. *A function $f : (X, \mu) \to (Y, \lambda, \mathcal{H})$ is (μ, λ)-continuous if and only if it is both $w_{(\mu,\lambda)}$-$\mathcal{H}$-c and $w^*_{(\mu,\lambda)}$-$\mathcal{H}$-c.*

Proof. Let $x \in X$ and V be any λ-open set of Y containing $f(x)$. Since f is (μ, λ)-continuous, there exist a μ-open set U containing x such that $f(U) \subset V \subset c^*_\lambda(V)$ and $f^{-1}(f^*_r(V))$ is μ-closed in (X, μ). Hence f is $w_{(\mu,\lambda)}$-$\mathcal{H}$-c and $w^*_{(\mu,\lambda)}$-$\mathcal{H}$-c. Conversely, let $x \in X$ and V be any λ-open set of Y containing $f(x)$, since f is $w_{(\mu,\lambda)}$-$\mathcal{H}$-c, there exist a μ-open set U containing

x such that $f(U) \subset c^*_\lambda(V)$. Now $f^*_r(V) = V^* - i_\lambda(V)$ and thus $f(x) \notin f^*_r(V)$. Hence $x \notin f^{-1}(f^*_r(V))$ and $U - f^{-1}(f^*_r(V))$ is μ-open set containing x since f is $w^*_{(\mu,\lambda)}$-$\mathcal{H}$-c. The proof will be complete when we show $f(U - f^{-1}(f^*_r(V))) \subset V$. Let $y \in U - f^{-1}(f^*_r(V))$. Then $y \in U$ and hence $f(y) \in f(U) \subset c^*_\lambda(V)$. But $y \notin f^{-1}(f^*_r(V))$ and thus $f(y) \notin f^*_r(V) = V^* - i_\lambda(V) = V^* - V$ which implies $f(y) \in V$. Therefore $f(U) \subset V$ hence f is (μ,λ)-continuity.

Remark 2.17. *The notions of $w_{(\mu,\lambda)}$-$\mathcal{H}$-c and $w^*_{(\mu,\lambda)}$-$\mathcal{H}$-c are independent.*

Example 2.18. *Let $X = Y = \{a,b,c,d\}$, $\mu = \{\emptyset, \{a,b\}, \{c\}, \{a,b,c\}, \{b,c,d\}, X\}$, $\lambda = \{\emptyset, \{a,b\}, \{a,c,d\}, Y\}$, and $\mathcal{H} = \{\emptyset, \{d\}\}$. The identity function $f : (X,\mu) \to (Y,\lambda,\mathcal{H})$ is $w_{(\mu,\lambda)}$-$\mathcal{H}$-c but not $w^*_{(\mu,\lambda)}$-$\mathcal{H}$-c.*

(i) Let $a \in X$. Then $V_1 = \{a,b\}$, $V_2 = \{a,c,d\}$ and Y are the λ-open sets containing $f(a)$ in (Y,λ). Now $V_1^* = V_2^* = Y$ and $c^*_\lambda(V_1) = c^*_\lambda(V_2) = Y$. There exist a μ-open set $U = \{a,b\}$ of (X,μ) containing a such that $f(U) \subset c^*_\lambda(V)$, where V is a λ-open set in (Y,λ) containing $f(a)$.

(ii) Let $b \in X$. Then $V = \{a,b\}$ and Y are the λ-open sets containing $f(b)$ in (Y,λ). Now $V^* = Y$ and $c^*_\lambda(V) = Y$. There exist a μ-open set $U = \{a,b\}$ of (X,μ) containing b such that $f(U) \subset c^*_\lambda(V)$.

(iii) Let $c \in X$. Then $V = \{a,c,d\}$ and Y are the λ-open sets containing $f(c)$ in (Y,λ). Now $V^* = Y$ and $c^*_\lambda(V) = Y$. There exist a μ-open set $U = \{a,b,c\}$ of (X,μ) containing c such that $f(U) \subset c^*_\lambda(V)$.

(iv) Let $d \in X$. Then $V = \{a,c,d\}$ and Y are the λ-open sets containing $f(d)$ in (Y,λ). Now $V^* = Y$ and $c^*_\lambda(V) = Y$. There exist a μ-open set $U = \{b,c,d\}$ of (X,μ) containing d such that $f(U) \subset c^*_\lambda(V)$.

By (i), (ii), (iii), and (iv), f is $w_{(\mu,\lambda)}$-$\mathcal{H}$-c. On the other hand, consider the λ-open set $V = \{a, c, d\}$ in (Y, λ). Now, $f_r^*(V) = V^* - i_\lambda(V) = \{b\}$. Since $f^{-1}(f_r^*(V)) = \{b\}$ and $\{b\}$ is not μ-closed in (X, μ), f is not $w^*_{(\mu,\lambda)}$-$\mathcal{H}$-c.

Example 2.19. *Let* $X = Y = \{a, b, c\}$, $\mu = \{\emptyset, \{a, b\}, \{c\}, X\}$, $\lambda = \{\emptyset, \{a\}, \{b, c\}, Y\}$ *and* $\mathcal{H} = \{\emptyset, \{c\}\}$. *The identity function* $f : (X, \mu) \to (Y, \lambda, \mathcal{H})$ *is* $w^*_{(\mu,\lambda)}$-$\mathcal{H}$*-c but not* $w_{(\mu,\lambda)}$-$\mathcal{H}$*-c.*

(i) Let $a \in X$. Then $V = \{a\}$ and Y are the λ-open sets containing $f(a)$ in (Y, λ). Now, $\{a\}^* = \{a\}$ and $f_r^*(V) = V^* - i_\lambda(V) = \emptyset$. Hence $f^{-1}(f_r^*(V)) = f^{-1}(\emptyset) = \emptyset$ and $\emptyset$ is μ-closed in (X, μ).

(ii) Let $b \in X$. Then $V = \{b, c\}$ and Y are the λ-open sets containing $f(b)$ in (Y, λ). Now, $(\{b, c\})^* = \{b, c\}$ and $f_r^*(V) = V^* - i_\lambda(V) = \emptyset$. Hence $f^{-1}(f_r^*(V)) = f^{-1}(\emptyset) = \emptyset$ and $\emptyset$ is μ-closed in (X, μ).

(iii) Let $c \in X$. Then $V = \{b, c\}$ and Y are the λ-open sets containing $f(c)$ in (Y, λ). Now, $(\{b, c\})^* = \{b, c\}$ and $f_r^*(V) = V^* - i_\lambda(V) = \emptyset$. Hence $f^{-1}(f_r^*(V)) = f^{-1}(\emptyset) = \emptyset$ and $\emptyset$ is μ-closed in (X, μ).

By (i), (ii), and (iii), f is $w^*_{(\mu,\lambda)}$-$\mathcal{H}$-c. On the other hand, consider the λ-open sets $V = \{a\}$ and (Y, λ) are containing $f(a)$ in Y. Now, $\{a\}^* = \{a\}$ and so, $c^*_\lambda(V) = \{a\}$. Note that the μ-open sets of (X, μ) containing a are $U = \{a, b\}$ and X. Further $f(U) = U \nsubseteq c^*_\lambda(V)$ and $f(X) = Y \nsubseteq c^*_\lambda(V)$. Therefore f is not $w_{(\mu,\lambda)}$-$\mathcal{H}$-c.

Definition 2.20. *A hereditary generalized topological space* $(X, \mu, \mathcal{H})$ *is said to be* $F_\mu\mathcal{H}^*$-space, *if* $c_\mu(U) \subseteq U^*$ *for every* μ*-open set* $U \subset X$.

Theorem 2.21. *Let* $(X, \mu, \mathcal{H})$ *be an* $F_\mu\mathcal{H}^*$*-space and* $A \in \mu$. *Then the following properties are hold:*

(1) $A^* = c^*_\mu(A) = (c_\mu(A))^* = c_\mu(A^*) = c^*_\mu(A^*)$,

(2) $c^*_\mu(c_\mu(A)) = c_\mu(c^*_\mu(A)) = c^*_\mu(A^*)$.

Proof. 1. Let $(X,\mu,\mathcal{H})$ be an $F_\mu\mathcal{H}^*$-space and $A \in \mu$. Then $c_\mu(A) \subseteq A^*$. Thus $(c_\mu(A))^* \subset (A^*)^* \subset A^*$ by Lemma 1.3. Also $A \subseteq c_\mu(A)$, $A^* \subseteq (c_\mu(A))^*$ by Lemma 1.3. Therefore $A^* = (c_\mu(A))^*$. By Lemma 1.3, $A^* = c_\mu(A^*)$. Since $(X,\mu,\mathcal{H})$ is an $F_\mu\mathcal{H}^*$-space, $A^* \subseteq c^*_\mu(A) \subseteq c_\mu(A) \subseteq A^*$. Thus, $A^* = c^*_\mu(A) = c_\mu(A)$. Now, $c^*_\mu(A^*) = c^*_\mu(c_\mu(A)) = c_\mu(A) \cup (c_\mu(A))^* = A^* \cup A^* = A^*$. Hence we obtain $A^* = c^*_\mu(A) = (c_\mu(A))^* = c_\mu(A^*) = c^*_\mu(A^*)$.

(2) Follows from (1).

Lemma 2.22. *If a hereditary generalized topological space $(Y,\lambda,\mathcal{H})$ is $F_\lambda\mathcal{H}^*$-space and a function $f:(X,\mu) \to (Y,\lambda,\mathcal{H})$ is $w_{(\mu,\lambda)}$-$\mathcal{H}$-c, then $c^*_\mu(f^{-1}(V)) \subset f^{-1}(c^*_\lambda(V))$ for each λ-open set $V \subset Y$.*

Proof. Let $x \in c^*_\mu(f^{-1}(V))$. Assume that $x \notin f^{-1}(c^*_\lambda(V))$. Then $f(x) \notin c^*_\lambda(V)$, we have $f(x) \notin V$ and $f(x) \notin V^*$. Since Y is $F_\lambda\mathcal{H}^*$-space, $f(x) \notin c_\lambda(V)$. Hence there exist a λ-open set W containing $f(x)$ such that $W \cap V = \emptyset$. Since V is λ-open, $V \cap c_\lambda(W) = \emptyset$ and hence we have $V \cap c^*_\lambda(W) = \emptyset$. Since f is $w_{(\mu,\lambda)}$-$\mathcal{H}$-c, there exist a μ-open set $U \subset X$ containing x such that $f(U) \subset c^*_\lambda(W)$. Thus we obtain $f(U) \cap V = \emptyset$. On the other hand, $x \in c^*_\mu(f^{-1}(V))$ and we have $x \in c_\mu(f^{-1}(V))$ and hence $U \cap f^{-1}(V) \neq \emptyset$. Thus $f(W) \cap V \neq \emptyset$, a contradiction so, $c^*_\mu(f^{-1}(V)) \subset f^{-1}(c^*_\lambda(V))$.

Theorem 2.23. *Let (X,μ) be a μ-regular space and $(Y,\lambda,\mathcal{H})$ be an $F_\lambda\mathcal{H}^*$-space. A function $f:(X,\mu) \to (Y,\lambda,\mathcal{H})$ is $\theta(\mu,\lambda)$-continuous if and only if it is $w_{(\mu,\lambda)}$-$\mathcal{H}$-c.*

Proof. Let f be $\theta(\mu,\lambda)$-continuous, $x \in X$ and V be any λ-open set of Y containing $f(x)$. Since f is $\theta(\mu,\lambda)$-continuous, there exists a μ-open neighbourhood U of x such that $f(c_\mu(U)) \subseteq c_\lambda(V)$. Since $(Y,\lambda,\mathcal{H})$ is an $F_\lambda\mathcal{H}^*$-space, $f(U) \subseteq f(c_\mu(U)) \subseteq c_\lambda(V) \subseteq V^* \subseteq V^* \cup V \subseteq c_\lambda^*(V)$. Thus f is $w_{(\mu,\lambda)}$-$\mathcal{H}$-c. Conversely, let f be $w_{(\mu,\lambda)}$-$\mathcal{H}$-c, $x \in X$ and V be any λ-open set of Y containing $f(x)$. Since f is $w_{(\mu,\lambda)}$-$\mathcal{H}$-c, there exists a μ-open neighbourhood U of x such that $f(U) \subseteq c_\lambda^*(V)$. Since $\lambda \subseteq \lambda^*$, $f(U) \subseteq c_\lambda^*(V) \subseteq c_\lambda(V)$. Since (X,μ) is a μ-regular space, there exists a μ-open neighbourhood W of x such that $x \in W \subseteq c_\mu(W) \subseteq U$ by Lemma 1.6. Then $f(c_\mu(W)) \subseteq f(U) \subseteq c_\lambda(V)$. Thus f is $\theta(\mu,\lambda)$-continuous.

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