Jordan Journal of Mathematics and Statistics (JJMS) 6(3), 2013, pp.169 -181

TENSOR PRODUCT OPERATORS INDUCE DYNAMICAL SYSTEM ON WEIGHTED LOCALLY CONVEX SPACE

D.SENTHILKUMAR [(1)], P.CHANDRA KALA [(2)] AND T.PRASAD [(3)]

Abstract. In this paper we obtained dynamical system induced by tensor product of composition and multiplication operators on tensor product of weighted locally convex space of cross-sections $LV_0(X)(or LV_b(X))$ and holomorphic functions $HV_b(X,Y)(or HV_0(X,Y)$.

1. Introduction

Let X and Y be Banach spaces and $H(X,Y)$ be the space of all holomorphic functions from X to Y. If $X = Y$ we write $H(X)$ for $H(X,Y)$. Let $X \otimes Y$ be the tensor product of X and Y. Then each mapping $\varphi : X \to X$ gives rise to a linear transformation H_φ from $H(X,Y)$ itself, defined as

$$C_\varphi f = f \circ \varphi, \text{ for every } f \in H(X,Y),$$

and it is called a composition operator on $H(X,Y)$ induced by φ. Let $\pi : X \to C$ be a mapping. Then the scalar multiplication gives rise to a linear transformation M_π from $H(X,Y)$ itself, defined as

$$M_\pi f = \pi f, \text{ for every } f \in H(X,Y),$$

2000 *Mathematics Subject Classification.* 47 B 37,47 B 38, 47 B 07,47 D 03, 30 H 05.

Key words and phrases. Locally convex space, Tensor product operators, multiplication operator, composition operators and dynamical systems.

Received: Oct. 21, 2011 Accepted : Feb. 16 , 2013 .

where the product of functions is defined pointwise and is called a multiplication operator on $H(X, Y)$. R.K.Singh and J.S.Manhas studied dynamical systems induced by multiplication operator and composition operator on continuous, holomorphic functions and cross-sections. For more details see [1, 3, 4, 5, 6, 8, 9, 10, 11]. Let X and Y be a non-zero complex Banach spaces. The single tensor product of $x \in X$ and $y \in Y$ is a conjugate bilinear functional $x \otimes y : X \otimes Y \to \mathbf{C}$ defined by

$$(x \otimes y)(u, v) =< x; u >< y; v > \text{ forall } (u, v) \in X \otimes Y.$$

The tensor product space is the completion of the inner product space consisting of all(finite) sums of single tensors, which is a Hilbert space with respect to the inner product

$$< \sum_i x_i \otimes y_i ; \sum_j w_j \otimes z_j >= \sum_i \sum_j < x_i; w_j >< y_i; z_j > \text{ forall } \sum_i x_i \otimes y_i \text{ and } \sum_j w_j \otimes z_j \text{ in } X \otimes Y,$$

(The norm on $X \otimes Y$ is the one generated by the above inner product.) By an operator on a normed space X we mean a bounded linear transformation of X into itself. Let $B[X]$ be the normed algebra(equipped with the induced uniform norm) of all operators on X. The tensor product of two operators A and B on X and Y is the transformation $A \otimes B$ defined by

$$(A \otimes B) \sum_i x_i \otimes y_i = \sum_i A_{x_i} \otimes B_{y_i} \text{ for every } \sum_i x_i \otimes y_i \in X \otimes Y,$$

which is an operator in $B[X \otimes Y]$. For an expository paper containing the essential properties of tensor products [2, 7]. Now, for $f \in H(X, Y)$, we have

$$\|f \otimes g\| = \sup\{\|f(x)\| \|g(y)\| : x \in X \text{ and } y \in Y\}.$$

Then it has basis of closed absolutely convex neighbourhoods of the origin of the form

$$B = \{f \otimes g : f, g \in H(X, Y) \ni \|f \otimes g\| \leq 1\}.$$

Let G be a topological group with e as the identity, let X be a topological space and $\pi : G \times X \to X$ be the continuous map such that (i) $\pi(e, x) = x$ for every $x \in X$. (ii) $\pi(s + t, x) = \pi(s, \pi(t, x))$ for every $t, s \in G, x \in X$. Then the triple (G, X, π) is called a transformation group, X is a state space. If $G = (\mathbb{R}, +)$ the corresponding transformation group is called a dynamical system. The transformation group $(\mathbb{R}, X, \pi)$ is known as continuous dynamical system. If X is a Banach space and $\pi(t, \alpha x + \beta y) = \alpha\pi(t, x) + \beta\pi(t, y)$for $t \in \mathbb{R}, \alpha, \beta \in \mathbb{C}, x, y \in X$ then $(\mathbb{R}^+, X, \pi)$ is called a linear dynamical system.

In this paper we studied a tensor product of two operators like composition operator and multiplication operator on holomorphic function spaces and tensor product of weighted locally convex space of cross-sections $LV_0(X)(or LV_b(X))$(For definition see section 2) and holomorphic functions $HV_b(X, Y)(or HV_0(X, Y)$(For definition see section 3). Then we proved that this operator induce dynamical system on holomorphic function spaces and tensor product of weighted locally convex space of cross-sections $LV_0(X)(or LV_b(X))$ and holomorphic functions $HV_b(X, Y)(or HV_0(X, Y)$.
This paper contains three sections, in the first section we obtained dynamical system induced by product of composition and multiplication operator on tensor product of holomorphic function spaces. Section two consists the product of composition and multiplication operator induce dynamical system on tensor product of weighted locally convex space of cross-sections. In the last section we wide up with the product of composition and multiplication operator induce dynamical system on tensor product of weighted locally convex space of holomorphic functions.

2. Tensor product of composition and multiplication operator on holomorphic function spaces

Let $\pi_t : X \to \mathbb{R}$ defined by $\pi_t(x) = e^{th(x)}$ for all $t \in \mathbb{R}$ and $x \in X$, where $h \in H_b(X, \mathbb{R})$ (the Banach algebra of all bounded analytic functions from X to $\mathbb{R}$)

and $\|h\|_\infty = \sup\{\|h(x)\| : x \in X\}$. Also $\varphi_t : \mathbb{R} \to \mathbb{R}$ is defined by $\varphi_t(\omega) = t + \omega$ the self-map.

Theorem 1.1 Let $\varphi : X \to X$ and $\pi_t : X \to \mathbf{C}$ be a continuous functions. Then $(C_\varphi \otimes M_{\pi_t})(f \otimes g)$ is bounded for every $t \in \mathbb{R}$, $f \otimes g \in H(X) \otimes H(X)$.

Proof. We shall show that $C_\varphi f \otimes M_{\pi_t} g$ is continuous at the origin. We claim that $C_\varphi \otimes M_{\pi_t}(B) \subseteq B$. We have

$$\begin{aligned}
&\|C_\varphi f \otimes M_{\pi_t} g\| \\
&\quad = \sup \|C_\varphi f(x)\| \|M_{\pi_t} g(x)\| \text{ for every } t \in \mathbb{R}, \text{ for every } x \otimes x \in X \otimes X \\
&\quad = \sup \|f \circ \varphi(x)\| \|\pi_t(x) g(x)\| \text{ for every } x \otimes x \in X \otimes X \\
&\quad \leq \sup \|f(\varphi(x))\| \|e^{th(x)}\| \|g(x)\| \\
&\quad \leq \sup \|f(x)\| e^{|t| \|h\|_\infty} \|g(x)\| \\
&\quad \leq \sup \|f(x)\| \|g(x)\| \text{ as } t \to 0 \\
&\quad \leq \|(f \otimes g)(x)\| \\
&\quad \leq 1.
\end{aligned}$$

Therefore $C_\varphi \otimes M_{\pi_t}$ is continuous at the origin. Hence proved. □

Theorem 1.2 Let $B(E)$ be the Banach algebra of all bounded linear operators on E. Let $H^\infty(X, B(E))$ be the space of bounded functions from X to $B(E)$. Let $h_\alpha(\varphi_{t_\alpha})$ converges to $h(\varphi_t)$ in $H^\infty(X, B(E))$ and let f_α be a sequence converging to f in $H(X, E)$. Then the product of $f_\alpha h_\alpha(\varphi_{t_\alpha})$ converges to $f h(\varphi_t)$ in $H(X, E) \otimes H(X, E)$.

Proof. Let $h_\alpha(\varphi_{t_\alpha})$ converges to $h(\varphi_t)$ in $H^\infty(X, B(E))$. Then

$$\begin{aligned}
&\|f_n h_n(\varphi_{t_n}) - f h(\varphi_t)\|_E \\
&\quad = \sup\{\|f_n(x)h_n(\varphi_{t_n}(y)) - f(x)h(\varphi_t(y))\| : x \otimes y \in X \otimes E\} \\
&\quad \leq \sup\{\|f_n(x)h_n(\varphi_{t_n}(y)) - f_n(x)h(\varphi_t(y))\| : x \otimes y \in X \otimes E\} \\
&\qquad + \sup\{\|f_n(x)h(\varphi_t(y)) - f(x)h(\varphi_t(y))\| : x \otimes y \in X \otimes E\} \\
&\quad \leq \sup\{\|f_n(x)\|\|h_n(\varphi_{t_n}(y)) - h(\varphi_t(y))\| : x \otimes y \in X \otimes E\} \\
&\qquad + \sup\{\|f_n(x) - f(x)\|\|h(\varphi_t(y))\| : x \otimes y \in X \otimes E\} \\
&\quad \leq \|h_n(\varphi_{t_n}) - h(\varphi_t)\|_\infty \|f_n\|_E + \|h(\varphi_t)\|_\infty \|f_n - f\|_E \to 0
\end{aligned}$$

$$\text{as } \|h_n(\varphi_{t_n}) - h(\varphi_t)\|_\infty \to 0 \; and \|f_n - f\|_E \to 0.$$

□

Theorem 1.3 Let $\bigtriangledown : \mathbb{R} \times H(\mathbb{R}) \otimes H(\mathbb{R}) \to H(\mathbb{R}) \otimes H(\mathbb{R})$ be the function defined by $\bigtriangledown(t, f \otimes g) = C_{\varphi_t} \otimes M_{\pi_t}(f \otimes g)$ for all $t \in \mathbb{R}$ and $f \otimes g \in H(\mathbb{R}) \otimes H(\mathbb{R})$. Then $\bigtriangledown$ is a linear dynamical system on $H(\mathbb{R}) \otimes H(\mathbb{R})$.

Proof. Since $C_{\varphi_t} \otimes M_{\pi_t}$ is a tensor product on $H(\mathbb{R}) \otimes H(\mathbb{R})$ for every $t \in \mathbb{R}$ and $f, g \in H(\mathbb{R}) \otimes H(\mathbb{R})$.It can be easily seen that $\bigtriangledown(0, f \otimes g) = f \otimes g$ and $\bigtriangledown(t + s, f \otimes g) = \bigtriangledown(t, \bigtriangledown(s, f \otimes g)$.In order to show that $\bigtriangledown(t, f \otimes g)$ is a dynamical system on $H(\mathbb{R}) \otimes H(\mathbb{R})$, it suffices to show that $\bigtriangledown$ is continuous.

Let $g_\alpha \to g$ and let $(t_\alpha, f_\alpha \otimes g_\alpha)$ be a net in $\mathbb{R} \times H(\mathbb{R}) \otimes H(\mathbb{R})$ such that $(t_\alpha, f_\alpha \otimes g_\alpha) \to$

$(t, f \otimes g)$. We shall show that $\bigtriangledown(t_\alpha, f_\alpha \otimes g_\alpha) \to \bigtriangledown(t, f \otimes g)$. Then

$$\begin{aligned}
\| \bigtriangledown (t_\alpha, f_\alpha \otimes g_\alpha) - \bigtriangledown(t, f \otimes g)\| \\
&= \|C_{\varphi_{t_\alpha}} f_\alpha \otimes M_{\pi_{t_\alpha}} g_\alpha - C_{\varphi_t} f \otimes M_{\pi_t} g\| \\
&= \sup\{\|C_{\varphi_{t_\alpha}} f_\alpha(\omega) M_{\pi_{t_\alpha}} g_\alpha(\xi) \\
&\quad -C_{\varphi_t} f(\omega) M_{\pi_t} g(\xi)\| \text{ for every } \omega \otimes \xi \in \mathbb{R} \otimes \mathbb{R} \ \} \\
&\leq \sup\{\|C_{\varphi_{t_\alpha}} f_\alpha(\omega) - C_{\varphi_t} f(\omega)\| \|M_{\pi_{t_\alpha}} g_\alpha(\xi)\| \\
&\quad : \forall \omega \otimes \xi \in \mathbb{R} \otimes \mathbb{R}\} \\
&\quad + \sup\{\|C_{\varphi_t} f(\omega)\| \|M_{\pi_{t_\alpha}} g_\alpha(\xi) - M_{\pi_t} g(\xi)\| \\
&\quad : \forall \omega \otimes \xi \in \mathbb{R} \otimes \mathbb{R}\} \\
&\leq \sup\{\|f_\alpha(\varphi_{t_\alpha}(\omega)) - f(\varphi_t(\omega))\| \|g_\alpha(\xi)\| \|\pi_{t_\alpha}(\xi))\| \\
&\quad : \forall \omega \otimes \xi \in \mathbb{R} \otimes \mathbb{R}\} \\
&\quad - \sup\{\|f(\varphi_t(\omega))\| \|\pi_{t_\alpha}(\xi)) g_\alpha(\xi) - \pi_{t_\alpha}(\xi)) g(\xi)\| \\
&\quad : \forall \omega \otimes \xi \in \mathbb{R} \otimes \mathbb{R}\} \\
\to 0
\end{aligned}$$

as $|g_\alpha - g| \to 0$ and $f(\varphi_{t_\alpha}) \to f(\varphi_t)$.

Therefore $\bigtriangledown(t, f \otimes g)$ is a dynamical system on $H(\mathbb{R}) \otimes H(\mathbb{R})$. □

Theorem 1.4 Let $\varphi : X \to X$ and $\pi_t : X \to \mathbf{C}$ be a continuous functions. Then $(M_{\pi_t} \otimes C_\varphi)(f \otimes g)$ is bounded for every $t \in \mathbb{R}$, $f \otimes g \in H(X) \otimes H(X)$.

Theorem 1.5 Let $\bigtriangledown : \mathbb{R} \times H(\mathbb{R}) \otimes H(\mathbb{R}) \to H(\mathbb{R}) \otimes H(\mathbb{R})$ be the function defined by $\bigtriangledown(t, f \otimes g) = M_{\pi_t} \otimes C_{\varphi_t}(f \otimes g)$ for all $t \in \mathbb{R}$ and $f \otimes g \in H(\mathbb{R}) \otimes H(\mathbb{R})$. Then $\bigtriangledown$

is a linear dynamical system on $H(\mathbb{R}) \otimes H(\mathbb{R})$.

Proof. Since $M_{\pi_t} \otimes C_{\varphi_t}$ is a tensor product on $H(\mathbb{R}) \otimes H(\mathbb{R})$ for every $t \in \mathbb{R}$ and $f \otimes g \in H(\mathbb{R}) \otimes H(\mathbb{R})$. It can be easily seen that $\bigtriangledown(0, f \otimes g) = f \otimes g$ and $\bigtriangledown(t + s, f \otimes g) = \bigtriangledown(t, \bigtriangledown(s, f \otimes g)$. In order to show that $\bigtriangledown(t, f \otimes g)$ is a dynamical system on $H(\mathbb{R}) \otimes H(\mathbb{R})$, it suffies to show that $\bigtriangledown$ is continuous. Let $(t_\alpha, f_\alpha \otimes g_\alpha)$ be a net in $R \times H(\mathbb{R}) \otimes H(\mathbb{R})$ such that $(t_\alpha, f_\alpha \otimes g_\alpha) \to (t, f \otimes g)$. We shall show that $\bigtriangledown(t_\alpha, f_\alpha \otimes g_\alpha) \to \bigtriangledown(t, f \otimes g)$. Then $\bigtriangledown(t, f \otimes g)$ is a dynamical system on $H(\mathbb{R}) \otimes H(\mathbb{R})$. Since by theorem 1.3. □

3. Dynamical system induced by tensor product operators on weighted spaces of cross-sections

Let X be a Hausdorff topological space.A vector-fibration over X is a pair $(X, (F_x))_{x \in X}$, where each F_x is a vector space over the field $\mathbb{K}$ (where $\mathbb{K}$=$\mathbb{R}$ (or)$\mathbb{C}$). A cross-section over X is then any element of the Cartesian product $\pi_{x \in X} F_x$. The Cartesian product $\pi_{x \in X} F_x$ is a made a vector space in the usual way and a vector space of cross-section over X is by definition any vector subspace of $\pi_{x \in X} F_x$.By a weight on X,we mean a function v on X such that $v(x)$ is a semi-norm on F_x for each $x \in X$. By $v \leq u$,we mean that $v_x \leq u_x$ for each $x \in X$. A set V of weights on X is said to be directed if, $\forall$ pair $u, v \in V$ and $\lambda > 0$, $\exists\ \omega \in V$ such that $\lambda u \leq \omega$ and $\lambda v \leq \omega$. Here after we assume that each set of weights is directed. We write $V > 0$, if given $x \in X$ and $y \in F_x$, there is some $v \in V$ for which $v_x(y) > 0$. A set V of weights on X which additionally satisfies $V > 0$ will be referred to as a system of weights on X. If f is a cross-section over X and v is a weight on X, then we will denote by $v[f]$ the positive-valued function on X which takes x into $v_x[f(y)]$. We denote by $L(X)$ a vector space of cross-sections over X. Now the weighted spaces of cross-sections over X with respect to the system of weights V on X are introduced as follows:

$LV_0(X) = \{f \otimes g \in L(Y) : v[f \otimes g]$ is upper semi continuous and vanishes at infinity on X for each $v \in V\}$
and

$$LV_b(X) = \{f \otimes g \in L(Y) : v[f \otimes g] \text{ is a bounded function on X for each } v \in V\}.$$

Then $LV_0(X)$ and $LV_b(X)$ are vector spaces and $LV_0(X) \subseteq LV_b(X)$. Now for $v \in V$ and $f \in L(X)$, if we put $\|f \otimes g\|_v = \sup\{v_y[f(x) \otimes g(x)] : x \in X\}$, then $\|.\|_v$ can be regarded as a seminorm on either $LV_0(X)$ or $LV_b(X)$, and the family of seminorms $\{\|.\|_v : v \in V\}$ defines a Hausdorff locally convex topology on each of these spaces. This topology is denoted by ι_v and the vector space endowed with ι_v are called weighted locally convex spaces of cross-sections. Since V is a directed set of weights, ι_v has a basis of closed absolutely convex neighborhood of the form

$$B_v = \{f \otimes g \in LV_b(X) : f, g \in LV_0(X, Y) \ni \|f\|_v \leq 1\}.$$

Theorem 2.1 Let $\varphi : X \to X$ and $\pi_t : X \to \mathbf{C}$ be two mappings. Then $C_\varphi f \otimes M_{\pi_t} g$ is bounded for every $t \in \mathbb{R}$, $f \otimes g \in LV_0(X) \otimes LV_0(X)$.

Theorem 2.2 Let $B(E)$ be the Banach algebra of all bounded linear operators on E. Let $L^\infty(X, B(E))$ be the space of bounded functions. Let $h_\alpha(\varphi_{t_\alpha})$ converges to $h(\varphi_t)$ in $L^\infty(X, B(E))$ and let f_α be a sequence converging to f in $LV_0(X, E)$. Then the product of $f_\alpha h_\alpha(\varphi_{t_\alpha})$ converges to $fh(\varphi_t)$ in $LV_0(X, E) \otimes LV_0(X, E)$.

Theorem 2.3 Let $\bigtriangledown : \mathbb{R} \times LV_0(\mathbb{R}) \otimes LV_0(\mathbb{R}) \to L(\mathbb{R}) \otimes H(\mathbb{R})$ be the function defined by $\bigtriangledown(t, f \otimes g) = C_{\varphi_t} \otimes M_{\pi_t}(f \otimes g)$ for all $t \in \mathbb{R}$ and $f \otimes g \in L(\mathbb{R}) \otimes LV_0(X, E)$. Then

$\bigtriangledown$ is a linear dynamical system on $LV_0(\mathbb{R}) \otimes LV_0(\mathbb{R})$.

Proof. Since $C_{\varphi_t} \otimes M_{\pi_t}$ is a tensor product on $LV_0(\mathbb{R}) \otimes LV_0(\mathbb{R})$ for every $t \in \mathbb{R}$ and $f, g \in LV_0(\mathbb{R}) \otimes LV_0(\mathbb{R})$. It can be easily seen that $\bigtriangledown(0, f \otimes g) = f \otimes g$ and $\bigtriangledown(t+s, f \otimes g) = \bigtriangledown(t, \bigtriangledown(s, f \otimes g)$. In order to show that $\bigtriangledown(t, f \otimes g)$ is a dynamical system on $LV_0(\mathbb{R}) \otimes LV_0(\mathbb{R})$, it suffies to show that $\bigtriangledown$ is continuous. Let $(t_\alpha, f_\alpha \otimes g_\alpha)$ be a net in $R \times LV_0(\mathbb{R}) \otimes LV_0(\mathbb{R})$ such that $(t_\alpha, f_\alpha \otimes g_\alpha) \to (t, f \otimes g)$. We shall show that $\bigtriangledown(t_\alpha, f_\alpha \otimes g_\alpha) \to \bigtriangledown(t, f \otimes g)$. Therefore $\bigtriangledown(t, f \otimes g)$ is a dynamical system on $LV_0(\mathbb{R}) \otimes LV_0(\mathbb{R})$. Since by theorem 1.3.

□

Theorem 2.4 Let $\varphi : X \to X$ and $\pi_t : X \to \mathbf{C}$ be a continuous functions. Then $M_{\pi_t} f \otimes C_\varphi g$ is bounded for every $t \in \mathbb{R}$, $f \otimes g \in LV_0(\mathbb{R}) \otimes LV_0(\mathbb{R})$.

Theorem 2.5 Let $\bigtriangledown : \mathbb{R} \times LV_0(\mathbb{R}) \otimes LV_0(\mathbb{R}) \to L(\mathbb{R}) \otimes L(\mathbb{R})$ be the function defined by $\bigtriangledown(t, f \otimes g) = M_{\pi_t} \otimes C_{\varphi_t}(f \otimes g)$ for all $t \in \mathbb{R}$ and $f \otimes g \in L(\mathbb{R}) \otimes L(\mathbb{R})$. Then $\bigtriangledown$ is a linear dynamical system on $L(\mathbb{R}) \otimes L(\mathbb{R})$.

Proof. Since $M_{\pi_t} \otimes C_{\varphi_t}$ is a tensor product on $L(\mathbb{R}) \otimes L(\mathbb{R})$ for every $t \in \mathbb{R}$ and $f \otimes g \in LV_0(\mathbb{R}) \otimes LV_0(\mathbb{R})$. It can be easily seen that $\bigtriangledown(0, f \otimes g) = f \otimes g$ and $\bigtriangledown(t+s, f \otimes g) = \bigtriangledown(t, \bigtriangledown(s, f \otimes g)$. In order to show that $\bigtriangledown(t, f \otimes g)$ is a dynamical system on $LV_0(\mathbb{R}) \otimes LV_0(\mathbb{R})$, it suffies to show that $\bigtriangledown$ is continuous. Let $(t_\alpha, f_\alpha \otimes g_\alpha)$ be a net in $R \times LV_0(\mathbb{R}) \otimes LV_0(\mathbb{R}))$ such that $(t_\alpha, f_\alpha \otimes g_\alpha) \to (t, f \otimes g)$. We shall show that $\bigtriangledown(t_\alpha, f_\alpha \otimes g_\alpha) \to \bigtriangledown(t, f \otimes g)$. Then $\bigtriangledown(t, f \otimes g)$ is a dynamical system on $H(\mathbb{R}) \otimes H(\mathbb{R})$. Since by theorem 1.3. □

Note 2.6

All results in thsi section hold if we replace $LV_b(X, E)$ instead of $LV_0(X, E)$.

4. Dynamical systems and tensor product operator on weighthed locally convex space of holomorphic functions

Let X be a Hausdorff topological space and $H(X, E)$ be the collection of holomorphic functions from X into E. Let V be a set of non-negative upper-semi continuous functions on X. If V is a set of weights on X such that given any $x \in X$, there is some $v \in V$ for which $v(x) > 0$. We write $V > 0$. A set V of weights on X is said to be directed upward provided for every pair u_1, u_2 in V and $\alpha > 0$ there exists $v \in V$ such that $\alpha u_i \leq v$ (pointwise on X) for $i = 1, 2$. By a system of weights,we mean a set V of weights on X with additionally satisfies $V > 0$. Let $cs(E)$ be the set of all continuous functions from X into E. If V is a system of weights on X, then the pair (X, V) is called the weighted topological system. Associated with each weighted topological system (X, V), we have the weighted spaces of holomorphic E-valued functions defined as:

$$HV_0(X, E) = \{f \otimes g \in H(X, E) \otimes H(X, E) : vf \otimes g \text{ vanishes at infinity on } X \text{ for each } v \in V\}$$

$$HV_b(X, E) = \{f \otimes g \in H(X, E) \otimes H(X, E) : vf(x) \otimes g(x) \text{ is bounded in } E \text{ for all } v \in V\}.$$

Let $v \in V$ $q \in cs(E)$ and $f \otimes g \in H(X, E) \otimes H(X, E)$. If we define

$$\|f \otimes g\|_v = \sup\{v(x)\|f(x)\|\|g(y)\| : x \in X \text{ and } y \in Y\},$$

then $\|.\|_v$ can be regarded as a seminorm on either $HV_0(X, E)$, $HV_b(X, E)$ and the family $\{\|.\|_{v,q} : v \in V, q \in cs(E)\}$ of seminorms defines a Hausdorff locally convex topology on each of these spaces. This topology will be denoted by ω_v and the vector spaces $HV_0(X, E)$ and $HV_b(X, E)$ endowed with ω_v are called the weighted locally convex space of vector-valued holomorphic functions. It has a basis of closed absolutely convex neighborhoods of the origin of the form,

$$B_{v,q} = \{f \otimes g : f, g \in H(X, Y) \ni \|f \otimes g\|_{v,q} \leq 1\}.$$

Also, $HV_0(X,E)$ is a closed subspace of $HV_b(X,E)$.

Theorem 3.1 Let $B(E)$ be the Banach algebra of all bounded linear operators on E. Let $H^\infty(X, B(E))$ be the space of bounded functions. Let $h_\alpha(\varphi_{t_\alpha})$ converges to $h(\varphi_t)$ in $H^\infty(X, B(E))$ and let f_α be a sequence converging to f in $HV_0(X,E)$. Then the product of $f_\alpha h_\alpha(\varphi_{t_\alpha})$ converges to $fh(\varphi_t)$ in $HV_0(X,E) \otimes HV_0(X,E)$.

Theorem 3.2 Let $\bigtriangledown : \mathbb{R} \times HV_0(\mathbb{R}) \otimes HV_0(\mathbb{R}) \to H(\mathbb{R}) \otimes H(\mathbb{R})$ be the function defined by $\bigtriangledown(t, f \otimes g) = C_{\varphi_t} \otimes M_{\pi_t}(f \otimes g)$ for all $t \in \mathbb{R}$ and $f \otimes g \in H(\mathbb{R}) \otimes HV_0(X,E)$. Then $\bigtriangledown$ is a linear dynamical system on $HV_0(\mathbb{R}) \otimes HV_0(\mathbb{R})$.

Proof. Since $C_{\varphi_t} \otimes M_{\pi_t}$ is a tensor product on $HV_0(\mathbb{R}) \otimes HV_0(\mathbb{R})$ for every $t \in \mathbb{R}$ and $f, g \in HV_0(\mathbb{R}) \otimes HV_0(\mathbb{R})$. It can be easily seen that $\bigtriangledown(0, f \otimes g) = f \otimes g$ and $\bigtriangledown(t+s, f \otimes g) = \bigtriangledown(t, \bigtriangledown(s, f \otimes g)$. In order to show that $\bigtriangledown(t, f \otimes g)$ is a dynamical system on $HV_0(\mathbb{R}) \otimes HV_0(\mathbb{R})$, it suffies to show that $\bigtriangledown$ is continuous. Let $(t_\alpha, f_\alpha \otimes g_\alpha)$ be a net in $R \times HV_0(\mathbb{R}) \otimes HV_0(\mathbb{R})$ such that $(t_\alpha, f_\alpha \otimes g_\alpha) \to (t, f \otimes g)$. We shall show that $\bigtriangledown(t_\alpha, f_\alpha \otimes g_\alpha) \to \bigtriangledown(t, f \otimes g)$. Therefore $\bigtriangledown(t, f \otimes g)$ is a dynamical system on $HV_0(\mathbb{R}) \otimes HV_0(\mathbb{R})$. Since by theorem 1.3.

□

Theorem 3.3 Let $\bigtriangledown : \mathbb{R} \times HV_0(\mathbb{R}) \otimes HV_0(\mathbb{R}) \to H(\mathbb{R}) \otimes H(\mathbb{R})$ be the function defined by $\bigtriangledown(t, f \otimes g) = M_{\pi_t} \otimes C_{\varphi_t}(f \otimes g)$ for all $t \in \mathbb{R}$ and $f \otimes g \in H(\mathbb{R}) \otimes H(\mathbb{R})$. Then $\bigtriangledown$ is a linear dynamical system on $H(\mathbb{R}) \otimes H(\mathbb{R})$.

Proof. Since $M_{\pi_t} \otimes C_{\varphi_t}$ is a tensor product on $H(\mathbb{R}) \otimes H(\mathbb{R})$ for every $t \in \mathbb{R}$ and $f \otimes g \in HV_0(\mathbb{R}) \otimes HV_0(\mathbb{R})$. It can be easily seen that $\bigtriangledown(0, f \otimes g) = f \otimes g$ and $\bigtriangledown(t+s, f \otimes g) = \bigtriangledown(t, \bigtriangledown(s, f \otimes g)$. In order to show that $\bigtriangledown(t, f \otimes g)$ is a dynamical system on $HV_0(\mathbb{R}) \otimes HV_0(\mathbb{R})$, it suffies to show that $\bigtriangledown$ is continuous. Let $(t_\alpha, f_\alpha \otimes g_\alpha)$ be a net in $R \times HV_0(\mathbb{R}) \otimes HV_0(\mathbb{R}))$ such that $(t_\alpha, f_\alpha \otimes g_\alpha) \to (t, f \otimes g)$. We shall show that $\bigtriangledown(t_\alpha, f_\alpha \otimes g_\alpha) \to \bigtriangledown(t, f \otimes g)$. Then $\bigtriangledown(t, f \otimes g)$ is a dynamical system on $H(\mathbb{R}) \otimes H(\mathbb{R})$. Since by theorem 1.3. □

Note 3.4

All results in this section hold if we replace $HV_b(X, E)$ instead of $HV_0(X, E)$.

Acknowledgment

The work was supported by Rajiv Gandhi National Fellowship under the University Grants Commission Fellowship and a bursary awarded to P.Chandra kala.

References

[1] C.Cowen and B.D.MacCluer, *Composition operator on spaces of analytic functions*, Studies in Advanced Mathematics,CRC press, Boca Raton, Fla, USA, 1995.

[2] C.S.Kubrusly, *A concise introduction to tensor product*, FJMS,22(2006),137-174.

[3] J.S.Manhas, *Weighted composition operators on weighted locally convex spaces of analytic functions*, Southeast Asian Bull.Math.29(2005), no.1, 127-140.

[4] J.S.Manhas, *Composition operators and Multiplication operator on weighted spaces of analytic functions*, Research article, International Journal of Mathematics and Mathematical sciences, vol 2007.

[5] J.S.Manhas, *Weighted composition operators on weighted spaces of vector-valued analytic functions*, J.Korean Math.soc.45(2008), no.5,1203-1220.

[6] Ruess.W.M and Summers.W.H, *Minimal sets of almost periodic motions*, Math.Ann., 276(1986), 145-158.

[7] Ryan.R.A, *Introduction to tensor products of Banach spaces*, Springer-Verlg, London, 2002.

[8] J.H.Shpiro , *Composition operators and classical function theory*, Universitext:Tracts in Mathematics, Springer, New York, NY, USA, 1993.

[9] R.K. Singh and J.S.Manhas, *Composition operators on function spaces*, North Holland Mathematics Studies, 1993.

[10] R.K. Singh and J.S.Manhas, *Multiplication operators and Dynamical systems on weighted spaces of cross-section*,Proc.AMS,vol 119, No 2, 1993.

[11] R.K. Singh and J.S.Manhas, *Weigted composition operators on weighted spaces of cross-sections*,Acta scientiarum Mathematicarum,vol 59,No 3-4,1994.

(1)Department of Mathematics, Government Arts College, Coimbatore-641 018, India

E-mail address, D.S: senthilsenkumhari@gmail.com

(2)Department of Mathematics, Government Arts College, Coimbatore-641 018, India

E-mail address, P.CK: kala.mathchandra@gmail.com

(3)Department of Mathematics, Government Arts College, Coimbatore-641 018, India

E-mail address, T.P: prasadvalapil@gmail.com