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TOPOLOGY ON GRILL M-SPACE

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Abstract. This paper is devoted to obtain a topology from a non topological space which is already in literature. Some characterizations of this topology will be discussed in detail.

1. Introduction

Study of ideal and grill on a topological space is going on from 1930 and 1947 respectively to till date. Mathematicians like Al-Omari and Noiri[1,2,3], Bandyopadhyay and Modak[4,13,14,15], Hamlett and Jankovic[8,10], Kuratowski[11], Vaidyanathaswamy[20], Natkaniec[16] Thron, Chattopadhyay, Njastad, and Choqet[5,6,7] had enriched this field and their contributions in this field is worthy. Recently Roy and Mukherjee[18] had used grill on topological space in different aspect. Using this concept, Noiri and Al-Omari in [2,1] had defined a new topology. More rscently Al-Omari and Noiri in [3] had introduced generalized space and in this space Modak et al[15] had made a new topology.

In this paper, we have introduced a new type of space which is the joint venture of Choqet's grill and Al-Omari and Noiri's generalized space. Also we shall define two

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operators and obtain a new topology. Further we shall characterize this topology with the help of topological properties.

2. Preliminaries

In this section we shall refer to some results and definitions which are relevant for this paper. We shall also prove some results which are preliminaries for this paper. At first we give formal definition of grill.

A subcollection $\mathcal{G}$ (not containing the empty set) of $\wp(X)$ is called a grill[7,19] on X if $\mathcal{G}$ satisfies the following conditions:

(1) $A \in \mathcal{G}$ and $A \subseteq B$ implies $B \in \mathcal{G}$.
(2) $A, B \subseteq X$ and $A \cup B \in \mathcal{G}$ implies that $A \in \mathcal{G}$ or $B \in \mathcal{G}$.

Definition 2.1.[3]. A subfamily $\mathcal{M}$ of the power set $\wp(X)$ of a nonempty set X is called a m-structure on X if $\mathcal{M}$ satisfies the following conditions:

(1) $\mathcal{M}$ contains ϕ and X,
(2) $\mathcal{M}$ is closed under the finite intersection.

The pair $(X, \mathcal{M})$ is called a m-space. An m-space $(X, \mathcal{M})$ with an grill $\mathcal{G}$ on X is called a grill m-space and is denoted as $(X, \mathcal{M}, \mathcal{G})$

Definition 2.2.[3]. A set $A \in \wp(X)$ is called a m-open set if $A \in \mathcal{M}$ and $B \in \wp(X)$ is called an m-closed set if $X \setminus B \in \mathcal{M}$. We define the m-interior of A and m-closure of A as follows $mInt(A) = \cup\{U : U \subseteq A,\ U \in \mathcal{M}\}$ and

$mCl(A) = \cap\{F : A \subseteq F, X \setminus F \in \mathcal{M}\}$.

Here we are mentioning two results which are used in this paper.

Theorem 2.3[15]. Let $(X, \mathcal{M})$ be an m-space. Then $x \in mCl(A)$ if and only if every m-open set U_x containing x, $U_x \cap A \neq \phi$.

Theorem 2.4[15]. Let $(X, \mathcal{M})$ be an m-space and $A \subseteq X$. Then $mInt(A) = X \setminus mCl(X \setminus A)$.

Theorem 2.5. Let $(X, \mathcal{M})$ be an m-space. Then for $G \in \mathcal{M}$, $G \cap mCl(A) \subseteq mCl(G \cap A)$.

Proof. Let $x \in G \cap mCl(A)$. Then $x \in G$ and $x \in mCl(A)$. Implies that $x \in G$ and for every m-open set U_x containing x, $U_x \cap A \neq \phi$. Again $G \cap U_x$ is an m-open set containing x, then $(G \cap U_x) \cap A \neq \phi$. Hence $x \in mCl(G \cap A)$. Therefore $G \cap mCl(A) \subseteq mCl(G \cap A)$. □

Definition 2.6[13]. Let $(X, \mathcal{M})$ be an m-space and $\mathcal{G}$ be a grill on X. Then a mapping $\varphi_{\mathcal{G}} : \wp(X) \rightarrow \wp(X)$ is defined by $\varphi_{\mathcal{G}}(A) = \varphi(A) = \{x \in X : A \cap U \in \mathcal{G}$ for all $U \in \mathcal{M}(\mathrm{x})\}$ and for each $A \in \wp(X)$, where $\mathcal{M}(\mathrm{x}) = \{U \in \mathcal{M}\text{: } x \in U\}$. The mapping φ is called the operator associated with the grill $\mathcal{G}$ and the m-structure $\mathcal{M}$ on X.

Remark 2.7[13]. Let $\mathcal{G}$ be a grill on the m-space $(X, \mathcal{M})$. We define a map $CL : \wp(X) \rightarrow \wp(X)$ by $CL(A) = A \cup \varphi(A)$, for all $A \in \wp(X)$. Then the map $'CL'$ is a Kuratowski closure operator. We will denote by $\tau_{\mathcal{MG}}$ the topology generated by CL, that is, $\tau_{\mathcal{MG}} = \{V \subseteq X : \ CL(X \setminus V) = X \setminus V\}$.

In this paper we shall denote the interior and the closure operator of $(X, \tau_{\mathcal{MG}})$ by $Int_{\mathcal{MG}}$ and $Cl_{\mathcal{MG}}$ respectively.

Theorem 2.8[13]. Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space. Then $\beta(\mathcal{M}, \mathcal{G}) = \{V \backslash \mathrm{G}\text{: } V \in \mathcal{M}, \mathrm{G} \notin \mathcal{G}\}$ is an open base for the topology $\tau_{\mathcal{MG}}$.

An important relation between interior and closure operator in topological space (X, τ) is; $IntA = X \backslash Cl(X \backslash A)$[11]. Similar, the relation also holds in m-space(Theorem 2.4). Hamlett and Jankovic in [8], Al-Omari and Noiri in [2] have defined similar relation using different types of operator on different spaces. We have already defined this type of relation in [13]. Here we also mention the following definition:

Definition 2.9[13]. Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space. An operator $\psi_{\varphi} : \wp(X) \rightarrow \mathcal{M}$

is defined as follows for every $A \in \wp(X)$, $\psi_\varphi(A) = \{x \in X$: there exists $U \in \mathcal{M}(\mathrm{x})$ such that $U \setminus A \notin \mathcal{G}\}$, hence we observe that $\psi_\varphi(A) = X \setminus \varphi(X \setminus A)$.

Now, we shall prove in the following some characterizations of grill m-spaces:

Theorem 2.10. Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space. Then $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$ if and only if $\varphi(X) = X$.

Proof. Suppose that $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$. It is obvious that $\varphi(X) \subseteq X$. For the reverse inclusion, let $x \in X$ but $x \notin \varphi(X)$. Then there exists $U \in \mathcal{M}(\mathrm{x})$, $U \cap X \notin \mathcal{G}$. Then $U \notin \mathcal{G}$, which is a contradiction with the fact that $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$. Hence, $\varphi(X) = X$. Conversely. Suppose that $\varphi(X) = X$. Let $\phi \neq V \in \mathcal{M}$. Then $V \cap X \neq \phi$. Since $\varphi(X) = X$, then $V \cap X \in \mathcal{G}$ which implies that $V \in \mathcal{G}$, and hence $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$. □

Corollary 2.11. Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space and $A \in \mathcal{M}$. Then $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$ if and only if $\varphi(A) = mCl(A)$.

Proof. Suppose that $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$. It is obvious that $\varphi(A) \subseteq mCl(A)$[13]. For the reverse inclusion, let $\alpha \in mCl(A)$. Then for every $U_\alpha \in \mathcal{M}(\alpha)$, $U_\alpha \cap A \neq \phi$ (from Theorem 2.3), implies that $U_\alpha \cap A \in \mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$. So $\alpha \in \varphi(A)$ and hence $\varphi(A) = mCl(A)$.

Converse part is obvious from Theorem 2.10. □

Joint result of the Theorem 2.10 and the Corollary 2.11 is:

Theorem 2.12. Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space. Then following properties are equivalent:

(1) $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$;
(2) For every $G \in \mathcal{M}$, $G \subseteq \varphi(G)$;
(3) $X = \varphi(X)$;
(4) If $A \in \mathcal{M}$, then $\varphi(A) = mCl(A)$.
(5) $\varphi(\psi_\varphi(A)) = mCl(\psi_\varphi(A))$.

Theorem 2.13. Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space and $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$. Then $\psi_\varphi(A) \setminus A = \phi$, for m-closed subset A.

Proof. Since, by Theorem 2.11 $\psi_\varphi(A) \setminus A = [X \setminus \varphi(X \setminus A] \setminus A = [X \setminus mCl(X \setminus A)] \setminus A = mInt(A) \setminus A = \phi$. □

Definition 2.14[12]. A subset A in a topological space (X, τ) is called semi-open if $A \subseteq cl(int(A))$.

The set of all semi-open sets in a topological space (X, τ) is denoted as $SO(X, \tau)$.

Definition 2.15[17]. A subset A in a topological space (X, τ) is called α-set if $A \subseteq int(cl(int(A)))$.

The set of all α-sets in a topological space (X, τ) is denoted as τ^α.

A topological space (X, τ) is said to be resolvable[9] if for a subset D of X both D and $X \setminus D$ are dense in (X, τ), otherwise it is said to be irresolvable.

The space of reals with usual topology provides an example of a resolvable space while any topological space with an isolated point furnishes for an irresolvable one.

3. ψ_φ-C set and Properties of $\psi_\varphi(X, \mathcal{M})$

This section is devoted to deal with a new type of set and its properties :

Definition 3.1. Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space. A subset A of X is called a ψ_φ-C set if $A \subseteq mCl(\psi_\varphi(A))$.

The collection of all ψ_φ-C sets in $(X, \mathcal{M}, \mathcal{G})$ is denoted as $\psi_\varphi(X, \mathcal{M})$.

Remark 3.2. It is obvious that $\mathcal{M} \subseteq \tau_{\mathcal{M}\mathcal{G}}$[13]$\subseteq \psi_\varphi(X, \mathcal{M})$, but the reverse inclusion does not hold in general.

Let $X = \{a, b, c, d\}$, $\mathcal{M} = \{\phi, X, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{G} = \{\{a\}, \{b\}, \{a, c\}, \{a, b\}, \{a, d\}, \{a, b, c\}, \{c, b, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c\}, \{b, d\}, \{b, c, d\}, X\}$. Consider $A = \{a, d\}$, then $\psi_\varphi(A) = \{a\}$. Thus $mCl(\psi_\varphi(A)) = \{a, d\}$. So $\{a, d\} \in \psi_\varphi(X, \mathcal{M})$.

But $\{a, d\} \notin \tau_{\mathcal{MG}}$.

Theorem 3.3. Let $\{A_\alpha : \alpha \in \Delta\}$ be a collection of nonempty ψ_φ-C sets in a grill m-space $(X, \mathcal{M}, \mathcal{G})$, then $\cup_{\alpha\in\Delta} A_\alpha \in \psi_\varphi(X, \mathcal{M})$.

Proof. For each $\alpha \in \Delta$, $A_\alpha \subseteq mCl(\psi_\varphi(A_\alpha)) \subseteq mCl(\psi_\varphi(\cup_{\alpha\in\Delta} A_\alpha))$[13]. This implies that $\cup_{\alpha\in\Delta} A_\alpha \subseteq mCl(\psi_\varphi(\cup_{\alpha\in\Delta} A_\alpha))$. Thus $\cup_{\alpha\in\Delta} A_\alpha \in \psi_\varphi(X, \mathcal{M})$. □

Remark 3.4. The intersection of two ψ_φ-C sets may not be a ψ_φ-C set in general. In Remark 3.1, consider $A = \{a, d\}$, and $B = \{b, c, d\}$, then $\varphi(X \setminus A) = \{b, c, d\}$ and $mCl(\psi_\varphi(A)) = \{a, d\}$, hence $A \in \psi_\varphi(X, \mathcal{M})$. Again $\varphi(X \setminus B) = \{a, d\}$ and $mCl(\psi_\varphi(B)) = \{b, c, d\}$, hence $B \in \psi_\varphi(X, \mathcal{M})$. Since $A \cap B = \{d\}$, $\varphi[X \setminus (A \cap B)] = X$ and $mCl(\psi_\varphi(A \cap B)) = \phi$ implies that $A \cap B \notin \psi_\varphi(X, \mathcal{M})$.

Theorem 3.5. Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space and $A \in \psi_\varphi(X, \mathcal{M})$. If $U \in \mathcal{M}$, then $U \cap A \in \psi_\varphi(X, \mathcal{M})$.

Proof. Let $U \in \mathcal{M}$ and $A \in \psi_\varphi(X, \mathcal{M})$. Then $U \cap A \subseteq U \cap mCl(\psi_\varphi(A))$. Since $\psi_\varphi(X, \mathcal{M})) \subseteq mCl[U \cap \psi_\varphi(A)]$, then by Theorem 2.5, $\psi_\varphi(X, \mathcal{M})) \subseteq mCl[U \cap \psi_\varphi(A)] \subseteq mCl[\psi_\varphi(U) \cap \psi_\varphi(A)]$[13] $= mCl[\psi_\varphi(U \cap A)]$. Hence the result. □

4. $\tau^\psi_{\mathcal{MG}}$-topology and Properties of $\tau^\psi_{\mathcal{MG}}$

In this section we shall introduce a new type of set whose collection forms a topology. Although this collection is used in section 3 does not form a topology.

Definition 4.1. Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space. Then a subset A of X is called a ψ_φ- set if $A \subseteq mInt(mCl(\psi_\varphi(A)))$.

The collection of all ψ_φ sets in $(X, \mathcal{M}, \mathcal{G})$ is denoted by $\tau^\psi_{\mathcal{MG}}$. This collection lies between $\mathcal{M}$ and $\psi_\varphi(X, \mathcal{M})$ i.e., $\mathcal{M} \subseteq \tau^\psi_{\mathcal{MG}} \subseteq \psi_\varphi(\mathrm{X}, \mathcal{M})$.

If we take $A = \{b, c, d\}$ in Remark 3.4., we can easily show that the reverse inclusion of $\tau^{\psi}_{\mathcal{MG}} \subseteq \psi_{\varphi}(\text{X}, \mathcal{M})$ does not hold.

In the following, we introduce some properties of $\tau^{\psi}_{\mathcal{MG}}$-topology.

Theorem 4.2. Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space. Then $\tau^{\psi}_{\mathcal{MG}} = \{A \subseteq X : A \subseteq mInt(mCl(\psi_{\varphi}(A)))\}$ forms a topology on X, where $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$.

Proof. (i). Since $\psi_{\varphi}(\phi) = X \setminus \varphi(X \setminus \phi) = \phi$. So, $\phi \in \tau^{\psi}_{\mathcal{MG}}$ and by Theorem 2.12, $\psi_{\varphi}(X) = X \setminus \varphi(X \setminus X) = X \setminus \phi = X$. Hence $X \subseteq mInt(mCl(\psi_{\varphi}(X)))$. Therefore $X \in \tau^{\psi}_{\mathcal{MG}}$.

(ii). Let $A_i \in \tau^{\psi}_{\mathcal{MG}}$ for all i. Now we show that $\cup_i A_i \in \tau^{\psi}_{\mathcal{MG}}$. Since $A_i \subseteq \cup_i A_i$, $\psi_{\varphi}(A_i) \subseteq \psi_{\varphi}(\cup_i A_i)$[13], then $mInt(mCl(\psi_{\varphi}(A_i))) \subseteq mInt(mCl(\psi_{\varphi}(\cup_i A_i)))$. So $A_i \subseteq mInt(mCl(\psi_{\varphi}(A_i))) \subseteq mInt(mCl(\psi_{\varphi}(\cup_i A_i)))$ for all i. Therefore $\cup_i A_i \in \tau^{\psi}_{\mathcal{MG}}$.

(iii). Let $A_1, A_2 \in \tau^{\psi}_{\mathcal{MG}}$. We show that $A_1 \cap A_2 \in \tau^{\psi}_{\mathcal{MG}}$. If $A_1 \cap A_2 = \phi$, we are done. Let $A_1 \cap A_2 \neq \phi$. Let $x \in A_1 \cap A_2$. Now $A_1 \subseteq mInt(mCl(\psi_{\varphi}(A_1)))$ and $A_2 \subseteq mInt(mCl(\psi_{\varphi}(A_2)))$, implies that $x \in mInt(mCl(\psi_{\varphi}(A_1))) \cap mInt(mCl(\psi_{\varphi}(A_2)))$. So $x \in mInt[mCl(\psi_{\varphi}(A_1)) \cap mCl(\psi_{\varphi}(A_2))]$, from Definition 2.2. Therefore there exists an m-open set V_x containing x such that $V_x \subseteq mCl(\psi_{\varphi}(A_1)) \cap mCl(\psi_{\varphi}(A_2))$. Let U_x be any m-open set containing x in $(X, \mathcal{M})$. Then $\phi \neq V_x \cap U_x \subseteq mCl(\psi_{\varphi}(A_1))$ and $V_x \cap U_x \subseteq mCl(\psi_{\varphi}(A_2))$. Let $y \in V_x \cap U_x$. Consider any m-open set G_y containing y. Without loss of generality we may suppose that $G_y \subseteq V_x \cap U_x$. So $G_y \cap (\psi_{\varphi}(A_1)) \neq \phi$. From the definition of $\psi_{\varphi}(A_1)$, there exists a $U \in \mathcal{M}(x)$ such that $U \subseteq G_y$ and $U \setminus A_1 \notin \mathcal{G}$. Again $U \subseteq mCl(\psi_{\varphi}(A_2))$, so there exists a nonempty m-open set $U' \subseteq U$ such that $U' \setminus A_2 \notin \mathcal{G}$. Now $U' \setminus (A_1 \cap A_2) = (U' \setminus A_1) \cup (U' \setminus A_2) \subseteq (U \setminus A_1) \cup (U' \setminus A_2) \notin \mathcal{G}$(from definition of grill). Hence from definition of ψ_{φ}, $U' \subseteq \psi_{\varphi}(A_1 \cap A_2)$. Since $U' \subseteq G_y$, $G_y \cap \psi_{\varphi}(A_1 \cap A_2) \neq \phi$, therefore $y \in mCl(\psi_{\varphi}(A_1 \cap A_2))$. Since y was any point of $V_x \cap U_x$, it follows that $V_x \cap U_x \subseteq mCl(\psi_{\varphi}(A_1 \cap A_2))$, implies that

$x \in mInt(mCl(\psi_\varphi(A_1 \cap A_2)))$. Thus $A_1 \cap A_2 \subseteq mInt(mCl(\psi_\varphi(A_1 \cap A_2)))$. Hence $A_1 \cap A_2 \in \tau^{\psi}_{\mathcal{MG}}$.

From (i), (ii) and (iii) $\tau^{\psi}_{\mathcal{MG}}$ forms a topology. □

Proposition 4.3. Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space with $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$. Then $\psi_\varphi(A) \neq \phi$ if and only if A contains a nonempty $\tau_{\mathcal{MG}}$-interior.

Proof. Let $\psi_\varphi(A) \neq \phi$. Then from definition of $\psi_\varphi(A)$, there exists a nonempty set $U \in \mathcal{M}$ such that $U \setminus A = P$, where $P \notin \mathcal{G}$. Now $U \setminus P \subseteq A$. By the Theorem 2.8., $U \setminus P \in \tau_{\mathcal{MG}}$ and A contains a nonempty $\tau_{\mathcal{MG}}$-interior.

Conversely suppose that A contains a nonempty $\tau_{\mathcal{MG}}$-interior. Hence there exists a $U \in \mathcal{M}$ and $P \notin \mathcal{G}$ such that $U \setminus P \subseteq A$. So $U \setminus A \subseteq P$. Let $H = U \setminus A \subseteq P$, then $H \notin \mathcal{G}$. Thus $\psi_\varphi(A) \neq \phi$.

Two topologies $\tau^{\psi}_{\mathcal{MG}}$ and $\tau_{\mathcal{MG}}$ have been obtained from $(X, \mathcal{M}, \mathcal{G})$ space. Now we shall discuss the resolvability of $\tau^{\psi}_{\mathcal{MG}}$ vis-a-vis resolvability of $\tau_{\mathcal{MG}}$. □

Theorem 4.4. If $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$ in $(X, \mathcal{M}, \mathcal{G})$, $\mathcal{D}(X, \tau_{\mathcal{MG}}) = \mathcal{D}(X, \tau^{\psi}_{\mathcal{MG}})$(where $D(X, \tau)$ denotes the collection of all dense subsets in a topological space (X, τ)).

Proof. Since $\tau_{\mathcal{MG}} \subseteq \tau^{\psi}_{\mathcal{MG}}$ then, $\mathcal{D}(X, \tau^{\psi}_{\mathcal{MG}}) \subseteq \mathcal{D}(X, \tau_{\mathcal{MG}})$————(i).

Next let $D \in \mathcal{D}(X, \tau_{\mathcal{MG}})$. We are to show that $D \in \mathcal{D}(X, \tau^{\psi}_{\mathcal{MG}})$. Let $\phi \neq A \in \tau^{\psi}_{\mathcal{MG}}$, so $\psi_\varphi(A) \neq \phi$. By proposition 4.3., A has a nonempty $\tau_{\mathcal{MG}}$-interior. Thus $Int_{\mathcal{MG}}(A) \neq \phi$. Now $Int_{\mathcal{MG}}(A) \cap D \subseteq A \cap D$, where $Int_{\mathcal{MG}}(A) \cap D \neq \phi$, since $D \in (X, \tau_{\mathcal{MG}})$. Thus $A \cap D \neq \phi$ so that $D \in \mathcal{D}(X, \tau^{\psi}_{\mathcal{MG}})$. Therefore, $\mathcal{D}(X, \tau_{\mathcal{MG}}) \subseteq \mathcal{D}(X, \tau^{\psi}_{\mathcal{MG}})$ ————(ii).

From (i) and (ii) we have $\mathcal{D}(X, \tau_{\mathcal{MG}}) = \mathcal{D}(X, \tau^{\psi}_{\mathcal{MG}})$. □

Theorem 4.5. Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space, where $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$. Then $(X, \tau^{\psi}_{\mathcal{MG}})$ is resolvable if and only if $(X, \tau_{\mathcal{MG}})$ is resolvable.

Proof. Since $\mathcal{D}(X, \tau_{\mathcal{MG}}) = \mathcal{D}(X, \tau^{\psi}_{\mathcal{MG}})$, it follows from definition of resolvability that $(X, \tau^{\psi}_{\mathcal{MG}})$ is resolvable if and only if $(X, \tau_{\mathcal{MG}})$ is resolvable.
Now, we shall give a representation of α-topology of $\tau_{\mathcal{MG}}$ with the help of ψ_{φ}-operator in following Theorems. $\square$

Theorem 4.6. Let $x \in X$. Then $\{x\} \in \psi_{\varphi}(X, \mathcal{M})$ if and only if $\{x\}$ is open in $(X, \tau_{\mathcal{MG}})$.

Proof. Let $\{x\} \in \psi_{\varphi}(X, \mathcal{M})$ then $\psi_{\varphi}(\{x\}) \neq \phi$. By Proposition 4.3, $\{x\}$ contain a nonempty $\tau_{\mathcal{MG}}$-interior. Therefore $\{x\}$ is open in $(X, \tau_{\mathcal{MG}})$. Conversely suppose that $\{x\}$ is open in $(X, \tau_{\mathcal{MG}})$, implies that $\{x\} \subseteq \psi_{\varphi}(\{x\})$[13]. Therefore $\{x\} \subseteq mCl(\psi_{\varphi}(\{x\}))$, that is $\{x\} \in \psi_{\varphi}(X, \mathcal{M})$. $\square$

Theorem 4.7. Let $x \in X$. Then $\{x\} \in \psi_{\varphi}(X, \mathcal{M})$ if and only if $\{x\} \in \tau^{\psi}_{\mathcal{MG}}$.

Proof. Let $\{x\} \in \psi_{\varphi}(X, \mathcal{M})$. Therefore $\{x\}$ is open in $(X, \tau_{\mathcal{MG}})$ (by above theorem). So $\{x\} \subseteq \psi_{\varphi}(\{x\})$[13] implies that $\{x\} \subseteq mInt(mCl(\psi_{\varphi}(\{x\})))$, since $\psi_{\varphi}(\{x\})$ is an m-open set. Thus $\{x\} \in \tau^{\psi}_{\mathcal{MG}}$. Conversely suppose that $\{x\} \in \tau^{\psi}_{\mathcal{MG}}$, then $\{x\} \subseteq mInt(mCl(\psi_{\varphi}(\{x\})))$, implying that $\{x\} \subseteq mCl(\psi_{\varphi}(\{x\}))$, hence $\{x\} \in \psi_{\varphi}(X, \mathcal{M})$. $\square$

From the above two theorems we get the following corollary:
Corollary 4.8. $\tau^{\psi}_{\mathcal{MG}}$ is exactly the collection such that A belongs to $\tau^{\psi}_{\mathcal{MG}}$ and B belongs to $\psi_{\varphi}(X, \mathcal{M})$ implies $A \cap B \in \psi_{\varphi}(X, \mathcal{M})$, where $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$.

Proof. Let $A \in \tau^{\psi}_{\mathcal{MG}}$ and $B \in \psi_{\varphi}(X, \mathcal{M})$. Now, we show that $A \cap B \in \psi_{\varphi}(X, \mathcal{M})$. If $A \cap B = \phi$, we are done. Let $A \cap B \neq \phi$. Let $x \in A \cap B$. This implies that $x \in mInt(mCl(\psi_{\varphi}(A)))$, therefore $x \in mCl(\psi_{\varphi}(A))$. So for every m-open set U_x containing x, $U_x \cap \psi_{\varphi}(A) \neq \phi$. Again $x \in B \subseteq mCl(\psi_{\varphi}(B))$, then for every m-open set V_x containing x, $V_x \cap \psi_{\varphi}(B) \neq \phi$. Therefore for m-open set $W_x = U_x \cap V_x$

containing x, $W_x \cap \psi_\varphi(A) \neq \phi$ and $W_x \cap \psi_\varphi(B) \neq \phi$. Again $W_x \cap \psi_\varphi(A) \subseteq W_x$ and $W_x \cap \psi_\varphi(B) \subseteq W_x$. Therefore $W_x \cap \psi_\varphi(A) \cap \psi_\varphi(B) \neq \phi$. So $x \in mCl[\psi_\varphi(A) \cap \psi_\varphi(B)]$, that is $x \in mCl[\psi_\varphi(A \cap B)]$[13], therefore $A \cap B \in \psi_\varphi(X, \mathcal{M})$.

Next we consider a subset A of X such that $A \cap B \in \psi_\varphi(X, \mathcal{M})$ for each $B \in \psi_\varphi(X, \mathcal{M})$. We have to show that $A \in \tau^\psi_{\mathcal{MG}}$, that is $A \subseteq mInt(mCl(\psi_\varphi(A)))$, that is $A \subseteq mInt(\varphi(\psi_\varphi(A)))$(by Theorem 2.14.). If possible suppose that $x \in A$ but $x \notin mInt(\varphi(\psi_\varphi(A)))$. Therefore $x \in A \cap [X \setminus mInt(\varphi(\psi_\varphi(A)))] = A \cap mCl[X \setminus \varphi(\psi_\varphi(A))]$(from Theorem 2.4.) $= A \cap mClC$, where $C = X \setminus \varphi(\psi_\varphi(A))$. It is obvious that C is an nonempty m-open set in $(X, \mathcal{M})$, since $\varphi(\psi_\varphi(A))$ is an m-closed set[13]. Since $x \in mClC$ then for all m-open set V_x containing x, $V_x \cap C \neq \phi$. Therefore $V_x \cap \psi_\varphi(C) \neq \phi$, since $C \subseteq \psi_\varphi(C)$[13]. This implies that

$x \in mCl(\psi_\varphi(C)) \subseteq mCl[\psi_\varphi(\{x\} \cup C)]$————–(i).

Hence $C \subseteq mCl(\psi_\varphi(C)) \subseteq mCl[\psi_\varphi(\{x\} \cup C)]$————-(ii).

From (i) and (ii) $\{x\} \cup C \subseteq mCl[\psi_\varphi(\{x\} \cup C)]$. Therefore $\{x\} \cup C \in \psi_\varphi(X, \mathcal{M})$. Now by hypothesis $A \cap (\{x\} \cup C)$ is a ψ_φ-C set. We show that $A \cap (\{x\} \cup C) = \{x\}$. If possible suppose that $y \in X$ and $x \neq y$ such that $y \in A \cap (\{x\} \cup C)$. So $y \in A$ and $y \in C$. Now $A = A \cap X$ and $X \in \psi_\varphi(X, \mathcal{M})$, again by hypothesis $A \in \psi_\varphi(X, \mathcal{M})$. Since $y \in A$, $y \in mCl(\psi_\varphi(A))$, a contradiction to the fact that $y \in C = [X \setminus \varphi(\psi_\varphi(A))]$ $= [X \setminus mCl(\psi_\varphi(A))]$. Thus $A \cap (\{x\} \cup C) = \{x\}$. Since $\{x\} \in \psi_\varphi(X, \mathcal{M})$, then $\{x\} \in \tau^\psi_{\mathcal{MG}}$(by Theorem 4.7). So $\{x\} \subseteq mInt(mCl(\psi_\varphi(\{x\}))) = mInt(mCl(\psi_\varphi(A \cap (\{x\} \cup C)))) \subseteq mInt(mCl(\psi_\varphi(A)))$. But $x \in mInt(mCl(\psi_\varphi(A)))$, a contradiction to the fact that $x \notin mInt[\varphi(\psi_\varphi(A))]$. Therefore we get $A \subseteq mInt(mCl(\psi_\varphi(A)))$ that is $A \in \tau^\psi_{\mathcal{MG}}$. The proof of the corollary is completed. □

Theorem 4.9. Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space, where $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$. Then $SO(X, \tau_{\mathcal{MG}}) = \{A \subseteq X : A \subseteq mCl(\psi_\varphi(A))\} = \psi_\varphi(X, \mathcal{M})$.

Proof. Let $A \in SO(X, \tau_{\mathcal{MG}})$. Then $A \subseteq Cl_{\mathcal{MG}}(Int_{\mathcal{MG}}(A)) = Cl_{\mathcal{MG}}(A \cap \psi_{\varphi}(A))$[13]$\subseteq Cl_{\mathcal{MG}}(\psi_{\varphi}(A)) = [\psi_{\varphi}(A) \cup \varphi(\psi_{\varphi}(A))] = \varphi(\psi_{\varphi}(A))$, since $\psi_{\varphi}(A) \in \mathcal{M}$. This implies that $A \subseteq mCl(\psi_{\varphi}(A))$. Hence $A \in \psi_{\varphi}(X, \mathcal{M})$.
So, $SO(X, \tau_{\mathcal{MG}}) \subseteq \psi_{\varphi}(X, \mathcal{M})$————(i).
For reverse inclusion, let $A \in \psi_{\varphi}(X, \mathcal{M})$. We show that $A \in SO(X, \tau_{\mathcal{MG}})$. Take $x \in A$. Consider $G_1 \in \beta(\mathcal{M}, \mathcal{G})$ such that $x \in G_1$. Then G_1 is of the form $G_1 = G \setminus E$, where $G \in \mathcal{M}$, $E \notin \mathcal{G}$. So $x \in G$. Since $A \subseteq mCl(\psi_{\varphi}(A))$ and $G \in \mathcal{M}$, $G \cap (\psi_{\varphi}(A)) \neq \phi$. Let $y \in G \cap (\psi_{\varphi}(A))$. Therefore there exists $O_y \in \mathcal{M}(y)$ such that $O_y \setminus A \notin \mathcal{G}$ by definition of $\psi_{\varphi}(A)$. Consider $\phi \neq G \cap O_y$. So $(G \cap O_y) \setminus A \notin \mathcal{G}$(from definition of grill). Let $G^{/} = G \cap O_y$. Then $G^{/} \neq \phi$, $G^{/} \in \mathcal{M}$ and $G^{/} \setminus A = P$ say, where $P \notin \mathcal{G}$ and so $G^{/} \setminus P \subseteq A$. Hence $G^{/} \setminus (E \cup P) \subseteq A$ where $G^{/} \setminus (E \cup P) \neq \phi$, since $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$. Write $M = G^{/} \setminus (E \cup P)$. Then $\phi \neq M \in \tau_{\mathcal{MG}}$ such that $M \subseteq A \cap (G \setminus E)$. Hence A contains a nonempty $\tau_{\mathcal{MG}}$-open set M contained in $G \setminus E = G_1$. Since x is an arbitrary point of A, we get $A \subseteq Cl_{\mathcal{MG}}(Int_{\mathcal{MG}}(A))$. Therefore $A \in SO(X, \tau_{\mathcal{MG}})$. Thus $\psi_{\varphi}(X, \mathcal{M}) \subseteq SO(X, \tau_{\mathcal{MG}})$————(ii).
From (i) and (ii), $\psi_{\varphi}(X, \mathcal{M}) = SO(X, \tau_{\mathcal{MG}})$. □

Remark 4.10. Let $x \in X$. Then $\{x\} \in SO(X, \tau_{\mathcal{MG}})$ if and only if $\{x\} \in \tau^{\psi}_{\mathcal{MG}}$, where $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$.

Proof. Obvious from Theorem 4.7. □

Theorem 4.11. $\tau^{\psi}_{\mathcal{MG}}$ is exactly the collection such that A belongs to $\tau^{\psi}_{\mathcal{MG}}$ and B belongs $SO(X, \tau_{\mathcal{MG}})$ implies $A \cap B \in SO(X, \tau_{\mathcal{MG}})$, where $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$.

Proof. Obvious from Corollary 4.8 and the Theorem 4.9.
Now, we shall discuss the relationship between $(\tau_{\mathcal{MG}})^{\alpha}$ and $\tau^{\psi}_{\mathcal{MG}}$ in the following results. □

Theorem 4.12.[17]. Let (X, τ) be a topological space. Then τ^α consists of exactly those sets A for which $A \cap B \in SO(X, \tau)$ for all $B \in SO(X, \tau)$.

Proof. Follows Theorems 4.12, 4.11. □

Corollary 4.13. Let $(X, \mathcal{M}, \mathcal{G})$ be a grill m-space, where $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$. Then $\tau^\psi_{\mathcal{M}\mathcal{G}}$ $= (\tau_{\mathcal{M}\mathcal{G}})^\alpha$.

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