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A DECOMPOSITION OF PAIRWISE CONTINUITY

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Abstract. In this paper, we introduce and study the notions of some weaker forms of τ_i - θ -open sets and some stronger forms of (i, j) -t-sets and (i, j) -B-sets in bitopological spaces. Also, we introduce various forms of pairwise continuity and using these we obtain some decompositions of pairwise continuity.

1. Introduction

In 1961, Levine [6] provided a decomposition of continuity. After his work many authors [1, 2, 8, 10, 12, 13, 14, 17, 18] obtained various decompositions of continuity in topological spaces. In 1990, Jelic [3, 4] obtained some decompositions of pairwise continuity in bitopological spaces. Ravi et al. [11] obtained a decomposition of $(1, 2)$ - $*$ -continuity and $(1, 2)$ - $*$ - α -continuity, in 2009. In this paper, we introduce and study the notions of some weaker forms of τ_i - θ -open sets and some stronger forms of (i, j) -t-sets and (i, j) -B-sets in bitopological spaces. Also, we introduce

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various forms of pairwise continuity and using these we obtain some decompositions of pairwise continuity.

2. Preliminaries

Throughout this paper, (X, τ_1, τ_2) and (Y, τ_1, τ_2) denote bitopological spaces on which no separation axioms are assumed unless explicitly stated. By τ_i-open set, we shall mean open set with respect to topology τ_i in X. We always use (i, j)- to denote the certain properties with respect to the topologies τ_i and τ_j respectively, where $i,\ j \in 1, 2$ and $i \neq j$. By τ_i-$int(A)$ and τ_i-$cl(A)$ we shall mean the interior and the closure of a subset A of X with respect to the topology τ_i. The complement of A is denoted by $X - A$ or A^c. A set A of (X, τ) is called θ-closed [19] if $A = cl_\theta(A)$, where $cl_\theta(A) = \{x \in X : A \cap cl(U) \neq \emptyset$ for all $U \in \tau(X, x)\}$. The complement of a θ-closed set is called θ-open, alternatively, a set A of (X, τ) is called θ-open if $A = int_\theta(A)$, where $int_\theta(A) = \{x \in X : cl(U) \subseteq A$ for some $U \in \tau(X, x)\}$. Now, we recall some definitions.

Definition 2.1. *A subset A of a bitopological space (X, τ_1, τ_2) is said to be*

(1) (i, j)*-semi-open* [7] *if* $A \subseteq \tau_j\text{-}cl(\tau_i\text{-}int(A))$,

(2) (i, j)-α*-open* [15] *if* $A \subseteq \tau_i\text{-}int(\tau_j\text{-}cl(\tau_i\text{-}int(A)))$,

(3) (i, j)*-pre-open* [3] *if* $A \subseteq \tau_i\text{-}int(\tau_j\text{-}cl(A))$,

(4) (i, j)-β*-open* [5] *if* $A \subseteq \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl(A)))$,

(5) (i, j)-t*-set* [16] *if* $\tau_i\text{-}int(A) = \tau_i\text{-}int(\tau_j\text{-}cl(A))$,

(6) (i, j)-B*-set* [16] *if* $A = U \cap V$, *where* U *is* τ_i*-open and* V *is an* (i, j)*-t-set.*

The complements of the above mentioned open sets in (X,τ_1,τ_2) are called their respective closed sets in (X,τ_1,τ_2).

Definition 2.2. [9] *A function* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is said to be* p*-continuous if the induced mappings* $f:(X,\tau_k)\to(Y,\sigma_k)$, *(*$k=1,2$*) are continuous.*

Definition 2.3. [16] *A function* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is said to be* p*-B-continuous if for every* $V\in\sigma_k$, $k=1,2$, $f^{-1}(V)$ *is an* (i,j)*-B-set.*

3. Weaker forms of τ_i-θ-open sets in bitopological spaces

Definition 3.1. *A subset* A *of a space* (X,τ_1,τ_2) *is said to be an*

(1) (i,j)*-*θ*-semi-open set if* $A\subseteq\tau_j\text{-}cl(\tau_i\text{-}int_\theta(A))$,

(2) (i,j)*-*θ*-pre open set if* $A\subseteq\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A))$,

(3) (i,j)*-*θ*-*α*-open set if* $A\subseteq\tau_i\text{-}int(\tau_j\text{-}cl(\tau_i\text{-}int_\theta(A)))$,

(4) (i,j)*-*θ*-*β*-open set if* $A\subseteq\tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)))$,

(5) (i,j)*-weakly* θ*-*β*-open set if* $A\subseteq\tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)))$.

Proposition 3.2. *Every* τ_i*-*θ*-open set is* (i,j)*-*θ*-pre open,* (i,j)*-*θ*-semi-open,* (i,j)*-*θ*-*α*-open,* (i,j)*-*θ*-*β*-open and* (i,j)*-weakly* θ*-*β*-open.*

Proof. Since A is τ_i-θ-open, we have

$$\begin{aligned}A&=\tau_i\text{-}int_\theta(A)\\&\subseteq\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A))\\&\subseteq\tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)))\\&\subseteq\tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A))).\end{aligned}$$

Thus A is (i,j)-θ-pre open, (i,j)-θ-β-open and (i,j)-weakly θ-β-open.

Now, $A=\tau_i\text{-}int_\theta(A)$

$\subseteq \tau_i\text{-}int(\tau_j\text{-}cl(\tau_i\text{-}int_\theta(A)))$

$\subseteq \tau_j\text{-}cl(\tau_i\text{-}int_\theta(A))$.

This shows that A is (i,j)-θ-α-open and (i,j)-θ-semi-open.

Proposition 3.3. *For a space (X, τ_1, τ_2), the following hold:*

(1) *Every (i,j)-θ-semi-open set is (i,j)-semi-open.*

(2) *Every (i,j)-pre open set is (i,j)-θ-pre open.*

(3) *Every (i,j)-θ-α-open set is (i,j)-α-open.*

(4) *Every (i,j)-β-open set is (i,j)-θ-β-open.*

Proof. 1. Let A be an (i,j)-θ-semi-open set. Then $A \subseteq \tau_j\text{-}cl(\tau_i\text{-}int_\theta(A)) \subseteq \tau_j\text{-}cl(\tau_i\text{-}int(A))$. This shows that A is (i,j)-semi-open.

2. Let A be an (i,j)-pre open set. Then $A \subseteq \tau_i\text{-}int(\tau_j\text{-}cl(A)) \subseteq \tau_i\text{-}int(\tau_j\text{-}cl_\theta(A))$. This shows that A is (i,j)-θ-pre open.

3. Let A be an (i,j)-θ-α-open set. Then

$A \subseteq \tau_i\text{-}int(\tau_j\text{-}cl(\tau_i\text{-}int_\theta(A)))$

$\subseteq \tau_i\text{-}int(\tau_j\text{-}cl(\tau_i\text{-}int(A)))$. This shows that A is (i,j)-α-open.

4. Let A be an (i,j)-β-open set. Then

$A \subseteq \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl(A)))$

$\subseteq \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)))$. This shows that A is (i,j)-θ-β-open.

Proposition 3.4. *For a space (X, τ_1, τ_2), the following hold:*

(1) *Every (i,j)-θ-semi-open set is (i,j)-θ-β-open.*

(2) *Every (i,j)-θ-pre open set is (i,j)-θ-β-open.*

(3) *Every (i,j)-θ-α-open set is (i,j)-θ-semi-open, (i,j)-θ-pre open and (i,j)-θ-β-open.*

Proof. 1. Let A be an (i,j)-θ-semi-open set. Then

$A \subseteq \tau_j\text{-}cl(\tau_i\text{-}int_\theta(A)) \subseteq \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)))$. This shows that A is (i,j)-θ-β-open.

2. Let A be an (i,j)-θ-pre open set. Then

$A \subseteq \tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)) \subseteq \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)))$.

This shows that A is (i,j)-θ-β-open.

3. Let A be an (i,j)-θ-α-open set.

Then $A \subseteq \tau_i\text{-}int(\tau_j\text{-}cl(\tau_i\text{-}int_\theta(A))) \subseteq \tau_j\text{-}cl(\tau_i\text{-}int_\theta(A))$. That is, A is (i,j)-θ-semi-open. Also, $A \subseteq \tau_i\text{-}int(\tau_j\text{-}cl(\tau_i\text{-}int_\theta(A))) \subseteq \tau_i\text{-}int(\tau_j\text{-}cl_\theta(A))$ $\subseteq \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)))$. This shows that A is (i,j)-θ-pre open and (i,j)-θ-β-open.

Proposition 3.5. *Every (i,j)-θ-pre open (resp. (i,j)-θ-semi-open, (i,j)-θ-α-open and (i,j)-θ-β-open) set is (i,j)-weakly θ-β-open.*

Proof. The proof is obvious.

Remark 3.6. *From the following examples, the converses of the above propositions need not be true.*

Example 3.7. *Let $X = \{a,b,c\}$ with topologies $\tau_1 = \{\emptyset, \{b\}, \{c\}, \{b,c\}, X\}$ and $\tau_2 = \{\emptyset, \{b\}, \{c\}, \{b,c\}, \{a,c\}, X\}$. Then the set $A = \{c\}$ is $(2,1)$-semi-open but it is not $(2,1)$-θ-semi-open.*

Example 3.8. *Let $X = \{a,b,c\}$ with topologies $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{a,b\}, \{b,c\}, X\}$ and $\tau_2 = \{\emptyset, \{c\}, \{a,b\}, X\}$. Then the set $A = \{a,c\}$ is $(2,1)$-θ-pre open and*

$(2,1)$-θ-β-open but it is not τ_2-θ-open, $(2,1)$-pre open, $(2,1)$-β-open and $(2,1)$-θ-α-open. Moreover, the set $B = \{b\}$ is $(1,2)$-α-open but it is not $(1,2)$-θ-α-open.

Example 3.9. *In* Example 3.8, *the set $A = \{a,b\}$ is $(1,2)$-θ-α-open but it is not τ_1-θ-open. In* Example 3.7, *the set $A = \{a,b\}$ is $(1,2)$-θ-β-open but it is not $(1,2)$-θ-semi-open. Moreover, the set $B = \{a,c\}$ is $(1,2)$-θ-β-open but it is not $(1,2)$-θ-pre open.*

Example 3.10. *Let $X = \{a,b,c,d\}$ with topologies $\tau_1 = \{\emptyset, \{d\}, \{a,d\}, \{b,c\}, \{b,c,d\}, X\}$ and $\tau_2 = \{\emptyset, \{d\}, \{a,b,c\}, X\}$. Then the set $A = \{a,d\}$ is $(2,1)$-θ-semi-open but it is not τ_2-θ-open and $(2,1)$-θ-α-open.*

Example 3.11. *In* Example 3.8, *the set $A = \{b\}$ is $(2,1)$-weakly θ-β-open but it is not $(2,1)$-θ-pre open, $(2,1)$-θ-semi-open, $(2,1)$-θ-α-open and $(2,1)$-θ-β-open.*

4. (i,j)-θ-T-SETS, (i,j)-θ_β-T-SETS AND (i,j)-STRONG θ_β-T-SETS

Definition 4.1. *A subset A of a space (X, τ_1, τ_2) is said to be an*

1. *(i,j)-θ-t-set if τ_i-$int(A) = \tau_i$-$int(\tau_j$-$cl_\theta(A))$,*
2. *(i,j)-θ_β-t-set if τ_i-$int(A) = \tau_j$-$cl(\tau_i$-$int(\tau_j$-$cl_\theta(A)))$,*
3. *(i,j)-strong θ_β-t-set if τ_i-$int(A) = \tau_j$-$cl_\theta(\tau_i$-$int(\tau_j$-$cl_\theta(A)))$.*

Proposition 4.2. *For a subset A of a space (X, τ_1, τ_2), the following hold:*

1. *If A is τ_j-θ-closed, then it is an (i,j)-θ-t-set.*
2. *A is an (i,j)-θ-t-set if and only if it is (j,i)-θ-semi-closed.*

Proof. 1. Since A is τ_j - θ -closed, we have $A = \tau_j\text{-}cl_\theta(A)$. Thus $\tau_i\text{-}int(A) = \tau_i\text{-}int(\tau_j\text{-}cl_\theta(A))$. This shows that A is an (i,j) - θ -t-set.

2. Let A be an (i,j) - θ -t-set. Then $\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)) = \tau_i\text{-}int(A) \subseteq A$. This implies A is (j,i) - θ -semi-closed.

Conversely, let A be (j,i) - θ -semi-closed, $\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)) \subseteq A$. Thus $\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)) \subseteq \tau_i\text{-}int(A) \subseteq \tau_i\text{-}int(\tau_j\text{-}cl_\theta(A))$. This implies $\tau_i\text{-}int(A) = \tau_i\text{-}int(\tau_j\text{-}cl_\theta(A))$ and so A is an (i,j) - θ -t-set.

Example 4.3. *The converse of* Proposition 4.2(1) *need not be true. In* Example 3.8, *the set* $A = \{c\}$ *is a* $(2,1)$ - θ *-t-set but not a* τ_1 - θ *-closed set.*

Proposition 4.4. *A subset* A *of a space* (X, τ_1, τ_2) *is* (i,j) - θ *-pre open and an* (i,j) - θ *-t-set if and only if* $\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)) = A$.

Proof. The proof is obvious.

Proposition 4.5. *For a space* (X, τ_1, τ_2) *, the following hold:*

(1) *Every* (i,j) - θ *-t-set is an* (i,j) *-t-set.*

(2) *Every* (i,j) - θ_β *-t-set is an* (i,j) - θ *-t-set.*

(3) *Every* (i,j) *-strong* θ_β *-t-set is an* (i,j) - θ *-t-set and an* (i,j) - θ_β *-t-set.*

Proof. 1. Let A be an (i,j) - θ -t-set.

Now, $\tau_i\text{-}int(\tau_j\text{-}cl(A)) \subseteq \tau_i\text{-}int(\tau_j\text{-}cl_\theta(A))$

$$= \tau_i\text{-}int(A)$$

$$\subseteq \tau_i\text{-}int(\tau_j\text{-}cl(A)).$$

This shows that A is an (i,j) -t-set.

2. Let A be an (i,j)-θ_β-t-set.

Now, τ_i-$int(\tau_j$-$cl_\theta(A)) \subseteq \tau_j$-$cl(\tau_i$-$int(\tau_j$-$cl_\theta(A)))$

$= \tau_i$-$int(A)$

$\subseteq \tau_i$-$int(\tau_j$-$cl_\theta(A))$.

This shows that A is an (i,j)-θ-t-set.

3. Let A be an (i,j)-strong θ_β-t-set.

Now, τ_i-$int(\tau_j$-$cl_\theta(A)) \subseteq \tau_j$-$cl(\tau_i$-$int(\tau_j$-$cl_\theta(A)))$

$\subseteq \tau_j$-$cl_\theta(\tau_i$-$int(\tau_j$-$cl_\theta(A)))$

$= \tau_i$-$int(A)$

$\subseteq \tau_i$-$int(\tau_j$-$cl_\theta(A))$

$\subseteq \tau_j$-$cl(\tau_i$-$int(\tau_j$-$cl_\theta(A)))$.

This shows that A is an (i,j)-θ-t-set and an (i,j)-θ_β-t-set.

Remark 4.6. *From the following examples, we see that the converses of the above proposition need not be true.*

Example 4.7. *Let* $X = \{a,b,c\}$ *with topologies* $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{a,b\}, \{b,c\}, X\}$ *and* $\tau_2 = \{\emptyset, \{c\}, \{b,c\}, X\}$. *Then the set* $A = \{c\}$ *is a* $(2,1)$*-t-set but it is not a* $(2,1)$-θ*-t-set.*

Example 4.8. *Let* $X = \{a,b,c,d\}$ *with topologies* $\tau_1 = \{\emptyset, \{d\}, \{a,c\}, \{a,c,d\}, X\}$ *and* $\tau_2 = \{\emptyset, \{a\}, \{d\}, \{a,d\}, X\}$. *Then the set* $A = \{d\}$ *is a* $(1,2)$-θ*-t-set but it is not a* $(1,2)$-θ_β*-t-set.*

Example 4.9. *In* Example 3.8, *the set* $A = \{c\}$ *is a* $(2,1)$-θ*-t-set and* $(2,1)$-θ_β*-t-set but it is not a* $(2,1)$*-strong* θ_β*-t-set.*

Proposition 4.10. *Intersection of two (i,j) - θ -t-sets is an (i,j) - θ -t-set.*

Proof. Let A and B be two (i,j) - θ -t-sets in (X, τ_1, τ_2).

Then, $\tau_i\text{-}int(A) = \tau_i\text{-}int(\tau_j\text{-}cl_\theta(A))$ and

$$\tau_i\text{-}int(B) = \tau_i\text{-}int(\tau_j\text{-}cl_\theta(B)).$$

Now,
$$\begin{aligned}\tau_i\text{-}int(A\cap B) &\subseteq \tau_i\text{-}int(\tau_j\text{-}cl_\theta(A\cap B))\\ &\subseteq \tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)\cap \tau_j\text{-}cl_\theta(B))\\ &= \tau_i\text{-}int(\tau_j\text{-}cl_\theta(A))\cap \tau_i\text{-}int(\tau_j\text{-}cl_\theta(B))\\ &= \tau_i\text{-}int(A)\cap \tau_i\text{-}int(B)\\ &= \tau_i\text{-}int(A\cap B).\end{aligned}$$

This implies $\tau_i\text{-}int(A\cap B) = \tau_i\text{-}int(\tau_j\text{-}cl_\theta(A\cap B))$. Therefore, $A\cap B$ is an (i,j) - θ -t-set.

Proposition 4.11. *Intersection of two (i,j) - θ_β -t-sets is an (i,j) - θ_β -t-set.*

Proof. Let A and B be two (i,j) - θ_β -t-sets in (X, τ_1, τ_2).

Then, $\tau_i\text{-}int(A) = \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)))$ and

$$\tau_i\text{-}int(B) = \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(B))).$$

Now,
$$\begin{aligned}\tau_i\text{-}int(A\cap B) &\subseteq \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A\cap B)))\\ &\subseteq \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)\cap \tau_j\text{-}cl_\theta(B)))\\ &= \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A))\cap \tau_i\text{-}int(\tau_j\text{-}cl_\theta(B)))\\ &\subseteq \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)))\cap \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(B)))\\ &= \tau_i\text{-}int(A)\cap \tau_i\text{-}int(B)\\ &= \tau_i\text{-}int(A\cap B).\end{aligned}$$

This implies $\tau_i\text{-}int(A\cap B) = \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A\cap B)))$.

Therefore, $A\cap B$ is an (i,j) - θ_β -t-set.

Proposition 4.12. *Intersection of two* (i,j) *-strong* θ_β *-t-sets is an* (i,j) *-strong* θ_β *-t-set.*

Proof. Let A and B be two (i,j)-strong θ_β-t-sets in (X,τ_1,τ_2).

Then, $\tau_i\text{-}int(A)=\tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)))$ and

$$\tau_i\text{-}int(B)=\tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(B))).$$

$$\begin{aligned}
\text{Now, } \tau_i\text{-}int(A\cap B) &\subseteq \tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A\cap B)))\\
&\subseteq \tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)\cap\tau_j\text{-}cl_\theta(B)))\\
&= \tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A))\cap\tau_i\text{-}int(\tau_j\text{-}cl_\theta(B)))\\
&\subseteq \tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)))\cap\tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(B)))\\
&= \tau_i\text{-}int(A)\cap\tau_i\text{-}int(B)\\
&= \tau_i\text{-}int(A\cap B).
\end{aligned}$$

This implies

$\tau_i\text{-}int(A\cap B)=\tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A\cap B)))$.

Therefore,

$A\cap B$ is an (i,j)-strong θ_β-t-set.

Remark 4.13. *Following examples show that in a bitopological space* (X,τ_1,τ_2),

(1) *Union of two* (i,j) *-* θ *-t-sets need not be an* (i,j) *-* θ *-t-set.*

(2) *Union of two* (i,j) *-* θ_β *-t-sets need not be an* (i,j) *-* θ_β *-t-set.*

(3) *Union of two* (i,j) *-strong* θ_β *-t-sets need not be an* (i,j) *-strong* θ_β *-t-set.*

Example 4.14. *In* Example 3.8, *the sets* $A=\{a\}$ *and* $B=\{c\}$ *are* $(2,1)$ *-* θ *-t-sets and* $(2,1)$ *-* θ_β *-t-sets but* $A\cup B=\{a,c\}$ *is neither a* $(2,1)$ *-* θ *-t-set nor a* $(2,1)$ *-* θ_β *-t-set.*

Example 4.15. *Let*

$X = \{a, b, c, d\}$ *with topologies* $\tau_1 = \{\emptyset, \{d\}, \{a, d\}, \{b, c\}, \{b, c, d\}, X\}$ *and* $\tau_2 = \{\emptyset, \{a, d\}, X\}$. *Then the sets* $A = \{b\}$ *and* $B = \{a, d\}$ *are* $(2, 1)$ *-strong* θ_β *-t-sets but* $A \cup B = \{a, b, d\}$ *is not a* $(2, 1)$ *-strong* θ_β *-t-set.*

5. (i, j) - θ -B-SETS, (i, j) - θ_β -B-SETS AND (i, j) -STRONG θ_β -B-SETS

Definition 5.1. *A subset* A *of a space* (X, τ_1, τ_2) *is said to be an*

(1) (i, j) - θ *-B-set if* $A = U \cap V$, *where* $U \in \tau_i$ *and* V *is an* (i, j) - θ *-t-set,*

(2) (i, j) - θ_β *-B-set if* $A = U \cap V$, *where* $U \in \tau_i$ *and* V *is an* (i, j) - θ_β *-t-set,*

(3) (i, j) *-strong* θ_β *-B-set if* $A = U \cap V$, *where* $U \in \tau_i$ *and* V *is an* (i, j) *-strong* θ_β *-t-set.*

Proposition 5.2. *Let* (X, τ_1, τ_2) *be a space. Then the following hold:*

(1) *Every* τ_j - θ *-closed set is* (i, j) - θ *-B-set.*

(2) *Every* τ_i *-open set is* (i, j) - θ *-B-set,* (i, j) - θ_β *-B-set and* (i, j) *-strong* θ_β *-B-set.*

(3) *Every* (i, j) - θ *-t-set is* (i, j) - θ *-B-set.*

(4) *Every* (i, j) - θ_β *-t-set is* (i, j) - θ_β *-B-set.*

(5) *Every* (i, j) *-strong* θ_β *-t-set is* (i, j) *-strong* θ_β *-B-set.*

Proof. The proof is straight forward.

Proposition 5.3. *Let* (X, τ_1, τ_2) *be a space. Then the following hold:*

(1) *Every* (i, j) - θ_β *-B-set is* (i, j) - θ *-B-set.*

(2) *Every* (i, j) - θ *-B-set is* (i, j) *-B-set.*

(3) *Every* (i, j) *-strong* θ_β *-B-set is* (i, j) - θ *-B-set and* (i, j) - θ_β *-B-set.*

Proof. The proof is straight forward.

Remark 5.4. *The converses of the above propositions need not be true as seen from the following examples.*

Example 5.5. *Let* $X = \{a, b, c\}$ *with topologies* $\tau_1 = \{\emptyset, \{c\}, \{a, b\}, X\}$ *and* $\tau_2 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$. *Then the set* $A = \{a, b\}$ *is a* $(1, 2)$ *-* θ *-B-set but it is not a* τ_2 *-* θ *-closed set. Moreover, the set* $A = \{a, b\}$ *is a* $(1, 2)$ *-* θ *-B-set and a* $(1, 2)$ *-* θ_β *-B-set but neither a* $(1, 2)$ *-* θ *-t-set nor a* $(1, 2)$ *-* θ_β *-t-set.*

Example 5.6. *In* Example 3.8, *the set* $A = \{c\}$ *is a* $(1, 2)$ *-* θ *-B-set,* $(1, 2)$ *-* θ_β *-B-set and a* $(1, 2)$ *-strong* θ_β *-B-set but it is not a* τ_1 *-open set.*

Example 5.7. In Example 4.7, *the set* $A = \{a, c\}$ *is a* $(2, 1)$ *-B-set but not a* $(2, 1)$ *-* θ *-B-set.*

Example 5.8. *Let* $X = \{a, b, c, d\}$ *with topologies* $\tau_1 = \{\emptyset, \{d\}, \{a, c\}, \{a, c, d\}, X\}$ *and* $\tau_2 = \{\emptyset, \{a\}, \{d\}, \{a, d\}, \{a, b, c\}, X\}$. *Then the set* $B = \{a, b, c\}$ *is a* $(1, 2)$ *-* θ *-B-set but it is not a* $(1, 2)$ *-strong* θ_β *-B-set.*

Example 5.9. *In* Example 5.8, *the set* $A = \{a, b, c\}$ *is a* $(1, 2)$ *-* θ *-B-set but it is not a* $(1, 2)$ *-* θ_β *-B-set. In* Example 4.8, *the set* $A = \{a, c, d\}$ *is a* $(1, 2)$ *-strong* θ_β *-B-set but it is not a* $(1, 2)$ *-strong* θ_β *-t-set.*

Example 5.10. *In* Example 5.5, *the set* $A = \{b, c\}$ *is a* $(1, 2)$ *-* θ_β *-B-set but it is not a* $(1, 2)$ *-strong* θ_β *-B-set.*

Proposition 5.11. *For a subset* A *of a space* (X, τ_1, τ_2), *the following are equivalent:*

(1) *A is τ_i-open.*

(2) *A is an (i,j)-θ-pre open set and an (i,j)-θ-B-set.*

(3) *A is an (i,j)-θ-β-open set and an (i,j)-θ_β-B-set.*

(4) *A is an (i,j)-weakly θ-β-open set and an (i,j)-strong θ_β-B-set.*

Proof. $(1) \Rightarrow (2)$, $(1) \Rightarrow (3)$ and $(1) \Rightarrow (4)$ are obvious, since X is (i,j)-θ-t-set, (i,j)-strong θ-t-set and (i,j)-θ_β-t-set and since $A \subseteq \tau_j\text{-}cl_\theta(A)$.

$(2) \Rightarrow (1)$. Let A be (i,j)-θ-pre open and an (i,j)-θ-B-set. Then we have $A = U \cap V$, where $U \in \tau_i$ and V is an (i,j)-θ-t-set. Now,

$A = U \cap A$

$\subseteq U \cap \tau_i\text{-}int(\tau_j\text{-}cl_\theta(A))$

$= U \cap \tau_i\text{-}int(\tau_j\text{-}cl_\theta(U \cap V))$

$\subseteq U \cap \tau_i\text{-}int(\tau_j\text{-}cl_\theta(U) \cap \tau_j\text{-}cl_\theta(V))$

$= U \cap \tau_i\text{-}int(\tau_j\text{-}cl_\theta(U)) \cap \tau_i\text{-}int(\tau_j\text{-}cl_\theta(V))$

$= U \cap \tau_i\text{-}int(\tau_j\text{-}cl_\theta(U)) \cap \tau_i\text{-}int(V)$ [since V is an (i,j)-θ-t-set]

$= U \cap \tau_i\text{-}int(V)$ [since $U = \tau_i\text{-}int(U) \subseteq \tau_i\text{-}int(\tau_j\text{-}cl_\theta(U))$]

That is, $A \subseteq U \cap \tau_i\text{-}int(V)$ and also $A = U \cap V \supseteq U \cap \tau_i\text{-}int(V)$.

Therefore $A = U \cap \tau_i\text{-}int(V) = \tau_i\text{-}int(A)$. Hence A is τ_i-open.

$(3) \Rightarrow (1)$. Let A be an (i,j)-θ-β-open set and an (i,j)-θ_β-B-set. Then we have $A = U \cap V$, where $U \in \tau_i$ and V is an (i,j)-θ_β-t-set. Now,

$A = U \cap A$

$\subseteq U \cap \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)))$

$= U \cap \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(U \cap V)))$

$\subseteq U \cap \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(U) \cap \tau_j\text{-}cl_\theta(V)))$

$= U \cap \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(U)) \cap \tau_i\text{-}int(\tau_j\text{-}cl_\theta(V)))$

$\subseteq U \cap \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(U))) \cap \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(V)))$

$= U \cap \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(U))) \cap \tau_i\text{-}int(V)$ [since V is an (i,j)-θ_β-t-set]

$= U \cap \tau_i\text{-}int(V)$ [since $U = \tau_i\text{-}int(U) \subseteq \tau_j\text{-}cl(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(U)))$]

That is, $A \subseteq U \cap \tau_i\text{-}int(V)$ and also $A = U \cap V \supseteq U \cap \tau_i\text{-}int(V)$.

Therefore $A = U \cap \tau_i\text{-}int(V) = \tau_i\text{-}int(A)$. Hence A is τ_i-open.

$(4) \Rightarrow (1)$. Let A be (i,j)-weakly θ-β-open and an (i,j)-strong θ_β-B-set. Then we have $A = U \cap V$, where $U \in \tau_i$ and V is an (i,j)-strong θ_β-t-set. Now,

$A = U \cap A$

$\subseteq U \cap \tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(A)))$

$= U \cap \tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(U \cap V)))$

$\subseteq U \cap \tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(U) \cap \tau_j\text{-}cl_\theta(V)))$

$= U \cap \tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(U)) \cap \tau_i\text{-}int(\tau_j\text{-}cl_\theta(V)))$

$\subseteq U \cap \tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(U))) \cap \tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(V)))$

$= U \cap \tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(U))) \cap \tau_i\text{-}int(V)$ [since V is an (i,j)-strong θ_β-t-set]

$= U \cap \tau_i\text{-}int(V)$ [since $U = \tau_i\text{-}int(U) \subseteq \tau_j\text{-}cl_\theta(\tau_i\text{-}int(\tau_j\text{-}cl_\theta(U)))$]

That is, $A \subseteq U \cap \tau_i\text{-}int(V)$ and also $A = U \cap V \supseteq U \cap \tau_i\text{-}int(V)$.

Therefore $A = U \cap \tau_i\text{-}int(V) = \tau_i\text{-}int(A)$. Hence A is τ_i-open.

Remark 5.12. *From the following examples we see that in a bitopological space* (X, τ_1, τ_2),

1. *The notions of* (i,j)-θ*-pre open sets and* (i,j)-θ*-B-sets are independent.* (Example 5.13)

(2) *The notions of* (i,j) *-* θ *-* β *-open sets and* (i,j) *-* θ_β *-B-sets are independent.* (Example 5.14 and Example 5.15)

(3) *The notions of* (i,j) *-weakly* θ *-* β *-open sets and* (i,j) *-strong* θ_β *-B-sets are independent.* (Example 5.14 and Example 5.15)

Example 5.13. *Let* $X = \{a,b,c,d\}$ *with topologies* $\tau_1 = \{\emptyset, \{d\}, \{a,c\}, \{a,c,d\}, X\}$ *and* $\tau_2 = \{\emptyset, \{a\}, \{d\}, \{a,d\}, \{a,c,d\}, X\}$. *Then the set* $A = \{a\}$ *is* $(1,2)$ *-* θ *-pre open but it is not a* $(1,2)$ *-* θ *-B-set. Moreover,* $B = \{c,d\}$ *is a* $(1,2)$ *-* θ *-B-set but it is not* $(1,2)$ *-* θ *-pre open.*

Example 5.14. *In* Example 5.8, *the set* $A = \{a,b,c\}$ *is a* $(1,2)$ *-weakly* θ *-* β *-open set but it is not a* $(1,2)$ *-strong* θ_β *-B-set. In* Example 3.11, *the set* $B = \{a\}$ *is a* $(1,2)$ *-* θ *-* β *-open set but it is not a* $(1,2)$ *-* θ_β *-B-set.*

Example 5.15. *In* Example 3.8, *the set* $A = \{c\}$ *is* $(1,2)$ *-* θ_β *-B-set but it is not* $(1,2)$ *-* θ *-* β *-open. Let* $X = \{a,b,c,d\}$ *with topologies* $\tau_1 = \{\emptyset, \{d\}, X\}$ *and* $\tau_2 = \{\emptyset, \{a\}, \{d\}, \{a,d\}, \{a,b,c\}, X\}$. *Then the set* $B = \{a,b,c\}$ *is a* $(1,2)$ *-strong* θ_β *-B-set but it is not* $(1,2)$ *-weakly* θ *-* β *-open.*

Remark 5.16. *From the above propositions, we have the following diagram. None of the implications is reversible.*

$$\begin{array}{ccccccc}
(i,j)\text{-strong } \theta_\beta\text{-B-set} & \Longrightarrow & (i,j)\text{-}\theta_\beta\text{-B-set} & \Longrightarrow & (i,j)\text{-}\theta\text{-B-set} & \Longrightarrow & (i,j)\text{-B-set} \\
\Uparrow & & \Uparrow & & \Uparrow & & \Uparrow \\
(i,j)\text{-strong } \theta_\beta\text{-t-set} & \Longrightarrow & (i,j)\text{-}\theta_\beta\text{-t-set} & \Longrightarrow & (i,j)\text{-}\theta\text{-t-set} & \Longrightarrow & (i,j)\text{-t-set}
\end{array}$$

6. Decompositions of Pairwise continuity

Definition 6.1. *A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be* p-θ-*pre-continuous if for every* $V \in \sigma_k$, $k = 1, 2$, $f^{-1}(V)$ *is an* (i, j)-θ-*pre open in* (X, τ_1, τ_2).

Definition 6.2. *A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be* p-θ-β-*continuous if for every* $V \in \sigma_k$, $k = 1, 2$, $f^{-1}(V)$ *is an* (i, j)-θ-β-*open set in* (X, τ_1, τ_2).

Definition 6.3. *A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be* p-*weakly* θ-*pre-continuous if for every* $V \in \sigma_k$, $k = 1, 2$, $f^{-1}(V)$ *is an* (i, j)-*weakly* θ-*pre open set in* (X, τ_1, τ_2).

Definition 6.4. *A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be* p-θ-*B-continuous if for every* $V \in \sigma_k$, $k = 1, 2$, $f^{-1}(V)$ *is an* (i, j)-θ-*B-set in* (X, τ_1, τ_2).

Definition 6.5. *A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be* p-θ_β-*B-continuous if for every* $V \in \sigma_k$, $k = 1, 2$, $f^{-1}(V)$ *is an* (i, j)-θ_β-*B-set in* (X, τ_1, τ_2).

Definition 6.6. *A function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be* p-*strong* θ_β-*B-continuous if for every* $V \in \sigma_k$, $k = 1, 2$, $f^{-1}(V)$ *is an* (i, j)-*strong* θ_β-*B-set in* (X, τ_1, τ_2).

Proposition 6.7. *For a function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, *the following hold:*

(1) *Every* p-θ-B-*continuous function is* p-B-*continuous.*

(2) *Every* p - θ_β - B *-continuous function is* p - θ - B *-continuous.*

(3) *Every* p *-strong* θ_β - B *-continuous function is* p - θ - B *-continuous and* p - θ_β - B *-continuous.*

Proof. The proof is obvious from Proposition 4.5.

Theorem 6.8. *For a function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, *the following are equivalent:*

(1) f *is p-continuous.*

(2) f *is* p - θ *-pre-continuous and* p - θ *-B-continuous.*

(3) f *is* p - θ - β *-continuous and* p - θ_β *-B-continuous.*

(4) f *is* p *-weakly* θ - β *-continuous and* p *-strong* θ_β *-B-continuous.*

Proof. This is an immediate consequence of Proposition 5.11.

Remark 6.9. *From the following examples we see that in a bitopological space* (X, τ_1, τ_2),

(1) *The notions of* p - θ *-pre-continuity and* p - θ *-B-continuity are independent.* (Example 6.10 and Example 6.11)

(2) *The notions of* p - θ - β *-continuity and* p - θ_β *-B-continuity are independent.* (Example 6.10 and Example 6.12)

(3) *The notions of* p *-weakly* θ - β *-continuity and* p *-strong* θ_β *-B-continuity are independent.* (Example 6.13 and Example 6.14)

Example 6.10. *Let* $X = \{a, b, c\}$ *with topologies* $\tau_1 = \{\emptyset, \{a\}, \{a, b\}, X\}$, $\tau_2 = \{\emptyset, \{b\}, \{a, b\}, \{b, c\}, X\}$ *and let* $Y = \{p, q, r\}$ *with topologies* $\sigma_1 = \{\emptyset, \{q\}, \{p, r\}, Y\}$ *and* $\sigma_2 = \{\emptyset, \{p, r\}, Y\}$. *Let* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *be a function defined* *as f(a) = p, f(b) = q and f(c) = r. Then f is* p-θ-*pre-continuous and* p-θ-β-*continuous but neither* p-θ-*B-continuous nor* p-θ_β-*B-continuous.*

Example 6.11. *Let* $X = \{a, b, c, d\}$ *with topologies* $\tau_1 = \{\emptyset, \{d\}, \{a, c\}, \{a, c, d\}, X\}$ *and* $\tau_2 = \{\emptyset, \{a\}, \{d\}, \{a, d\}, \{a, b, c\}, X\}$. *and let* $Y = \{p, q, r\}$ *with topologies* $\sigma_1 = \{\emptyset, \{p\}, Y\}$ *and*
$\sigma_2 = \{\emptyset, \{q\}, Y\}$. *Let* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *be a function defined as f(a) = f(b) = f(c) = p and f(d) = q. Then f is* p-θ-*B-continuous but not* p-θ-*pre-continuous.*

Example 6.12. *Let* $X = \{a, b, c\}$ *with topologies* $\tau_1 = \{\emptyset, \{c\}, \{a, b\}, X\}$, $\tau_2 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$ *and let* $Y = \{p, q, r\}$ *with topologies* $\sigma_1 = \{\emptyset, \{p\}, Y\}$ *and* $\sigma_2 = \{\emptyset, \{p, q\}, Y\}$. *Let* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *be a function defined as f(a) = p, f(b) = q and f(c) = r. Then f is* p-θ_β-*B-continuous but not* p-θ-β-*continuous.*

Example 6.13. *Let* $X = \{a, b, c, d\}$ *with topologies*
$\tau_1 = \{\emptyset, \{d\}, \{a, d\}, \{b, c\}, \{b, c, d\}, X\}$, $\tau_2 = \{\emptyset, \{d\}, \{a, b, c\}, X\}$ *and let* $Y = \{p, q, r\}$ *with topologies* $\sigma_1 = \{\emptyset, \{p, q\}, Y\}$ *and* $\sigma_2 = \{\emptyset, \{p, r\}, Y\}$. *Let* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *be a function defined as f(a) = r, f(b) = q and f(c) = f(d) = p. Then f is* p-*weakly* θ-β-*continuous but not* p-*strong* θ_β-B-*continuous.*

Example 6.14. *Let* $X = \{a, b, c, d\}$ *with topologies* $\tau_1 = \{\emptyset, \{d\}, X\}$ *and* $\tau_2 = \{\emptyset, \{a\}, \{d\}, \{a, d\}, \{a, b, c\}, X\}$ *and let* $Y = \{p, q\}$ *with topologies* $\sigma_1 = \{\emptyset, \{p\}, Y\}$ *and* $\sigma_2 = \{\emptyset, \{q\}, Y\}$. *Let* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *be a function defined as f(a) = f(b) = f(c)= p, and f(d) = q. Then f is* p-*strong* θ_β-B-*continuous but not* p-*weakly* θ-β-*continuous.*

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