Piezoelectric Contributions to Parametric Amplification of Acoustical Phonons in Magnetized n-InSb Crystal

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Abstract: Using the hydrodynamic model of semiconductor-plasmas and following the coupled-mode approach, we develop a theoretical formulation to study piezoelectric contributions to parametric amplification of acoustical phonons in magnetized n-InSb crystal. The origin of nonlinear interaction is assumed to lie in effective second-order optical susceptibility arising due to nonlinear induced current density and electrostrictive polarization of the medium. Expressions are obtained for threshold pump amplitude for the onset of parametric process and parametric gain coefficient (well above the threshold pump field) in the presence and absence of piezoelectricity and/or externally applied magnetostatic field. Numerical analysis is made for n-InSb crystal irradiated by 10.6 m pulsed CO₂ laser. The piezoelectric contributions to three-wave parametric amplification process are only in the presence of magnetostatic field. The parametric gain coefficient is independent of doping concentration for the cases when either/both piezoelectricity and magnetostatic field are absent. Around resonance (electron-cyclotron frequency ~ pump frequency), the parametric gain coefficient in the presence of piezoelectricity is 10² times higher than that in the absence of piezoelectricity. The analysis establishes the technological potentiality of transversely magnetized n-InSb as the host for parametric devices, like parametric amplifiers and oscillators.

Keywords: Parametric amplification, Acoustical phonons, Piezoelectricity, Electrostriction, n-InSb crystal.

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Introduction

Parametric interaction (PI) processes have played a distinctive role in nonlinear optics. Optical parametric amplifiers, optical parametric oscillators, optical phase conjugation, pulse narrowing, squeezed state generation, … etc. [1] are some of the outcomes of PI processes in a nonlinear medium. Among these, optical parametric amplifiers are of special interest owing to their successful application in laser technology, laser spectroscopy, optical communication, photo-physics, photo-chemistry, material processing, … etc. [2, 3]. The basic underlying principle of optical parametric amplifiers lies in optical parametric amplification. It is a three-wave mixing process and its origin lies in second-order optical susceptibility $\chi^{(2)}$ of the nonlinear medium. For a comprehensive overview, the reader is addressed to [4].

Literature survey reveals that the manipulations of threshold pump field and gain coefficient have been important issues to
improve the efficiency and functionality of optical parametric devices. It has been found that crystalline nonlinear media offer greatest potential for fabrication of optoelectronic devices [5]. The reason for this is of two folds: First, it can be shown that $\chi^{(2)}$ is zero unless the medium lacks a centre of symmetry [6]. Second, the birefringence of a crystalline medium could be used to match the phase velocities of fundamental and harmonic radiation by compensating material dispersion [7]. These promising results are a considerable spur to activity and at about this time, a search began for suitable nonlinear optical materials. The qualities looked for are many and demanding. Despite the considerable success of this research, disappointingly few materials have proved capable of satisfying the requirements; thus, the development of parametric devices is still material-limited [8, 9].

It has been found that n-InSb crystal is transparent to photons of energy below the band gap [10-12] and thus proved to be an advantageous host for fabrication of optoelectronic devices based on PI processes. Moreover, in this crystal, different types of coherent modes, such as plasmon, acoustical phonon (AP), optical phonon (OP), polaron, polariton modes, … etc. can be excited at the expense of the pump wave. The plasmon mode arises due to plasma oscillations. AP and OP modes arise due to lattice and molecular vibrations, respectively. The polaron mode originates due to conduction electron together with its self-induced polarization. The polariton mode originates due to strong interaction of pump field with an electric and magnetic dipole-carrying excitation. Under appropriate physical conditions and using the coupled-mode scheme, a strong tunable electromagnetic Stokes mode with a considerable growth rate may be achieved as a signal wave.

Up to now, the parametric amplification caused by different optically excited coherent modes, in n-InSb crystal, has been reported by several research groups [13-17]. By considering the AP mode as an idler wave and scattered Stokes mode as signal wave, parametric amplification in n-InSb crystal has been reported by Singh et al. [18]. In all these earlier studies, the piezoelectric contributions to parametric amplification of APs have been ignored. However, n-InSb crystal, being non-centrosymmetric, exhibits a weak piezoelectric behavior, which is expected to modify $\chi^{(2)}$ and hence, the related phenomena [19].

In addition to piezoelectricity, the effect of externally applied electric/magnetic fields on parametric amplification of APs in n-InSb crystal has not been explored at length. Singh et al. [20] observed a large enhancement in $\chi^{(2)}$ in various III-V semiconductors, including n-InSb crystal, when placed in an external magnetostatic field of 14.2 T. Therefore, an enhancement in parametric gain coefficient (dependent on $\chi^{(2)}$) seems to be possible in n-InSb crystal.

Motivated by the above discussion, in this paper, we present an analytical investigation of parametric amplification of APs in n-InSb crystal. The influences of piezoelectricity and externally applied magnetostatic field on threshold pump amplitude and gain coefficient of the parametric process are studied in detail. Weakly piezoelectric n-type n-InSb crystal is chosen as nonlinear optical material subjected to a pump wave. It is worth pointing out that in n-InSb crystals, the free carrier absorption is quite small and can be neglected [21]. In order to study the parametric process, nonlinear mechanisms taken into account in the present analysis are: (i) scattering of pump wave by the nonlinear carrier density; and (ii) nonlinear polarization that is the cause of the nonlinear coupling between the pump electromagnetic wave and AP mode on account of electrostriction. Using the coupled-mode theory for semiconductor-plasma, the complex effective second-order optical susceptibility $\chi^{(2)}_{\text{eff}}$ of the crystal and consequent threshold pump amplitude $E_{0,\text{th}}$ for the onset of parametric process and parametric gain coefficient $g_{\text{para}}$ well above the threshold field ($E_0 > E_{0,\text{th}}$) are obtained in the presence and absence of piezoelectricity and/or externally applied magnetostatic field.

## Theoretical Formulations

We consider the well-known hydrodynamic model of homogeneous n-type semiconductor-plasma at 77 K (i.e., liquid nitrogen temperature) with electrons as carriers subjected to an intense pump wave and an external magnetostatic field under thermal equilibrium. This model is
suitable for the present theoretical formulation, as it simplifies our analysis, without any loss of significant information, by replacing the streaming electrons with an electron fluid described by a few macroscopic parameters, like average carrier density, average velocity, etc. However, it restricts our analysis to be valid only in the limit \( k_a l \ll 1; k_a \) being the acoustic wave number and \( l \) the carrier mean free path. This means that the time in which the AP mode travels one wavelength, the carriers have undergone many collisions and hence, the average thermal velocity of carrier vanishes. Thus, the total velocity is equal to the average drift velocity. In n-InSb crystal under a magnetostatic field, the piezoelectric and electrostrictive coefficients are no longer isotropic and therefore off-diagonal components of the susceptibility tensor are non-zero. For simplicity, we consider generation of longitudinal AP mode in a cubic media possessing 43-\( m \) symmetry and for such mode, the piezoelectric and electrostrictive tensors may reduce to a single component [22].

Effective Second-order Optical Susceptibility

The three-wave coupled-mode scheme has been employed to obtain an expression for \( \chi_{\text{eff}}^{(2)} \). The origin of \( \chi_{\text{eff}}^{(2)} \) lies in coupling between the pump wave and signal waves via density perturbations in the crystal. We consider the parametric coupling among three waves:

(i) The input strong pump wave

\[
E_0(x,t) = E_0 \exp[i(k_0x - \omega_0 t)],
\]

(ii) The induced AP mode (idler)

\[u(x,t) = u_0 \exp[i(k_0x - \omega_0 t)] \] and

(iii) The scattered Stokes component of pump electromagnetic wave (signal)

\[E_s(x,t) = E_s \exp[i(k_sx - \omega_s t)].\]

The momentum and energy conservation relations for these modes should satisfy the phase matching conditions: \( \hbar k_0 = \hbar k_s + \hbar k_a \) and \( \hbar \omega_0 = \hbar \omega_s + \hbar \omega_a \). We consider \( \tau_p > \Gamma_a^{-1} \), where \( \tau_p \) is the pump pulse duration and \( \Gamma_a (\approx 10^{-11} \text{ SI units}) \) the AP mode damping time. This condition allows the nonlinear interaction to be treated as a quasi-steady state process [23]. We consider the n-InSb crystal to be immersed in a transverse magnetostatic field \( \vec{B}_z = zB_z \) (i.e., perpendicular to the direction of the input pump beam).

The time-varying pump electric field produces density perturbations and is thus capable of deriving AP mode in the crystal. Let \( u(x,t) \) be the deviation of a point \( x \) from its equilibrium position, so that the strain along the direction of pump wave is \( \frac{\partial u}{\partial x} \). The equation of motion of \( u(x,t) \) can be expressed as [24]:

\[
\frac{\partial^2 u}{\partial t^2} - \frac{c \partial^2 u}{\partial x^2} + 2I_a \frac{\partial u}{\partial t} = \frac{F}{\rho},
\]

where \( \rho \) is the mass density of the crystal. \( C \) represents the linear elastic modulus of the crystal such that the AP mode velocity is given by \( v_a = (C / \rho)^{1/2} \). The term \( 2 \Gamma_a \frac{\partial u}{\partial t} \) is introduced phenomenologically to include the acoustical damping, in which \( \Gamma_a (\approx 10^{-11} \text{ SI units}) \) [25] is the phenomenological damping parameter of the AP mode. \( F \) represents the effective force per unit volume experienced by the medium due pump wave electric field; it can be expressed as:

\[
F = F^{(1)} + F^{(2)},
\]

where \( F^{(1)} = -\beta \frac{\partial E_s}{\partial x} \) and \( F^{(2)} = \frac{\gamma}{2} \frac{\partial}{\partial x} (E_s E_s^*) \) represent the first- and second-order forces due to the piezoelectric and electrostrictive properties of the crystal, respectively. Here, \( \beta \) and \( \gamma \) are the piezoelectric and electrostriction coefficients of the medium. It is worth pointing out that in the presence of the time-varying pump electric field, the ions within the lattice move into non-symmetrical position, usually producing a contraction in the direction of the field and an expansion across it. The electrostrictive force thus produced is the origin of the electrostriction in the medium. The coupling of pump electric field and elastic properties of the lattice gives rise to piezoelectric force in the medium. \( E_s \) is the space-charge electric field of the medium.

In previously reported works, the origin of the PI processes has been taken into second-order forces; the contributions arising due to first-order forces (i.e., piezoelectric property of the crystal) have normally been ignored. In the present analysis, we included the piezoelectric...
forces to study the important phenomenon of parametric amplification in the n-InSb crystal. After including the effects of piezoelectricity, the modified equation of motion for \( u(x, t) \) of lattice vibrations in the crystal is given as [25]:
\[
\frac{\partial^2 u}{\partial t^2} - \frac{c}{\rho} \frac{\partial^2 u}{\partial x^2} = 2 \Gamma_a \frac{\partial u}{\partial t} - \frac{\beta \delta E_{15}}{\rho} + \frac{\nu}{2\rho} \frac{\partial}{\partial x} (E_0 E_{15}),
\]
(1b)

The other basic equations in the formulation of \( \chi^{(3)}_{\text{eff}} \) are:
\[
\frac{\partial \bar{E}}{\partial t} + v \frac{\partial \bar{v}}{\partial x} + \left( \bar{v}_0 \frac{\partial}{\partial x} \right) \bar{v}_0 = -\frac{e}{m} \left[ \bar{B}_0 + (\bar{v}_0 \times \bar{B}_0) \right] = -\frac{e}{m} (\bar{E}_0 + \bar{v}_0 \times \bar{B}_0)
\]
(2)
\[
\frac{\partial \bar{v}}{\partial t} + \frac{\partial \bar{v}_1}{\partial x} + \left( \bar{v}_1 \frac{\partial}{\partial x} \right) \bar{v}_1 = -\frac{e}{m} \left[ \bar{E}_1 + (\bar{v}_1 \times \bar{B}_0) \right]
\]
(3)
\[
\frac{\partial n}{\partial t} + n_0 \frac{\partial \bar{v}_1}{\partial x} + n_1 \frac{\partial \bar{v}_0}{\partial x} + \nu_0 \frac{\partial n}{\partial x} = 0
\]
(4)
\[
\bar{P}_{\text{es}} = -\gamma \frac{\partial \bar{v}}{\partial x}
\]
(5)
\[
\frac{\partial^2 \bar{E}_{15}}{\partial x^2} + \frac{\bar{E}_0}{\epsilon} \frac{\partial^2 \bar{u}^*}{\partial x^2} + \frac{\gamma^2}{\epsilon} \frac{\partial^2 \bar{u}^*}{\partial x^2} (E_0) = -\frac{n_1 e}{\epsilon}
\]
(6)

Eqs. (2) and (3) are the zeroth and first-order electron momentum transfer equations, in which \( \bar{v}, \bar{v}_1 \) and \( v \) are the first-order oscillatory fluid velocity and collision frequency of an electron, respectively. \( \bar{v}_0 \) is the oscillatory fluid velocity of an electron of charge \(-e\) and effective mass \( m \) at pump wave frequency \( \omega_0 \). In Eq. (2), \( \bar{E}_0 \) represents the effective electric field which includes the Lorentz force \((\bar{v}_0 \times \bar{B}_0)\) due to external magnetic field. In Eq. (3), we have neglected the effect due to \((\bar{v}_0 \times \bar{B}_0)\) by assuming that the propagating AP mode is producing a longitudinal electric field. Eq. (4) is the continuity equation for electrons, in which \( n_0 \) and \( n_1 \) are equilibrium and perturbed electron density, respectively. Eq. (5) illustrates that the AP mode generated via electrostrictive strain modulates the dielectric constant of the semiconductor medium and gives rise to the nonlinear induced polarization \( P_{\text{es}} \). Poisson’s Eq. (6) gives the space charge field \( E_{15} \), in which \( \epsilon(=\varepsilon_s \varepsilon_r) \) is the dielectric constant of the semiconductor medium, where \( \varepsilon_s \) is the static dielectric constant of the medium. The asterisk (*) stands for the conjugate of a complex entity.

The piezoelectric and electrostrictive forces generate a carrier density perturbation in the medium which can be obtained by using the method adopted by one of the present authors [18].

Differentiating Eq. (4) and then putting the first-order derivatives of \( n_0 \) and \( n_1 \) from Eqs. (2) and (3) and \( E_0 \) from Eq. (6), we obtain:
\[
\frac{\partial^2 n_1}{\partial t^2} + n \frac{\partial n_1}{\partial t} + \omega_p^2 n_1 + \frac{n_0 e k^2 u^*}{m \varepsilon_1} \left( \frac{\beta \psi \delta E_{15}}{E_0} + \gamma^2 \right) E_0 E_{15}^* = i n_1 k_s \bar{E},
\]
(7)
where \( \bar{E} = \epsilon / m (\bar{E}_0 + \bar{v}_0 \times \bar{B}_0) \), \( \delta_1 = 1 - \frac{\omega_0^2}{(\omega_0 - \omega_0^*)}, \alpha_p = \frac{n \omega_p}{(\omega_0^2 + \omega_0^*)^{1/2}}, \omega_c = \frac{e B_0}{m} \) (electron-cyclotron frequency) and \( \omega_0 = (\frac{n_0 e^2}{m \varepsilon_1})^{1/2} \) (electron-plasma frequency).

The perturbed electron concentration \( n_1 \) can be expressed as: \( n_1 = n_1 \_f (\omega_0) + n_1 \_s (\omega_0) \), where \( n_1 \_f \) (slow component) oscillates at AP mode frequency \( \omega_0 \), and \( n_1 \_s \) (high component) is associated with electromagnetic wave frequencies \( \omega_0 \pm \omega_s \). The higher-order terms at frequencies \( \omega_0 \pm \omega_p \), for \( p = 2, 3, \ldots \), being off-resonant, are neglected. In the forthcoming formulation, we consider only the first-order Stokes component of the back-scattered electromagnetic wave. Under rotating-wave approximation (RWA), Eq. (7) leads to the following coupled equations:
\[
\frac{\partial^2 n_1 f}{\partial t^2} + \frac{\partial n_1 f}{\partial t} + \omega_p^2 n_1 f + \frac{n_0 e k^2 u^*}{m \varepsilon_1} \left( \frac{\beta \psi \delta E_{15}}{E_0} + \gamma^2 \right) E_0 E_{15}^* = -i n_1 f k_s \bar{E},
\]
(8a)
and
\[
\frac{\partial^2 n_1 s}{\partial t^2} + \frac{\partial n_1 s}{\partial t} + \omega_p^2 n_1 s = i n_1 s f k_s \bar{E}.
\]
(8b)

Eqs. (8a) and (8b) reveal that the slow and fast components \((n_1 \_f, n_1 \_s)\) of the density perturbations are coupled to each other via the pump electric field \((\bar{E})\). Thus, it is clear that for the PI process to occur, the presence of the pump field is the fundamental necessity.
By solving these equations and using Eq. (1b), the expression for $n_s$ is obtained as:

$$n_{s} = \frac{\varepsilon_{0}\alpha_{s}\kappa_{s}}{2p_{s}\delta_{s}(\alpha_{s}^{2}+2i\Gamma_{s}\omega_{n})} \left( \frac{\gamma_{s}\delta_{s}^{2}}{E_{0}} + \gamma^{2} \right) E_{0} E_{s}^{*}. $$

(9)

where $\Omega_{e}^{2} = \omega_{p}^{2} - k_{s}^{2}v_{e}^{2}$, $\delta_{e} = 1 - \frac{k_{s}^{2}e^{2}}{(\alpha_{s}^{2}v_{e}^{2})(\alpha_{s}^{2}+i\Gamma_{s}\omega_{n})}$, in which $\Omega_{p}^{2} = \omega_{p}^{2} - \omega_{s}^{2}$ and $\Omega_{p}^{2} = \omega_{p}^{2} - \omega_{q}^{2}$.

The nonlinear induced current density can be obtained as:

$$J_{cd}(\omega_{s}) = n_{s}^{*}e_{0}v_{0}$$

$$= \frac{\varepsilon_{0}\alpha_{s}\kappa_{s}^{2}v_{e}(\omega_{s}-i\omega_{0})}{2p_{s}\delta_{s}(\alpha_{s}^{2}+2i\Gamma_{s}\omega_{n})} \left( \frac{\gamma_{s}\delta_{s}^{2}}{E_{0}} + \gamma^{2} \right) E_{0} E_{s}^{*}. $$

(10)

The time integral of induced current density yields nonlinear induced polarization as:

$$P_{cd}(\omega_{s}) = \int J_{cd}(\omega_{s}) dt$$

$$= \frac{\varepsilon_{0}\kappa_{s}^{2}v_{e}(\omega_{s}-i\omega_{0})}{2p_{s}\delta_{s}(\alpha_{s}^{2}+2i\Gamma_{s}\omega_{n})} \left( \frac{\gamma_{s}\delta_{s}^{2}}{E_{0}} + \gamma^{2} \right) E_{0} E_{s}^{*}. $$

(11)

Comparing Eq. (11) with the well-known relation $P_{cd}(\omega_{s}) = e_{0}\chi_{cd}^{(2)} E_{0} E_{s}^{*}$, the second-order optical susceptibility $\chi_{cd}^{(2)}$ due to induced current density is given by:

$$\chi_{cd}^{(2)} = \frac{\varepsilon_{0}\kappa_{s}^{2}v_{e}(\omega_{s}-i\omega_{0})}{2p_{s}\delta_{s}(\alpha_{s}^{2}+2i\Gamma_{s}\omega_{n})} \left( \frac{\gamma_{s}\delta_{s}^{2}}{E_{0}} + \gamma^{2} \right). $$

(12)

Here, it is worth pointing out that besides $P_{cd}(\omega_{s})$, the system possesses electrostrictive polarization $P_{es}(\omega_{s})$ that arises due to coherent interaction of the pump wave with the AP mode generated in the medium. This is due to the fact that the scattering of light from the AP mode provides a convenient means of controlling the frequency, intensity and direction of an optical beam. This type of control makes possible a large number of applications involving the transmission, display and processing of information. The electrostrictive polarization can be obtained from Eq. (5) and using Eq. (1b) as:

$$P_{es}(\omega_{s}) = \frac{k_{r}\kappa_{r}^{2}v_{e}^{2}}{2p_{r}(\alpha_{r}^{2}+2i\Gamma_{r}\omega_{n})} \left( \frac{\gamma_{r}\delta_{r}^{2}}{E_{0}} + \gamma^{2} \right) E_{0} E_{s}^{*}. $$

(13)

Comparing Eq. (13) with the well-known relation $P_{es}(\omega_{s}) = e_{0}\chi_{es}^{(2)} E_{0} E_{s}^{*}$, the second-order optical susceptibility $\chi_{es}^{(2)}$ due to electrostrictive polarization is given by:

$$\chi_{es}^{(2)} = \frac{k_{r}\kappa_{r}^{2}v_{e}^{2}}{2e_{0}p_{r}(\alpha_{r}^{2}+2i\Gamma_{r}\omega_{n})} \left( \frac{\gamma_{r}\delta_{r}^{2}}{E_{0}} + \gamma^{2} \right). $$

(14)

The effective second-order optical susceptibility at Stokes frequency in the n-InSb crystal due to nonlinear current density and electrostrictive nonlinearity is given by:

$$\chi_{eff}^{(2)} = \chi_{cd}^{(2)} + \chi_{es}^{(2)} = \frac{k_{r}\kappa_{r}^{2}v_{e}^{2}}{2e_{0}p_{r}(\alpha_{r}^{2}+2i\Gamma_{r}\omega_{n})} \left( \frac{\gamma_{r}\delta_{r}^{2}}{E_{0}} + \gamma^{2} \right). $$

(15)

where $\delta_{r} = \left(1 + \frac{\gamma_{r}\delta_{r}^{2}}{E_{0}} \right)$. The complex $\chi_{eff}^{(2)}$ may be expressed as:

$$\chi_{eff}^{(2)} = [\chi_{eff}^{(2)} + i\chi_{eff}^{(2)}], $$

(16)

where $[\chi_{eff}^{(2)}$ and $i\chi_{eff}^{(2)}$, represent the real and imaginary parts of complex $\chi_{eff}^{(2)}$, respectively.

Rationalizing Eq. (15), we get:

$$[\chi_{eff}^{(2)} r] = \frac{k_{r}\kappa_{r}^{2}v_{e}^{2}}{2e_{0}p_{r}(\alpha_{r}^{2}+2i\Gamma_{r}\omega_{n})} \left( \frac{\gamma_{r}\delta_{r}^{2}}{E_{0}} + \gamma^{2} \right), $$

(17a)

and

$$[\chi_{eff}^{(2)} i] = -\frac{k_{r}\kappa_{r}^{2}v_{e}^{2}}{2e_{0}p_{r}(\alpha_{r}^{2}+2i\Gamma_{r}\omega_{n})} \left( \frac{\gamma_{r}\delta_{r}^{2}}{E_{0}} + \gamma^{2} \right). $$

(17b)

The above equations reveal that $[\chi_{eff}^{(2)}$, and $i\chi_{eff}^{(2)}$, are influenced by piezoelectric coefficient $\beta$ (via parameter $\delta_{r}$), externally applied transverse magnetostatic field $B_{o}$ (via parameter $\omega_{s}$) and doping concentration $n_{b}$ (via parameter $\omega_{r}$, and hence $\delta_{r}$).

Here, it is worth pointing out that $[\chi_{eff}^{(2)}$, is responsible for parametric dispersion, while $i\chi_{eff}^{(2)}$, give rise to parametric amplification/
attenuation and oscillation. The present paper deals with the study of parametric amplification of APs in transversely magnetized n-InSb crystal only. The study of parametric dispersion and parametric oscillation is the future plan research work.

Threshold Pump Field for the Onset of Parametric Amplification

As it is well-known, parametric amplification can be achieved at excitation intensities above a certain threshold value. This threshold nature can be obtained by setting \( |\chi^{(2)}_{\text{off}}| = 0 \). This condition can be used to obtain threshold pump amplitude for the onset of parametric process to occur for different cases of interest:

(i) In the presence of magnetostatic field \((B_0 \neq 0; \text{i.e., } \omega_c \neq 0)\):

\[
|E_{0,\text{th}}|_{B_0 \neq 0} = \frac{m}{\varepsilon r_h} \left[ 1 - \frac{\omega_0^2}{\omega_P^2} \right] [(\Omega_{ps}^2 + v^2 \omega_0^2)]^{1/4}.
\]

(18a)

(ii) In the absence of magnetostatic field \((B_0 = 0; \text{i.e., } \omega_c = 0)\):

\[
|E_{0,\text{th}}|_{B_0 = 0} = \frac{m}{\varepsilon r_h} \left[ (\Omega_{ps}^2 + v^2 \omega_0^2) \right]^{1/4}.
\]

(18b)

We observed from Eqs. (18a) and (18b) that \( |E_{0,\text{th}}| \) is strongly influenced by the presence of externally applied magnetostatic field and doping concentration, but is independent of piezoelectric property of the crystal.

Parametric Gain Coefficient

In order to obtain the three-wave parametric amplification/gain coefficient \( \alpha_{\text{para}} \) in a magnetized n-InSb crystal, we employ the relation [18]:

\[
g_{\text{para}} = \frac{\omega_0}{\eta c} |\chi^{(2)}_{\text{off}}|.
\]

(19)

The nonlinear parametric gain of the signal as well as the idler waves can be possible only if \( \alpha_{\text{para}} \) is negative for pump field \(|E_0| > |E_{0,\text{th}}|\).

Eq. (19) can be used to obtain the parametric gain coefficient for different cases of interest:

(i) The parametric gain coefficient due to the presence of both piezoelectricity \((\beta \neq 0; \text{i.e., } \delta_4 \neq 0)\) and magnetostatic field \((B_0 \neq 0; \text{i.e., } \omega_c \neq 0)\) is given by:

\[
g_{\text{para}}(\beta \neq 0, B_0 \neq 0) = \frac{-k_B k_B \omega_0^2 \omega_P^2 |E_0|}{\eta c \varepsilon r_h \left[ (\omega_0^2 - \omega_0^2) + 4 \nu^2 \omega_0^2 \right]} \left( \frac{1 + \omega_0^2}{\omega_0^2} \right). \]

(20a)

(ii) The parametric gain coefficient due to finiteness of piezoelectricity \((\beta \neq 0; \text{i.e., } \delta_4 \neq 0)\) and absence of magnetostatic field \((B_0 = 0; \text{i.e., } \omega_c = 0)\) is given by:

\[
g_{\text{para}}(\beta \neq 0, B_0 = 0) = \frac{-k_B k_B \omega_0^2 \omega_P^2 |E_0|}{\eta c \varepsilon r_h \left[ (\omega_0^2 - \omega_0^2) + 4 \nu^2 \omega_0^2 \right]} \left( \frac{1 + \omega_0^2}{\omega_0^2} \right). \]

(20b)

(iii) The parametric gain coefficient due to absence of piezoelectricity \((\beta = 0; \text{i.e., } \delta_4 = 0)\) and presence of magnetostatic field \((B_0 \neq 0; \text{i.e., } \omega_c \neq 0)\) is given by:

\[
g_{\text{para}}(\beta = 0, B_0 \neq 0) = \frac{-k_B k_B \omega_0^2 \omega_P^2 |E_0|}{\eta c \varepsilon r_h \left[ (\omega_0^2 - \omega_0^2) + 4 \nu^2 \omega_0^2 \right]} \left( \frac{1 + \omega_0^2}{\omega_0^2} \right). \]

(20c)

(iv) The parametric gain coefficient due to absence of both piezoelectricity \((\beta = 0; \text{i.e., } \delta_4 = 0)\) and magnetostatic field \((B_0 = 0; \text{i.e., } \omega_c = 0)\) is given by:

\[
g_{\text{para}}(\beta = 0, B_0 = 0) = \frac{-k_B k_B \omega_0^2 \omega_P^2 |E_0|}{\eta c \varepsilon r_h \left[ (\omega_0^2 - \omega_0^2) + 4 \nu^2 \omega_0^2 \right]} \left( \frac{1 + \omega_0^2}{\omega_0^2} \right). \]

(20d)

It can be observed that Eqs. (20b) and (20d) are identical; i.e., parametric gain coefficients due to (a) finiteness of piezoelectricity \((\beta \neq 0; \text{i.e., } \delta_4 \neq 0)\) and absence of magnetostatic field \((B_0 = 0; \text{i.e., } \omega_c = 0)\) and (b) absence of both piezoelectricity \((\beta = 0; \text{i.e., } \delta_4 = 0)\) and magnetostatic field \((B_0 = 0; \text{i.e., } \omega_c = 0)\) are equal in magnitude. It can be inferred that when \( \omega_c = 0, \delta_4 = 0 \) and hence \( \delta_4 = 0 \), which is the case in absence of piezoelectricity, the piezoelectric contributions to three-wave parametric amplification are only in the presence of magnetostatic field. Moreover, (i) the
Results and Discussion

To have a numerical appreciation of the results obtained in the analysis, the n-InSb crystal is assumed to be irradiated by 10.6-m pulsed CO$_2$ laser. The other parameters are [18]:

\[
  m = 0.0145m_0 \quad (m_0 \text{ being the free mass of electron}), \quad \varepsilon_r = 15.8, \quad v_p = 4 \times 10^{13} \text{ ms}^{-1},
\]

\[
  \beta = 0.054 \text{ Cm}^{-2}, \quad \gamma = 5 \times 10^{-10} \text{ s}^{-1}, \quad \Gamma_a = 2 \times 10^{10} \text{ s}^{-1}, \quad \rho = 5.8 \times 10^3 \text{ kgm}^{-3}, \quad n_0 = 10^{21} - 10^{24} \text{ m}^{-3},
\]

\[
  \omega_0 = 2 \times 10^{11} \text{ s}^{-1}, \quad \nu = 4 \times 10^{11} \text{ s}^{-1} \text{ and } \omega_0 = 1.78 \times 10^{14} \text{ s}^{-1}.
\]

The main focus of this paper is to study threshold and gain characteristics of parametric amplification under the above-mentioned four different cases.

Using the physical constants for n-InSb given above, the nature of dependence of the threshold pump electric field $E_{0,th}$ necessary for the onset of parametric process and parametric gain coefficient $g_{para}$ on different parameters such as acoustical wave number $k_a$, externally applied magnetostatic field $B_0$, doping concentration $n_0$, etc. may be studied from Eqs. (18a) and (18b) respectively and a common dip at $B_0 = 10.6$ T, respectively and a common dip at $B_0 = 14.2$ T. This behaviour can be explained as follows: (i) For $n_0 = 10^{20} \text{ m}^{-3}$, the dip at $B_0 = 10.6$ T arises due to parameter $\Omega^2_{pa}$ (i.e., $\overline{\omega}_p^2 \sim \omega_0^2$, resonance between coupled cyclotron-plasmon frequency and Stokes frequency) [in confirmatory with Eq. (18a)]. (ii) For $n_0 = 10^{22} \text{ m}^{-3}$, the dip at $B_0 = 4.8$ T arises due to parameter $\Omega^2_{pa}$ (i.e., $\overline{\omega}_p^2 \sim \omega_0^2$, resonance between coupled cyclotron frequency and AP mode frequency) [in confirmatory with Eq. (18a)]. The common dip at $B_0 = 14.2$ T arises due to the factor $1 - \frac{\omega_0^2}{\omega_0^2}$ (i.e., $\omega_0^2 \sim \omega_0^2$, resonance between electron cyclotron frequency and pump wave frequency) [in confirmatory with Eq. (18a)]. Moreover, at $B_0 = 14.2$ T, $E_{0,th}$ becomes independent of doping concentration.
The parametric gain characteristics are shown in Fig. 3. The nature of dependence of parametric gain coefficient $g_{\text{para}}$ on acoustical wave number $k_a$ for different cases, viz. (i) $\beta \neq 0$, $B_0 = 14.2$ T, (ii) $\beta = 0$, $B_0 = 14.2$ T, (iii) $\beta \neq 0$ or $\beta = 0$, $B_0 = 0$ T for $n_0 = 10^{22}$ m$^{-3}$ and $n_0 = 10^{20}$ m$^{-3}$. This indicates the threshold pump amplitude $E_{0,m}$ with acoustical wave number $k_a$ in the absence ($B_0 = 0$ T) and presence of magnetostatic field ($B_0 = 14.2$ T) with $n_0 = 10^{22}$ m$^{-3}$.
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\[ E_0 = 7.5 \times 10^4 \text{ V m}^{-1} (\rangle E_{\text{th}}). \]  

Here, \( g_{\text{para}} \) is found to be very sensitive to the dispersion characteristics of the AP mode. It can be observed that in all the cases, \( g_{\text{para}} \) is smaller for \( k_a < 5 \times 10^7 \text{ m}^{-1} \) (i.e., \( \omega_a > k_a v_a \), anomalous acoustical wave dispersion regime). For a particular value of \( k_a = 5 \times 10^7 \text{ m}^{-1} \) (i.e., \( \omega_a = k_a v_a \), dispersion-less acoustical wave regime), \( g_{\text{para}} \) shows a peak and becomes independent of \( \beta \) and \( B_0 \). For \( k_a > 5 \times 10^7 \text{ m}^{-1} \) (i.e., \( \omega_a < k_a v_a \), normal acoustical wave regime), \( g_{\text{para}} \) decreases sharply and then gradually increases with further increasing \( k_a \).

This behaviour may be attributed to the fact that \( g_{\text{para}} \propto (\Omega_a^2 + 4\Gamma_a^2 \omega_s^2)^{-1} \) [Eqs. (20a) – (20d)]. A comparison among all the above three cases reveals that in anomalous and normal acoustical wave dispersion regimes, \( g_{\text{para}} \) is comparatively smaller, satisfying the inequality condition: \( g_{\text{para}}(\beta \neq 0, B_0 = 14.2 \text{ T}) > g_{\text{para}}(\beta = 0, B_0 = 14.2 \text{ T}) > g_{\text{para}}(\beta \neq 0 \text{ or } \beta = 0, B_0 = 0 \text{ T}) \). In dispersion-less acoustical wave regime \( g_{\text{para}} \), being independent of \( \beta \) and \( B_0 \), is about 10^4 times larger than in anomalous and normal acoustical wave dispersion regimes.

![Fig. 3](image-url)

**Fig. 3.** Nature of dependence of parametric gain coefficient \( g_{\text{para}} \) on acoustical wave number \( k_a \) for different cases, viz. (i) \( \beta \neq 0 \), \( B_0 = 14.2 \text{ T} \), (ii) \( \beta = 0 \), \( B_0 = 14.2 \text{ T} \), (iii) \( \beta \neq 0 \) or \( \beta = 0 \), \( B_0 = 0 \text{ T} \) for \( n_0 = 10^{27} \text{ m}^{-3} \) and \( E_0 = 7.5 \times 10^4 \text{ V m}^{-1} (\rangle E_{\text{th}}). \)

![Fig. 4](image-url)

**Fig. 4.** Nature of dependence of parametric gain coefficient \( g_{\text{para}} \) on magnetostatic field \( B_0 \) for two different cases, viz. (i) \( \beta \neq 0 \) (presence of piezoelectricity) and (ii) \( \beta = 0 \) (absence of piezoelectricity) for \( n_0 = 10^{27} \text{ m}^{-3} \), \( E_0 = 7.5 \times 10^4 \text{ V m}^{-1} (\rangle E_{\text{th}}) \) and \( \omega_a = k_a v_a \). For the case when \( \beta \neq 0 \), we observed sharp peaks around \( B_0 = 10.6 \text{ T} \) (due to resonance condition \( \omega_p \sim \omega_s \); \( g_{\text{para}} \propto \delta_4 \) in confirmatory with Eq. (20a)) and \( B_0 = 14.2 \text{ T} \) (due to resonance condition \( \omega_a \sim \omega_s \); \( g_{\text{para}} \propto (\omega_a^2 - \omega_s^2)^2 \) in confirmatory with Eq. (20a)). However, for the case when \( \beta = 0 \), we
observed only a single peak at $B_0 = 14.2 \text{T}$ (due to resonance condition $\omega_0 \approx \omega_0'$; $g_{\text{para}} \propto (\omega_0' - \omega_i')^{-2}$ in confirmatory with Eq. (20c)). A departure from resonance conditions causes a sharp fall in $g_{\text{para}}$. A comparison between the two cases reveals that

$$g_{\text{para}}(B_0 = T > 0) > g_{\text{para}}(B_0 = 0)$$

for the regime of magnetostatic field considered. Around resonance ($\omega_0 \approx \omega_0'$), we obtain:

$$\frac{(g_{\text{para}})_{B_0 = 0}}{(g_{\text{para}})_{B_0 = T}} \approx 10^2.$$  

![FIG. 4. Nature of dependence of parametric gain coefficient $g_{\text{para}}$ on magnetostatic field $B_0$ for two different cases, viz. (i) $\beta \neq 0$ (presence of piezoelectricity) and (ii) $\beta = 0$ (absence of piezoelectricity) for $n_0 = 10^{32} \text{ m}^{-3}$, $E_0 = 7.5 \times 10^6 \text{ V m}^{-1}$ ($> E_{0,\text{th}}$) and $\omega_a = k_a v_\text{drift}$.](image)

It is thus clear from this study that the applied magnetostatic field can be used as a control parameter to enhance the parametric gain coefficient around resonance. Here, it should be pointed out that around resonance, the magnetic field dependent drift velocity becomes many times larger than AP mode velocity and as a result, more energy is transferred from carrier wave to the AP mode and eventually, the AP mode gets amplified. In turn, the amplified AP mode strongly interacts with pump wave and as a result, the parametric gain coefficient enhances substantially.

Fig. 5 shows the nature of dependence of parametric gain coefficient $g_{\text{para}}$ on doping concentration $n_0$ for different cases, viz. (i) $\beta \neq 0$, $B_0 = 14.2 \text{T}$, (ii) $\beta = 0$, $B_0 = 14.2 \text{T}$, (iii) $\beta \neq 0$ or $\beta = 0$, $B_0 = 0 \text{T}$ for $E_0 = 7.5 \times 10^6 \text{ V m}^{-1}$ ($> E_{0,\text{th}}$) and $\omega_a = k_a v_\text{drift}$. We observed that for case (i), $g_{\text{para}}$ shows a sharp peak at a particular value of $n_0 = 5.8 \times 10^{32} \text{ m}^{-3}$. This peak arises due to parameter $\frac{\delta_1}{\delta_2}$ [Eq. 20a)]. However, for cases (ii) and (iii), $g_{\text{para}}$ is independent of $n_0$. While making a comparison among the above three cases, we observed that over the large range of doping concentration, the gain coefficients satisfy the following inequality:
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\[ (g_{\text{para}})_{\beta \neq 0, B_0 = 14.2 \text{T}} > (g_{\text{para}})_{\beta = 0, B_0 = 14.2 \text{T}} > (g_{\text{para}})_{\beta \neq 0 \text{ or } \beta = 0, B_0 = 0 \text{T}}. \]

Around \( n_0 = 5.8 \times 10^{22} \) m\(^{-3}\), we obtain the following ratio:

\[ (g_{\text{para}})_{\beta \neq 0, B_0 = 14.2 \text{T}} : (g_{\text{para}})_{\beta = 0, B_0 = 14.2 \text{T}} : (g_{\text{para}})_{\beta \neq 0 \text{ or } \beta = 0, B_0 = 0 \text{T}} = 1: 10^{-4}, 10^{-5}. \]

**Fig. 5.** Nature of dependence of parametric gain coefficient \( g_{\text{para}} \) on doping concentration \( n_0 \) for different cases, viz. (i) \( \beta \neq 0, B_0 = 14.2 \text{T} \), (ii) \( \beta = 0, B_0 = 14.2 \text{T} \), (iii) \( \beta \neq 0 \) or \( \beta = 0, B_0 = 0 \text{T} \) for

\[ E_0 = 7.5 \times 10^5 \text{ Vm}^{-1} (> E_{0,\text{th}}) \text{ and } \omega_a = k_a v_a. \]

Thus, higher pump field yields higher parametric gain coefficient. Here, it should be noted that we cannot increase the pump field in an arbitrary manner, as it may damage the n-InSb crystal. The damage threshold of the crystal can be increased either by the mechanism of free carrier absorption or by irradiating the crystal by short-duration pulsed laser.
FIG. 6. Nature of dependence of parametric gain coefficient \( g_{\text{para}} \) on pump amplitude \( E_0 (\geq E_{\text{th}}) \) for different cases, viz. (i) \( \beta \neq 0, \ B_0 = 14.2 \text{T} \), (ii) \( \beta = 0, \ B_0 = 14.2 \text{T} \), (iii) \( \beta \neq 0 \) or \( \beta = 0, \ B_0 = 0 \text{T} \) for \( n_0 = 10^{22} \) and \( \omega_c = k_a v_a \).

Conclusions

In the present study, using hydrodynamic model of semiconductor-plasmas and following the coupled-mode approach, the piezoelectric contributions to parametric amplification of APs in magnetized n-InSb crystal have been considered. The analysis enables us to draw the following conclusions:

1. The threshold pump amplitude for the onset of optical parametric amplification in n-InSb crystal can be lowered by proper selection of either/both doping concentration and magnetostatic field \( (n_0 = 10^{30} \text{ m}^{-3}, \ B_0 = 10.6 \text{T} \) or \( n_0 = 10^{22} \text{ m}^{-3}, \ B_0 = 4.8 \text{ T} \) ) or magnetostatic field only \( (B_0 = 14.2 \text{T}) \).

2. The parametric gain coefficients due to (a) finiteness of piezoelectricity and absence of magnetostatic field and (b) absence of both piezoelectricity and magnetostatic field are both equal in magnitude; i.e., piezoelectric contributions to three-wave parametric amplification are only in the presence of magnetostatic field. Moreover, (i) the electrostrictive property is a primary requirement for parametric amplification to occur; (ii) the parametric gain coefficient is independent of doping concentration for the cases when either/both piezoelectricity and magnetostatic field are absent.

3. The parametric gain coefficients become independent of presence of piezoelectricity and/or magnetostatic field and enhance by properly adjusting the acoustical wave number in dispersion-less regime. Around resonance (electron-cyclotron frequency ~ pump frequency), the parametric gain coefficient in the presence of piezoelectricity is \( 10^2 \) times higher than in the absence of piezoelectricity.

4. The parametric gain coefficient depends on doping concentration in the finiteness of both piezoelectricity and magnetostatic field. When either piezoelectricity or magnetostatic field or both is/are absent, the parametric gain coefficient becomes independent of doping concentration.

5. The technological potentiality of a transversely magnetized n-InSb crystal as the host for parametric devices, like parametric amplifiers and oscillators, is established.
the n-InSb crystal, parametric amplification and oscillation in the infrared regime appear quite promising under the resonance conditions and replace the conventional idea of using high-power pulsed lasers.

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